Multielement stable resonators containing a variable lens

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A unified formulation for the analysis of linear stable resonators containing a lens of variable focal length, which represents the rod of a solid-state laser, and other intracavity optical systems is presented. The stability, the mode spot sizes, the dynamical stability, and the misalignment sensitivity are investigated, and general properties that are valid for any resonator are derived. Some important practical consequences for resonator design are discussed.

1. INTRODUCTION

The thermal gradient that is necessarily established inside the rod of a solid-state laser to permit heat dissipation at the surface makes the rod a lens of considerable focal length (even shorter than 20 cm for input powers exceeding a few kilowatts). Since the dioptric power of the rod is proportional to the lamp input power, in addition to the gain of the active medium, all the resonator characteristics, the mode structure, and the sensitivity to misalignment depend on the pumping rate. Consequently, the analysis of such resonators is generally rather complicated and laborious, and the design is a demanding task, especially when a fundamental transverse mode with a large cross section that adequately fills the rod is required to extract the stored power (or energy) in a TEM_{00} beam.

Since the early days of solid-state lasers, great efforts have been made to design stable resonators that can counteract or compensate for the thermal focusing of the rod. At first, this compensation was achieved by means of a negative lens ground directly on the end faces of the rod or through a convex mirror of suitable curvature.[1–3] An important step forward was made with the introduction of dynamically stable resonators, which are designed to have the spot size in the rod stationary with respect to the rod focal-length variations.[4–6] The concept of dynamical stability has also been successfully applied to resonators with intracavity telescopes, which present the advantages of easy adjustment for different pump powers, of short resonator lengths, and of sufficiently large spot sizes on the mirrors.[7–9] In addition, a number of other different solutions for obtaining a large mode volume in the rod or for improving the divergence of the output beam have been proposed.[10,11]

Besides the dynamical stability, the mechanical stability, or the sensitivity to mirror misalignment of stable[12–15] and unstable[16–18] resonators, has been the subject of many papers, which, however, mostly concern peculiar resonator configurations. The effects of phase aberrations (both phase tilt and curvature) in unstable cavities have also been extensively studied to evaluate the aberration sensitivity and to determine the suitable adaptive-optics compensation by means of deformable mirrors.[19–21]

In recent papers[22,23] the sensitivity to mirror misalignment has been shown to be a key factor in the design of stable resonators for solid-state lasers, since dynamically stable resonators might still be unreliable because of the alignment difficulties.[22,23] A detailed analysis of a generic simple resonator with an intracavity focusing rod was performed, and some basic properties were demonstrated theoretically and experimentally.[24,25] As a result, a novel design procedure for optimized dynamically stable resonators has been devised and applied to a cw Nd:YAG laser, yielding a considerable improvement of mechanical stability and TEM_{00} output power, especially in the mode-locking regime.[25]

Now, a natural question is whether multielement stable resonators more complicated than telescopic ones, or perhaps containing many lenses, may further improve the performances of solid-state lasers. To solve this problem, a unified formulation that permits the analysis of complicated resonators containing a lens of variable focal length and an arbitrary number of other optical elements is necessary. The aim of this work is to provide such a formulation. The problem is approached with the background of Refs. 26–28, in which the resonator modes are treated as Gaussian beams and the optical elements are described by using their ray-transfer matrices. Basic properties regarding the resonator stability, the mode spot sizes on the mirrors and in the rod, and the misalignment sensitivity are derived. First, we consider the first-order effects of misalignment and analyze a few optical systems, which are of particular interest for resonators. Next, the misalignment sensitivity of a multielement stable resonator is examined, and equations to calculate the position of the mode axis are given. In the same section some general features of stable resonators are briefly reviewed and phrased in a manner closer to this context, and then linear resonators containing a focusing rod of variable focal length and other arbitrary optical systems are treated. It is shown that all the properties valid for a bare resonator containing only the variable lens can also be formulated for complicated resonators containing any optical systems.

2. FIRST-ORDER MISALIGNED OPTICAL SYSTEMS

In one dimension a misaligned system can be described either by an augmented 3×3 matrix instead of the 2×2 matrix appropriate for an aligned system or through a 2×1 vector in addition to the usual 2×2 matrix.[29–31] The differ-

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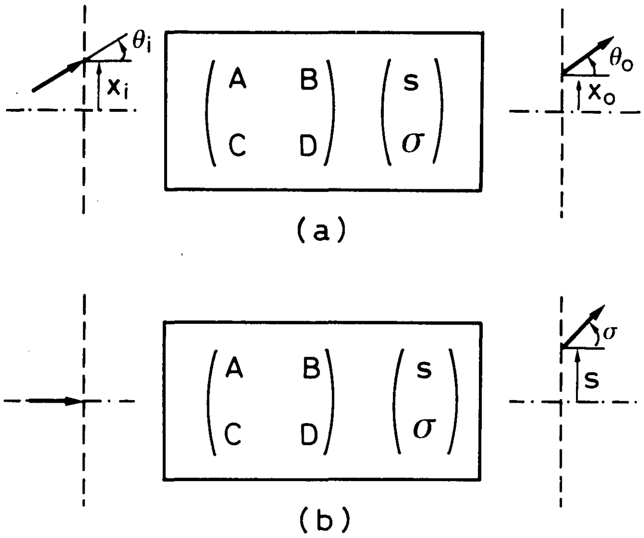


Fig. 1. Misaligned optical system. (a) The matrix and the vector relate the position and the slope of the rays at the output plane to those at the input plane. (b) Interpretation of the elements of the misalignment vector for an input ray coincident with the reference axis.

ence between the two methods is merely formal, and here we adopt the latter, which appears to be more suitable for our purposes.

Consider the generic optical system shown in Fig. 1(a), for which the position and the slope of the rays at the input and output planes are measured with reference to the optical axis of the aligned system. When some element of the system is misaligned, the position and the slope of the ray at the output plane, $\binom{x_0}{\theta_0}$, are related to the corresponding parameters at the input plane, $\binom{x_i}{\theta_i}$, by an equation that is linear, as for perfect alignment, but no longer homogeneous:

$$\begin{pmatrix} x_0 \\ \theta_0 \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} x_i \\ \theta_i \end{pmatrix} + \begin{pmatrix} s \\ \sigma \end{pmatrix}. \tag{1}$$

In this equation the matrix is the same pertaining to the aligned system, while the vector $\binom{s}{\sigma}$ describes the misalignment. The physical meaning of the elements of the misalignment vector becomes evident if one takes, as shown in Fig. 1(b), an input ray coincident with the reference axis of the system: for $x_i = 0$ and $\theta_i = 0$ the position and the slope of the output ray are $x_0 = s$ and $\theta_0 = \sigma$, respectively, whereas for the perfect alignment the output ray would still be superposed to the reference axis (the optical axis of the system). Because of linearity, the principle of superposition can be used to evaluate the effect of the misalignment of each simple element composing a more complicated system. Thus the misalignment vector will be the sum of the output ray vectors obtained by assuming an input ray coincident with the reference axis and taking one misaligned element at a time. In practice, s and σ result in linear combinations of tilting angles and displacements of the various decentered elements.

A few examples of ray-transfer matrices and misalignment vectors for simple systems are shown in Table 1. Note that, as is known, tilting of a thin lens does not produce first-order effects. The overall misalignment vector of a cascade of elementary subsystems can easily be obtained by means of Eq. (1), considering the output of each element as the input to the next one.

Let us consider now a generic system closed at one end by a spherical mirror so that the rays pass through the system in the two directions before emerging. Without lack of generality, the mirror can be considered planar and always perpendicular to the reference axis, as shown in Fig. 2(a): in fact, the mirror can be resolved into a lens and a plane reflector, and its power and misalignment can thus be attributed to the optics in front of it. If the transfer matrix $\mathbf{T}$ and the misalignment vector $\mathbf{m}$ for propagation from the reference plane to the mirror are

$$\mathbf{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{2a}$$

and

$$\mathbf{m} = \begin{pmatrix} s \\ \sigma \end{pmatrix}, \tag{2b}$$

then the matrix and the vector for opposite propagation are

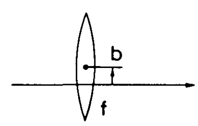
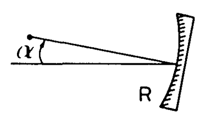
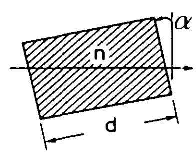
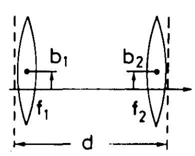
$$\begin{bmatrix} D & B \\ C & A \end{bmatrix}, \quad \begin{bmatrix} D & B \\ C & A \end{bmatrix} \begin{pmatrix} -s \\ \sigma \end{pmatrix}.$$

Thus, by multiplying, we obtain, for the overall matrix $\mathbf{T}_r$ and the vector $\mathbf{m}_r$ relative to the path from the reference plane to the mirror and back to the reference plane,

$$\mathbf{T}_r = \begin{bmatrix} 2AD - 1 & 2BD \\ 2AC & 2AD - 1 \end{bmatrix} \tag{3a}$$

and

Table 1. Ray-Transfer Matrices and Misalignment Vectors

	Optics	Matrix	Vector
a)		$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{b}{f} \end{pmatrix}$
b)		$\begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 2\alpha \end{pmatrix}$
c)		$\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \alpha d(1-\frac{1}{n}) \\ 0 \end{pmatrix}$
d)		$\begin{pmatrix} 1-\frac{1}{f_1} & d \\ -\frac{1}{f_1}-\frac{1}{f_2}+\frac{d}{f_1 f_2} & 1-\frac{d}{f_2} \end{pmatrix}$	$\begin{pmatrix} b_1\frac{d}{f_1} \\ \frac{b_1}{f_1}+\frac{b_2}{f_2} \end{pmatrix}$

a) Thin lens
b) Mirror
c) Dielectric block
d) Two lenses combination

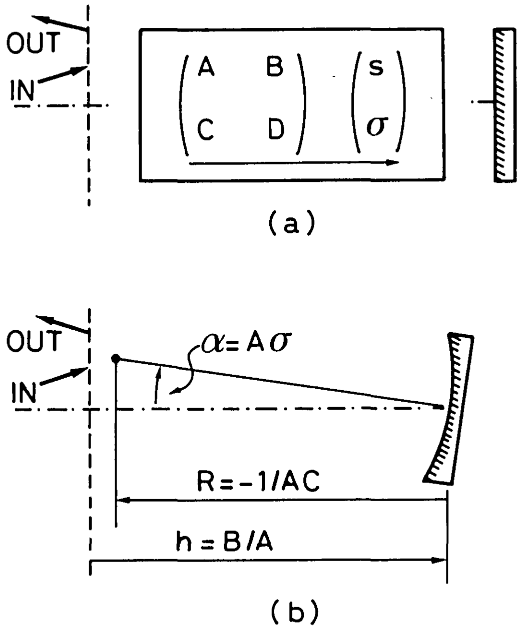


Fig. 2. Optical system closed at one side by a mirror. (a) Generic system: the transfer matrix and the misalignment vector are associated with the ray path from the entrance plane to the mirror and also include the possible mirror curvature and misalignment. (b) Equivalent system made by a misaligned spherical mirror.

$$\mathbf{m}_r = 2\sigma \begin{pmatrix} B \\ A \end{pmatrix}. \tag{3b}$$

It is remarkable that the diagonal elements of the matrix $\mathbf{T}_r$ are equal and that only the second element of the misalignment vector **m** produces an effect. It follows that a generic system ending at one side with a mirror [Fig. 2(a)] results in an equivalent spherical mirror of radius $R = -1/AC$ placed at a distance $h = B/A$ from the reference plane and tilted by an angle $\alpha = A\sigma$, as shown in Fig. 2(b), since both systems have the same round-trip transfer matrix. This equivalence is the formal reason that allows to extend most of the properties valid for simple resonators containing only a variable lens to more complicated resonators made by the variable lens sandwiched between two arbitrary optical systems. On the basis of this equivalence many general relationships may be obtained as an extension of the results of Ref. 22. However, here we shall derive the properties of these resonators by using the transfer matrices directly, since by using equivalent mirrors we could not calculate the spot size on the actual mirrors and, moreover, in certain conditions the distance h might become negative and we would consequently have to consider the puzzling case of an equivalent resonator with the variable lens outside the mirrors.

3. STABILITY AND MISALIGNMENT OF LINEAR RESONATORS

In this section we discuss, in terms of geometrical optics, first some particular characteristics of linear resonators at the edge between stability and instability and then the misalignment sensitivity, which presents a behavior closely related to those peculiarities. The results will also serve as a basis for analyzing a resonator containing a variable lens.

A generic linear resonator made by two spherical mirrors (M_1 and M_2) with an intracavity optical system (IOS) is shown in Fig. 3(a). It is convenient, however, to resolve the curved mirrors into the combination of a plane mirror and a thin lens. By using this procedure a generic resonator can be described, as shown schematically in Fig. 3(b), simply by a pair of plane mirrors (P_1 and P_2) and by the transfer matrix **T**, given in Eq. (2a), now representing the ray propagation from mirror P_1 to mirror P_2. The misalignment of the spherical mirrors, as well as that of any other cavity element, is included in the vector **m**, given in Eq. (2b), so that the plane mirrors can be considered as always perpendicular to the reference axis. The results obtained with the equivalent resonator of Fig. 3(b) will also be formulated for the actual resonator of Fig. 3(a) to make the application to real systems more immediate.

The resonator stability condition is[32] $0 < AD < 1$, which, since **T** is unitary, can also be expressed as

$$ABCD < 0. \tag{4}$$

Therefore the edge between stability and instability is simply determined by the vanishing of one of the elements of the matrix **T**. The spot sizes on the mirrors, w_1 and w_2, have the following expressions[32]:

$$w_1^4 = -\left(\frac{\lambda}{\pi}\right)^2 \frac{BD}{AC}, \tag{5}$$

$$w_2^4 = -\left(\frac{\lambda}{\pi}\right)^2 \frac{AB}{CD}, \tag{6}$$

where λ is the laser wavelength. Thus, at the stability edges, the spot on the mirrors is either null or infinite, and the Gaussian modes degenerate in spherical or plane waves. One can see that at the stability limits either the center of one mirror or its center of curvature is conjugated through the IOS to one of the corresponding points of the other mirror. Consider, for instance, the case $A = 0$. In the resonator shown in Fig. 3(b) for $A = 0$, mirror P_2 lies in a focal plane of the optical system represented by the matrix **T**

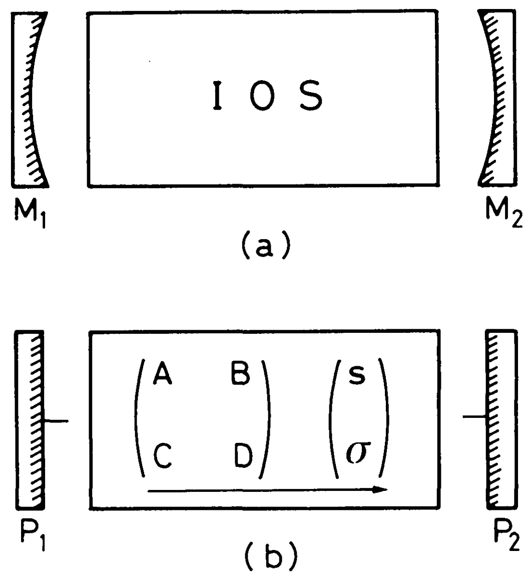


Fig. 3. (a) Linear resonator with curved mirrors (M_1 and M_2) and an IOS. (b) Description of the resonator by plane mirrors (P_1 and P_2) and by a transfer matrix and a misalignment vector representing the ray path from P_1 and P_2.

Table 2. Resonators at the Stability Limits

Stability Limit	Pair of Conjugated Points[a]		Spot Size on Mirrors	
	Mirror 1	Mirror 2	w_1	w_2
$A = 0$	c.c.	m.c.	∞	0
$D = 0$	m.c.	c.c.	0	∞
$B = 0$	m.c.	m.c.	0	0
$C = 0$	c.c.	c.c.	∞	∞

[a] c.c., Mirror center of curvature; m.c., mirror center.

(Ref. 28); consequently, a plane wave entering from the left is transformed into a spherical wave converging onto the plane of mirror P_2. Since these waves reproduce themselves after one round trip, they constitute the resonator mode. The mode in the actual resonator with curved mirrors [Fig. 3(a)] is obtained by attaching to the plane mirrors the appropriate lenses, which have been previously included in the matrix **T**: the mode on mirror M_1 becomes a spherical wave converging toward its center of curvature. Therefore, for $A = 0$, the center of curvature of mirror M_1 is imaged by the IOS onto the center of mirror M_2. The pairs of conjugated points through the IOS and the values of the spot sizes at each stability limit are listed in Table 2.

To treat the misalignment sensitivity, we consider first the resonator with plane mirrors shown in Fig. 3(b). The axis of the resonator modes is the ray that retraces itself after one round trip around the resonator. For a perfectly aligned resonator this ray obviously coincides with the optical axis of the system between mirrors and is perpendicular to the mirror surfaces. In case of misalignment the position and the slope of the axis can be calculated by means of an equation that represents the self-consistency of a ray after one round trip. The round-trip matrix and the misalignment vector for the path starting (and ending) at the plane of mirror P_1 are given by Eqs. (3); thus if we denote by $\binom{x_1}{\theta_1}$ the position and the slope of the axis at the surface of mirror P_1, the self-consistency equation reads as

$$\begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix} = \begin{bmatrix} 2AD-1 & 2BD \\ 2AC & 2AD-1 \end{bmatrix} \begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix} + 2\sigma \begin{pmatrix} B \\ A \end{pmatrix}. \tag{7}$$

The solution of this equation is

$$\begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix} = \begin{pmatrix} -\sigma/C \\ 0 \end{pmatrix}. \tag{8}$$

The fact that the slope θ_1 of the axis is zero confirms the circumstance that the axis is always normal to the mirror surface and also that there are no ray paths that close in one round trip except the mode axis. Such closed ray paths are possible only at the stability limit $B = 0$; if C also vanishes, the resonator is degenerate, and any ray retraces itself after one round trip.[33] Let us now go back to the resonator with curved mirrors in Fig. 3(a). One can easily see that the axis, or its prolongation, passes through the center of curvature of each mirror and hence through the images (real or virtual) of these points produced by the optics IOS. For $C = 0$ the position of the axis is no longer defined because x_1 tends to infinity. Since in this case the centers of curvature of the mirrors are conjugated through the IOS, as indicated in Table 1, even a small misalignment causes the axis to become perpendicular to the optical axis of the aligned system (reference axis). Since the focal length of any system is equal to $-1/C$, for $C = 0$ the optics between the plane mirrors is an afocal system, such as a telescope focused at infinity. Therefore concentric resonators, as well as resonators made by a focused telescope between two plane mirrors, are difficult to align and definitely unstable from a mechanical point of view.

4. RESONATORS WITH A THERMAL LENS

The rod of a solid-state laser, which has a parabolic radial refractive-index profile and a dioptric power proportional to the pump power, can be treated as a thin lens provided that the distances are measured with reference to the principal planes of the rod, whose positions are independent of the pump power.[22,34] Therefore a generic resonator of a solid-state laser can be modeled, as shown in Fig. 4, by two plane mirrors that enclose a lens of variable focal length f between two generic optical systems. The transfer matrices and the misalignment vectors shown in Fig. 4 are associated with the ray paths from the lens to the mirrors and also include the possible mirror curvature and misalignment. To make the equations of this section more readable, we define the following variables:

$$\eta = \frac{1}{f} - \frac{1}{2}\left(\frac{A_1}{B_1} + \frac{C_1}{D_1} + \frac{A_2}{B_2} + \frac{C_2}{D_2}\right), \tag{9}$$

$$u = \frac{1}{2B_1D_1} - \frac{1}{2B_2D_2}, \tag{10}$$

$$v = -\frac{1}{2B_1D_1} - \frac{1}{2B_2D_2}. \tag{11}$$

With this notation the transfer matrix **T** from mirror 1 to mirror 2 is

$$\mathbf{T} = -\begin{bmatrix} D_1B_2(\eta + u) & B_1B_2(\eta + v) \\ D_1D_2(\eta - v) & B_1D_2(\eta - u) \end{bmatrix}. \tag{12}$$

Note that the only variable that depends on f, i.e., on the pump power, is η and that the matrix elements are linear functions of $1/f$.

A. Stability and Spot Sizes

The stability limits as a function of η are obtained by equating to zero each of the elements of the matrix **T** and solving for η: by inspection of Eq. (12) we immediately obtain $\eta = \pm u$ and $\eta = \pm v$. The expressions in terms of lens dioptric power and the relevant vanishing matrix elements are listed in Table 3. After inserting the elements of the matrix **T** [Eq. (12)] into the stability condition [Eq. (7)], we conclude that

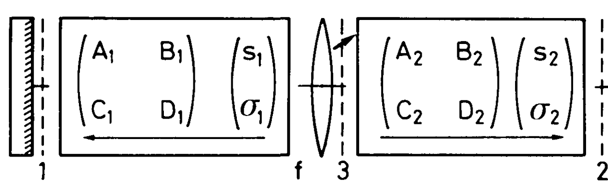


Fig. 4. Linear resonator with an internal lens of variable focal length f and other intracavity optical systems. The arrows indicate that the matrices and the vectors represent the paths from the lens to the mirrors. The dashed lines are reference planes.

Table 3. Stability Limits of a Resonator Containing a Variable Lens

Stability Limit	Value of η	Rod Dioptric Power	Stability Zone $uv > 0$	Stability Zone $uv < 0$
$A = 0$	$-u$	$\frac{C_1}{D_1} + \frac{A_2}{B_2}$	I	II
$D = 0$	u	$\frac{A_1}{B_1} + \frac{C_2}{D_2}$	II	I
$B = 0$	$-v$	$\frac{A_1}{B_1} + \frac{A_2}{B_2}$	I	I
$C = 0$	v	$\frac{C_1}{D_1} + \frac{C_2}{D_2}$	II	II

the resonator is stable if $|\eta|$ belongs to the interval $\overline{|u||v|}$. Thus, as a function of the rod dioptric power, there are always two stability zones that are symmetrically located about the zero of the η axis and that have the same width, given by

$$\Delta\eta = \Delta \frac{1}{f} = \min(|u + v|, |u - v|) \tag{13a}$$

$$= \min\left(\left|\frac{1}{B_1 D_1}\right|, \left|\frac{1}{B_2 D_2}\right|\right). \tag{13b}$$

It might be emphasized that the stability zones are crossed, in principle, simply by varying the input power to the pump lamp. For a reason related to the misalignment sensitivity, which is discussed below, we denote by zone I the stability interval limited at one of the extrema by $\eta = -v$ ($B = 0$) and by zone II that interval limited by $\eta = v$ ($C = 0$). The second stability limit ($+u$ or $-u$) can be determined immediately by noting that the stability intervals are symmetric with respect to $\eta = 0$, and thus in each zone η has a constant sign. The zone corresponding to each limit is also indicated in Table 3 for both $uv > 0$ and $uv < 0$.

The spot size w_3 of the TEM_{00} mode on the lens, calculated by assuming that a Gaussian beam reproduces itself after one round trip, is given by

$$w_3^4 = -\left(\frac{2\lambda}{\pi}\right)^2 \frac{\eta^2}{(\eta^2 - u^2)(\eta^2 - v^2)}. \tag{14}$$

This equation makes it apparent that the spot size goes to infinity at the stability limits and hence reaches a minimum in each stability zone. In correspondence to the minimum of w_3 the resonator is dynamically stable since the spot size in the rod is, at the first order, insensitive to variations of the rod focal length. The condition for the dynamical stability can be obtained by solving for η the equation

$$\frac{\mathrm{d}w_3^4}{\mathrm{d}\eta^2} = 0, \tag{15}$$

which gives

$$\eta = \pm(|uv|)^{1/2}. \tag{16}$$

The value w_{30} of the spot size w_3 for both these values of η is

$$w_{30}^2 = \frac{2\lambda}{\pi} \frac{1}{\Delta \frac{1}{f}}, \tag{17}$$

where $\Delta(1/f)$ is given by Eqs. (13). The above relationship indicates that, independently of the resonator configuration, the volume of the TEM_{00} mode in the rod at the dynamical stability is inversely proportional to the range of rod dioptric power for which the resonator is stable. The proportionality coefficient depends only on the laser wavelength. A more practical formulation of this property is obtained by expressing the rod focal length as a function of the pump power. If the heat is uniformly generated within the rod, its focal length is given by[34]

$$\frac{1}{f} = \frac{k}{\pi r^2} P_{\mathrm{in}}, \tag{18}$$

where P_{in} is the input power to the lamp, r is the rod radius, and k is a coefficient that depends on the physical properties of the laser material and on the pumping efficiency but is generally independent of the rod length. Combining Eq. (18) with Eq. (17) yields

$$\Delta P_{\mathrm{in}} = \frac{2\lambda}{k} \left(\frac{r}{w_{30}}\right)^2. \tag{19}$$

For single-transverse-mode operation, on the assumption that the rod represents the limiting aperture of the resonator, the ratio r/w_{30} ranges at most from 1.2 to 2.[4,6,15,25] Therefore the pump power stability range for dynamically stable TEM_{00} lasers is a characteristic of the laser material that does not depend on the resonator configuration. As an example, for the Nd:YAG laser, using a typical dependence reported in Refs. 34 and 35 of 0.5 D/kW for a rod 6.3 mm in diameter and assuming that $r/w_{30} = 1.5$, we calculate $\Delta P_{\mathrm{in}} \cong$ 300 W. The stability range might be extended by increasing the ratio r/w_{30}, which implies either multimode operation or an inefficient utilization of the rod volume. This makes it evident that no optical compensation of the resonator, no matter how it is done, can effectively increase the stability range of a dynamically stable TEM_{00} solid-state laser. The solution should be based on the use of athermal rod materials or on the reduction of the unused radiation absorbed by the rod. It should be noted, however, that since the relative stability range $\Delta P_{\mathrm{in}}/P_{\mathrm{in}}$ decreases as the average pump power increases, the limited stability range may become a problem mainly for cw or high-repetition-rate lasers. For a given w_{30} the stability range is wider by a factor of 2 when the two zones join, because $u = 0$ or $v = 0$. The resonator in both cases is dynamically stable for $\eta = 0$, exactly in the center of the overall stability zone. However, as shown in the next paragraph, for $u = 0$ the misalignment sensitivity diverges at one edge of the overall stability zone and the resonator can still be used; on the contrary, for $v = 0$ the divergence occurs exactly at the center of the stability zone in correspondence to the dynamical stability, which prevents any practical utilization of such resonator configurations.

Another important parameter that must be taken into account in the resonator design is the spot size on the mirrors because of possible damage at high intensity. By using Eqs. (12), (5), and (6), these spot sizes can be expressed as

$$w_1^4 = -\left(\frac{\lambda B_1}{\pi D_1}\right)^2 \frac{(\eta - u)(\eta + v)}{(\eta + u)(\eta - v)}, \tag{20}$$

$$w_2{}^4 = -\left(\frac{\lambda B_2}{\pi D_2}\right)^2 \frac{(\eta + u)(\eta + v)}{(\eta - u)(\eta - v)}. \tag{21}$$

To proceed further, we can assume, without loss of generality, that $uv > 0$, which is equivalent to $|B_1D_1| > |B_2D_2|$; in the opposite case, the results will be obtained simply by interchanging the indices 1 and 2. The values of the spot sizes at the stability limits are given in Table 2. With the help of Table 3 it can be seen that the TEM_{00} beam is brought to a focus on mirror 2 at both edges of zone I, whereas it diverges at both limits of zone II. On the other hand, the spot size on mirror 1 in both zones is zero at one extremum and infinite at the other one. Thus, as a function of the rod dioptric power, the derivative of w_1 is always different from zero, whereas w_2 presents a maximum in zone I and a minimum in zone II, in correspondence to the same rod focal lengths for which the resonator is dynamically stable. It should be noted that this is a general feature of zones I and II, with respect to either mirror 1 or mirror 2, depending on the sign of uv. At the point of dynamical stability, i.e., for η given by Eq. (16), the spot sizes w_{10} on mirror 1 are identical in both zones and are given by

$$w_{10}{}^2 = \frac{\lambda}{\pi}\left|\frac{B_1}{D_1}\right|. \tag{22}$$

Those on mirror 2, w_{20}, are

$$w_{20}{}^2 = \frac{\lambda}{\pi}\frac{1}{D_2{}^2}\left[|B_1D_1| \mp \sqrt{(B_1D_1)^2 - (B_2D_2)^2}\right], \tag{23}$$

where the minus before the radical must be used for zone I (maximum of w_2) and the plus for zone II (minimum of w_2). The qualitative behavior of the TEM_{00} mode spot size on the mirrors and in the rod are shown in Fig. 5 as function of η, assuming that $|u| < |v|$. The picture corresponding to $|u| > |v|$ can easily be obtained on the basis of the previous discussion with the help of Table 3. Figure 5 shows that inside the stability zone the spot sizes have a quite smooth dependence on η, whereas they rapidly diverge or drop to zero as η approaches the stability limits. Therefore the values of the spot sizes at the dynamical stability can be considered representative of the mode dimension in the whole stability zone; thus Eqs. (17), (22), and (23) can also be conveniently used for approximate evaluations also when the resonator is not dynamically stable.

B. Misalignment Sensitivity

The misalignment sensitivity of resonators with a variable lens can be investigated by applying the results obtained in the previous section. In particular, Eq. (8), which gives the position of the mode axis on mirror 1, indicates that at one edge of zone II, where $C = 0$, i.e., $\eta = v$, an arbitrary small misalignment makes the mode axis rotate by 90 deg. However, to go a little more deeply on this subject it is convenient to examine the axis position on the plane of the lens rather than on the mirrors. In fact, the additional power losses introduced by misalignment of some component of the resonator essentially arise from the displacement of the mode axis, and of the field pattern, on the plane of the limiting (mode-selecting) aperture, which in solid-state laser having a mode that fully utilizes the active material is generally constituted by the rod cross section. The position and the slope $\binom{x_3}{\theta_3}$ of the mode axis in the plane of the lens (marked by 3 in Fig. 4) can be obtained from Eq. (8), making the ray propagate from the mirror to the lens. Using the notation of Fig. 4, the result can be expressed as

$$\begin{pmatrix} x_3 \\ \theta_3 \end{pmatrix} = -\frac{1}{C}\begin{bmatrix} D_2\sigma_1 + D_1\sigma_2 \\ -C_2\sigma_1 + (C_1 - D_1/f)\sigma_2 \end{bmatrix}, \tag{24}$$

where $C = -D_1D_2(\eta - v)$ is the element 2, 1 of the matrix **T** given in Eq. (12). The detailed expression of the position of the mode axis as a function of tilting and displacement of each decentered element can obviously be calculated only

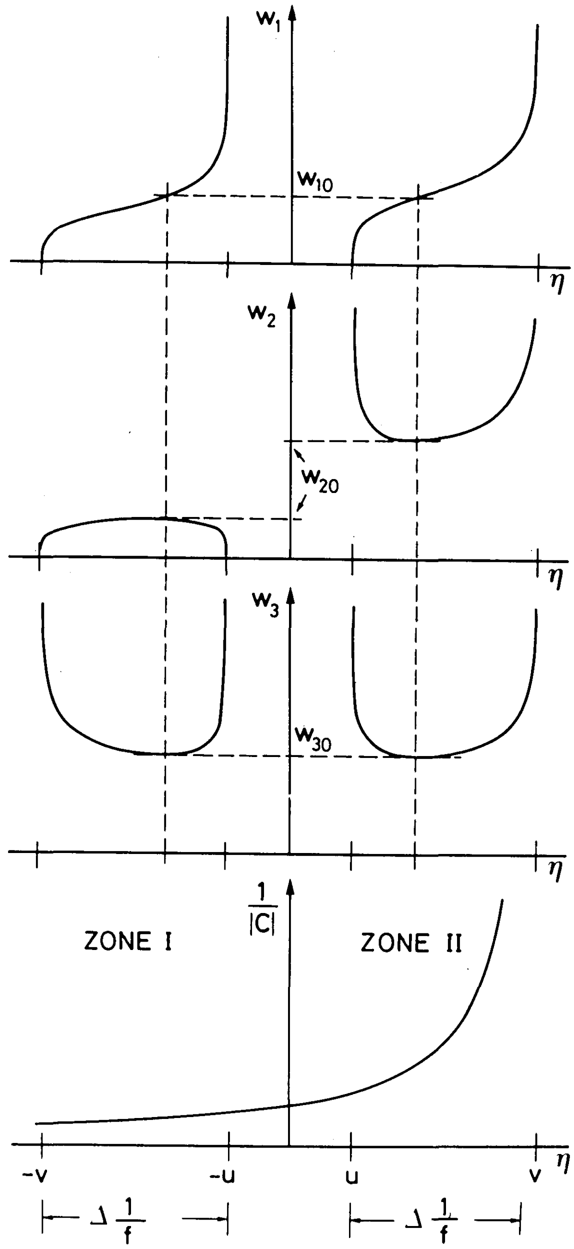


Fig. 5. Spot sizes and misalignment sensitivity of a linear resonator with an internal variable lens for $uv > 0$ and $|u| < |v|$ as a function of η (dioptric power of the lens shifted by a constant amount). (a) Spot size on mirror 1. (b) Spot size on mirror 2. (c) Spot size on the lens. (d) Absolute value of the focal length of the optics between mirrors (including the mirrors' power), which determines the misalignment sensitivity. The dashed vertical lines correspond to the dynamical stability.

when a particular resonator configuration is specified. However, it can easily be shown that the effects of the mirror misalignment are given directly by Eq. (24) by substituting the tilt angles of mirror 1 and 2 for σ_1 and σ_2, respectively. It is obvious that, whichever element is misaligned, the dependence of the axis displacement on the rod focal length is always contained only in the denominator C. The behavior, as a function of η, of $|1/C|$, which is the absolute value of the focal length of the optics between mirrors (mirrors' power included), is also shown in Fig. 5. From this figure it is apparent that zone II is more sensitive than zone I to misalignment, and the presence of the asymptote might be troublesome, especially when the stability range is small, as, for instance, in high-power cw lasers. Note that if $1/f$ is equal to zero, Eq. (24) is still valid and permits us to calculate the mode axis in a generic plane of a resonator, provided that the appropriate matrix elements are used. Moreover, since Eq. (24) has been derived on the basis of geometrical optics, the results are also valid for unstable resonators.

The previous discussion has made it clear that high mechanical stability generally requires operation of the resonator in zone I. In this condition, however, the mode is focused on one of the mirrors at both stability limits, and the spot size may be very small inside the zone. Thus high mechanical stability seems to be unavoidably connected with small spot sizes on the mirrors. At this point it is worthwhile to describe another interesting result. Let the optical system between mirror 1 and the lens (see Fig. 4) be replaced by another system that has the same transfer matrix for the path from the lens to the mirror and back to the lens. The two systems are equivalent in the sense stated in Section 2 since they correspond to the same spherical mirror, and the substitution leaves the overall round-trip matrix around the resonator unchanged. It can be shown that in this case the sensitivity to misalignment of mirror 1 in the two resonators changes proportionally to the spot size on mirror 1. More precisely, the ratio $(x_3/\alpha_1)/w_1$, where α_1 is the tilt angle of mirror 1 that causes the mode axis on the lens to move by x_3, is the same for both resonators.

5. CONCLUSIONS

A general formulation to analyze multielement stable resonators for solid-state lasers containing a rod of variable focal length and other optical systems has been presented. Linear solid-state laser resonators, in which the rod acts as a variable lens, are stable when the rod dioptric power is inside two intervals of the same width, called stability zones, whose limits are simply related to the matrix elements of the intracavity optics. Within each zone there is a point of dynamical stability, where the spot size on the rod is insensitive, to the first order, to variations of the rod focal length. At such a point the cross-sectional area of the TEM_{00} mode in the rod is inversely proportional to the width of the stability zones. Practically, this has two important consequences: (1) in dynamically stable lasers the volume of the portion of the rod filled by the TEM_{00} mode is inversely proportional, through a coefficient independent of the resonator configuration, to the range of input power for which the resonator is stable; and (2) if the TEM_{00} mode fills the rod entirely, the input power stability range depends only on the physical properties of the rod material and on the pumping efficiency. As for the misalignment, the resonator in one of the two stability zones (zone I) is in general much less sensitive than in the other one (zone II), where the misalignment sensitivity may diverge.

Further investigations are in progress, in particular on ring resonators, that present substantially the same characteristics but only one stability zone of wider extension and some other interesting peculiarities.

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REFERENCES

1. C. M. Stickley, "Laser brightness gain and mode control by compensation for thermal distortion," IEEE J. Quantum Electron. **QE-2**, 511–518 (1966).
2. L. M. Osterink and L. D. Foster, "Thermal effects and transverse mode control in a Nd:YAG laser," Appl. Phys. Lett. **12**, 128–131 (1968).
3. F. A. Levine, "TEM_{00} enhancement in cw Nd:YAG by thermal lensing compensation," IEEE J. Quantum Electron. **QE-7**, 170–172 (1971).
4. R. B. Chesler and D. Maydan, "Convex-concave resonators for TEM_{00} operation of solid-state ion lasers," J. Appl. Phys. **43**, 2254–2257 (1972).
5. J. Steffen, J. P. Lörtscher, and G. Herziger, "Fundamental mode radiation with solid-state lasers," IEEE J. Quantum Electron. **QE-8**, 239–245 (1972).
6. J. P. Lörtscher, J. Steffen, and G. Herziger, "Dynamic stable resonators: a design procedure," Opt. Quantum Electron. **7**, 505–514 (1975).
7. P. H. Sarkies, "A stable YAG resonator yielding a beam of very low divergence and high output energy," Opt. Commun. **31**, 189–192 (1979).
8. D. C. Hanna, C. G. Sawyers, and M. A. Yuratich, "Telescopic resonators for large-volume TEM_{00}-mode operation," Opt. Quantum Electron. **13**, 493–507 (1981).
9. A. J. Berry, D. C. Hanna, and C. G. Sawyers, "High power, single frequency operation of a Q-switched TEM_{00} mode Nd:YAG laser," Opt. Commun. **40**, 54–58 (1981).
10. R. Iffländer, H. P. Kortz, and H. Weber, "Beam divergence and refractive power of directly coated solid-state lasers," Opt. Commun. **29**, 223–226 (1979).
11. H. P. Kortz, R. Iffländer, and H. Weber, "Stability and beam divergence of multimode lasers with internal variable lenses," Appl. Opt. **20**, 4124–4134 (1981).
12. A. G. Fox and T. Li, "Modes in a maser interferometer with curved and tilted mirrors," Proc. IEEE **51**, 80–89 (1963).
13. R. L. Sanderson and W. Streifer, "Laser resonators with tilted reflectors," Appl. Opt. **8**, 2241–2248 (1969).
14. J. A. Arnaud, "Degenerate optical cavities. II: Effect of misalignment," Appl. Opt. **8**, 1909–1917 (1969).
15. R. Hauck, H. P. Kortz, and H. Weber, "Misalignment sensitivity of optical resonators," Appl. Opt. **19**, 598–601 (1980).
16. M. J. Konopnicki and M. E. Smithers, "Unstable resonator with multiple outputs," Appl. Opt. **22**, 947–951 (1983).
17. E. Sklar, "The advantages of a negative branch unstable resonator for use with free-electron lasers," IEEE J. Quantum Electron. **QE-22**, 1088–1094 (1986).
18. W. F. Krupke and W. R. Sooy, "Properties of an unstable confocal resonator CO_2 laser system," IEEE J. Quantum Electron. **QE-5**, 575–586 (1969).
19. K. E. Oughstun, "Intracavity adaptive optic compensation of

phase aberrations. I: Analysis," J. Opt. Soc. Am. 71, 862–872 (1981).

20. K. E. Oughstun, "Intracavity compensation of quadratic phase aberrations," J. Opt. Soc. Am. 72, 1529–1537 (1982).
21. K. E. Oughstun, "Aberration sensitivity of unstable-cavity geometries," J. Opt. Soc. Am. A 3, 1113–1141 (1986).
22. V. Magni, "Resonators for solid-state lasers with large-volume fundamental mode and high alignment stability," Appl. Opt. 25, 107–117 (1986).
23. S. De Silvestri, P. Laporta, and V. Magni, "Misalignment sensitivity of solid-state laser resonators with thermal lensing," Opt. Commun. 59, 43–48 (1986).
24. S. De Silvestri, P. Laporta, and V. Magni, "Novel stability diagrams for continuous wave solid-state lasers," Opt. Lett. 11, 513–515 (1986).
25. S. De Silvestri, P. Laporta, and V. Magni, "14-W continuous-wave mode-locked Nd:YAG laser," Opt. Lett. 11, 785–787 (1986).
26. H. Kogelnik, "Imaging of optical modes—resonators with internal lenses," Bell Syst. Tech. J. 44, 455–494 (1965).
27. H. Kogelnik and T. Li, "Laser beams and resonators," Appl. Opt. 5, 1550–1567 (1965).
28. K. Halbach, "Matrix representation of Gaussian optics," Am. J. Phys. 32, 90–108 (1964).
29. A. Gerrard and J. M. Burch, *Introduction to Matrix Methods in Optics* (Wiley, London, 1975), pp. 106–108, 286–291.
30. J. A. Arnaud, *Beam and Fiber Optics* (Academic, New York, 1976), Chap. 4.
31. M. Nazarathy, A. Hardy, and J. Shamir, "Misaligned first-order optics: canonical operator theory," J. Opt. Soc. Am. A 3, 1360–1369 (1986).
32. P. Baues, "Huygens' principle in inhomogeneous, isotropic media and a general integral equation applicable to optical resonators," Opto-Electronics 1, 37–44 (1969).
33. J. A. Arnaud, "Degenerate optical cavities," Appl. Opt. 8, 189–195 (1969).
34. W. Koechner, "Thermal lensing in a Nd:YAG laser rod," Appl. Opt. 9, 2548–2553 (1970).
35. W. Koechner, *Solid-State Laser Engineering* (Springer-Verlag, New York, 1976), p. 355.