

Spatial entanglement and optimal single-mode coupling

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The challenge of optimizing the emission into single spatial modes of photons from spontaneous parametric down-conversion is addressed from the perspective of spatial entanglement. It is shown that single-mode coupling is most efficient in the absence of entanglement. Evidence of the relationship between spatial entanglement and pump focusing is revealed through experimental results, and numerical simulations show that spatial entanglement and single-mode coupling are optimized under nearly identical pump parameters.

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I. INTRODUCTION

Photon pairs produced in the process of spontaneous parametric down-conversion (SPDC) have been used extensively for fundamental studies of quantum mechanics and figure prominently in quantum technologies such as quantum key distribution, quantum metrology, and quantum computing. From the outset, a significant challenge for experimentalists has been the efficient collection of SPDC photons. The difficulty arises from the fact that, in general, photons are emitted into a multitude of spatial modes, only a fraction of which can be coupled to the collection optics. The number of spatial modes can be greatly reduced by causing SPDC to occur in a waveguide; however, bulk SPDC sources are still widely used.

A number of works in recent years have addressed the challenge of single-mode coupling by taking the very pragmatic approach of determining the values of various parameters that maximize the number of photons coupled into a single-mode system (meaning a single mode for the signal and a single mode for the idler, not necessarily the same mode) [1–6]. The general result is that focusing the pump improves the coupling efficiency, although in certain circumstances, a focused pump can lead to spectral and spatial asymmetries [7–9]. Here, we take a more fundamental approach and show that the conditions which maximize the single-mode coupling efficiency and the conditions which minimize the amount of spatial entanglement are nearly the same. We support this claim in three stages: first, with a brief theoretical treatment establishing a direct link between the amount of spatial entanglement and the fraction of emitted photons that can be coupled into a single-mode system; second, with experimental evidence showing that the conditions that lead to better single-mode coupling—namely, a focused pump—also lead to reduced spatial entanglement; and third, with numerical simulations showing that optimal

coupling is achieved with nearly the same pump focusing that yields the lowest amount of spatial entanglement.

II. SPATIAL ENTANGLEMENT AND SINGLE-MODE COUPLING

The notion of optimizing single-mode emission by looking to spatial entanglement can be motivated by the following simple example. Suppose SPDC photons are to be coupled into the two spatial modes $|\beta\rangle$ and $|\beta'\rangle$. It is clear that coupling is optimized by a source emitting the two-photon state $|\Psi\rangle = |\beta\rangle|\beta'\rangle$. While this statement may at first seem trivial, it should be noted that $|\Psi\rangle$ is a pure product state, and therefore, the two photons are not spatially entangled. This suggests that the overlap between the two-photon state and the collection modes may be limited by the amount of spatial entanglement.

Firmer footing can be found by expressing the two-photon state in terms of its Schmidt modes. Any pure bipartite state can be uniquely written as a “diagonal” superposition of mode pairs, $|\Psi\rangle = \sum_j \sqrt{p_j} |u_j\rangle_s |u'_j\rangle_i$, where s and i denote signal and idler, respectively, and p_j is the probability of emitting a pair of photons into the modes $|u_j\rangle_s, |u'_j\rangle_i$. The total probability of emitting a photon pair is $\langle\Psi|\Psi\rangle = \sum_j p_j \equiv p_{\text{emit}}$. In writing such expressions, we implicitly consider only the spatial degree of freedom and we assume that $|\Psi\rangle$ is a pure state (a similar argument can be made using a statistical mixture of states). The probability that the signal photon couples into some spatial mode $|\beta\rangle$ is

$$\begin{aligned}
 p_{|\beta\rangle} &\equiv |\langle\beta|\Psi\rangle|^2 \\
 &= \left| \sum_j \sqrt{p_j} \langle\beta|u_j\rangle_s |u'_j\rangle_i \right|^2 \\
 &= \sum_j p_j |\langle\beta|u_j\rangle_s|^2 \\
 &\leq p_{\text{max}} \sum_j |\langle\beta|u_j\rangle_s|^2 = p_{\text{max}}, \quad (1)
 \end{aligned}$$

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where p_{\max} is the largest Schmidt probability, and where we have made use of the fact that $\{|u_j\rangle_s\}$ and $\{|u'_j\rangle_i\}$ form complete bases for the signal and idler photons, respectively.

The inequality in Eq. (1) shows that the probability of coupling even one of the photons to a single spatial mode $|\beta\rangle$ can be no larger than that of the most probable Schmidt mode. The right side of the inequality attains a value of unity only in the case of a single Schmidt mode, that is, when there is no spatial entanglement. On the other hand, if the two-photon state has a high degree of spatial entanglement, emission is distributed across many pairs of modes, so that p_{\max} is small. A simple upper bound on p_{\max} can be obtained in terms of the effective number of modes

$$N \equiv \frac{(\sum_j p_j)^2}{\sum_j p_j^2}. \quad (2)$$

Since $N \leq (\sum_j p_j)^2 / p_{\max}^2$, the coupling efficiency is

$$\frac{p_{|\beta\rangle}}{p_{\text{emit}}} \leq \frac{1}{\sqrt{N}}. \quad (3)$$

Here the term ‘‘coupling efficiency’’ means the fraction of emitted photons that are collected; other authors have used coupling efficiency to mean the fraction of collected photons whose partners are also collected.

The expression in Eq. (3) shows that there is a direct link between the coupling efficiency and the amount of spatial entanglement: a high degree of spatial entanglement ($N \gg 1$) imposes a low coupling efficiency and a low degree of spatial entanglement ($N \sim 1$) admits a high coupling efficiency. Note that to realize high coupling efficiency in the latter case, it is furthermore necessary that the collection modes match the dominant Schmidt modes. In the next section we show how the pump focus can be used to adjust N without significantly changing the total emission probability. In this case, relation (3) amounts to a relationship between source brightness ($p_{|\beta\rangle}$) and spatial entanglement.

III. SPATIAL ENTANGLEMENT AND PUMP FOCUS

The setup depicted in Fig. 1 was used to establish a relationship between pump focusing and spatial entanglement. A 10-mm periodically poled potassium titanyl phosphate (PPKTP) crystal poled for collinear type II emission was pumped by the frequency doubled output of a Ti:sapphire laser configured for mode-locked operation. Pulse durations were 1–3 ps and the center wavelength was 785 nm. A polarization beamsplitter was used to separate the signal and idler photons, which were then directed to two fiber-coupled single-photon detectors. Large coupling lenses and multimode fibers were used in order to collect light from a very large range of emission angles. Narrow slits on translation stages were placed in front of each collection lens to limit the collected light to ~ 1.5 mrad. A 1-nm bandpass filter (not shown) was placed in one arm to restrict the experiment to the spatial domain.

By measuring the coincidence rate as the slit positions are varied, it is possible to observe correlations in the photons’ emission directions, or transverse momenta. If the two-photon state is free of spatial entanglement, then the emission direction of, say, the signal photon will be completely independent

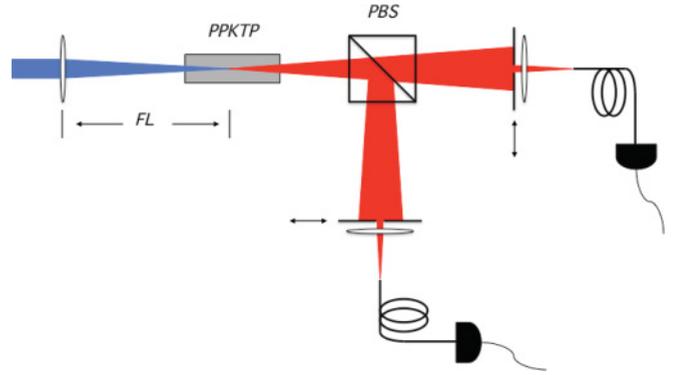


FIG. 1. (Color online) Experimental setup. The pump lens, when present, is placed so as to focus the pump beam at the center of the crystal. The signal and idler photons are split by the polarization beamsplitter (PBS) and sent to two large-area detectors (multimode fiber and coupling lens). Movable slits are placed in front of the lenses.

of the direction of the idler. On the other hand, a high degree of spatial entanglement will be manifested as a strong correlation between emission directions. By mapping the emission patterns for several different pump configurations, that is, with several different lenses in the pump beam, the influence of pump focusing on spatial entanglement is revealed.

The results of these scans are shown in Fig. 2, along with insets showing the magnitudes of the first several coefficients of Schmidt decompositions of the coincidence distributions (the corresponding Schmidt modes are different for the three scans and are not shown). For a collimated pump (no focusing lens), the signal and idler photons are highly correlated in their emission directions. That is, coincidence counts are observed only for specific combinations of slit positions. As shown in the inset, the emission is distributed across many modes. This correlation is less pronounced when the pump is focused, as seen in the middle and lower panels. For the strongest focusing used, the emission directions are essentially uncorrelated and only the first Schmidt coefficient has appreciable amplitude. As discussed previously, this indicates that the photons are unentangled in the transverse momentum.¹

The experiment illustrates how the properties of the pump influence the transverse momentum correlations between the signal and idler photons. These correlations are determined not only by the pump, but also by the phase-matching properties of the SPDC medium. The most general expression for the two-photon state is

$$|\Psi\rangle = \int d^2\mathbf{k}_s d^2\mathbf{k}_i \alpha(\mathbf{k}_s + \mathbf{k}_i) \phi(\mathbf{k}_s, \mathbf{k}_i) \hat{a}_{\mathbf{k}_s}^\dagger \hat{a}_{\mathbf{k}_i}^\dagger |0\rangle, \quad (4)$$

where $\hat{a}_{\mathbf{k}_\mu}^\dagger$ is the raising operator for a photon with transverse wave vector \mathbf{k}_μ . The probability amplitude is a product of a phase-matching function, $\phi(\mathbf{k}_s, \mathbf{k}_i)$, and a pump function $\alpha(\mathbf{k}_s + \mathbf{k}_i)$. The pump function describes the pump field and represents the range of $\mathbf{k}_p = \mathbf{k}_s + \mathbf{k}_i$ available for down-conversion, while the phase-matching function represents all

¹In principle there could be entanglement in the phase distribution, but that is not expected in our setup.

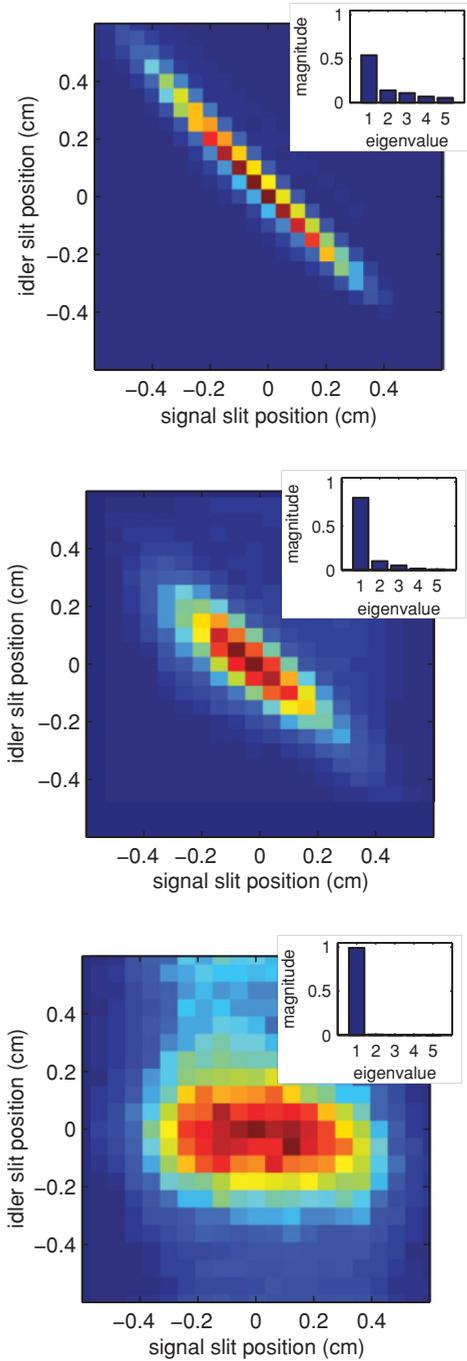


FIG. 2. (Color online) Joint spatial distribution under various degrees of pump focus. (upper) collimated pump; (middle) focused with a 200-mm lens; (lower) focused with a 50-mm lens. The insets show the amplitudes of the first several Schmidt coefficients for the coincidence distributions.

of the ways that the \mathbf{k}_p can be distributed to the signal and idler photons. The phase-matching function takes the form

$$\phi(\mathbf{k}_s, \mathbf{k}_i) \propto \frac{\sin[\Delta k_z L/2]}{\Delta k_z L/2}, \quad (5)$$

where L is the length of the crystal along the z direction and $\Delta k_z = k_{pz} - k_{sz} - k_{iz}$ is the wave mismatch.

Figure 3 shows calculated density plots of $|\phi(\mathbf{k}_{sy}, \mathbf{k}_{iy})|^2$ and $|\alpha(\mathbf{k}_{sy} + \mathbf{k}_{iy})|^2$ for the experimental conditions described in previous sections, where, for simplicity, attention is restricted to just one transverse component. The orientation of the phase-matching function suggests a positive correlation between the transverse momenta of the signal and idler. The reason for this is revealed through Taylor expansions of the longitudinal components of the wave vector:

$$k_z \simeq k_z^{(0)} + \frac{\partial k_z}{\partial k_y} \delta k_y + \frac{\partial^2 k_z}{\partial k_y^2} \delta k_y^2, \quad (6)$$

where δk_y is the displacement of the y component of the wave vector relative to the nominal direction. If the interaction is phase-matched in the nominal direction, then $k_{pz}^{(0)} - k_{sz}^{(0)} - k_{iz}^{(0)} = 0$. The wave mismatch is further simplified in the absence of spatial walkoff, that is, when $\partial k_z / \partial k_y = 0$. In this case, the phase mismatch becomes

$$\Delta k_z = \frac{1}{2} \left(\frac{\partial^2 k_{pz}}{\partial k_{py}^2} (\delta k_{sy} + \delta k_{iy})^2 - \frac{\partial^2 k_{sz}}{\partial k_{sy}^2} \delta k_{sy}^2 - \frac{\partial^2 k_{iz}}{\partial k_{iy}^2} \delta k_{iy}^2 \right). \quad (7)$$

For most optical materials, the geometric contribution to $\partial^2 k_z / \partial k_y^2$ is much larger than the material contribution; and, for small angles, $\partial^2 k_z / \partial k_y^2 \simeq -1/k_z$. This gives

$$\begin{aligned} \Delta k_z &= \frac{1}{2} \left(-\frac{(\delta k_{sy} + \delta k_{iy})^2}{k_{pz}} + \frac{\delta k_{sy}^2}{k_{sz}} + \frac{\delta k_{iy}^2}{k_{iz}} \right) \\ &= \frac{1}{2} \frac{1}{k_{pz}} (\delta k_{sy} - \delta k_{iy})^2, \end{aligned} \quad (8)$$

where we have used the fact that $k_{pz} \simeq 2k_{sz} \simeq 2k_{iz}$ for degenerate SPDC in most materials. This relationship results in positively correlated transverse momenta ($\delta k_{sy} = \delta k_{iy}$), since the phase-matching function has appreciable value only when Δk_L is near zero. The pump function, on the other hand, tends to yield negatively correlated momenta ($\delta k_{sy} = -\delta k_{iy}$). Depending on the relative widths of the pump and phase-matching functions, their product, or the joint transverse momentum distribution, can exhibit correlations that are either positive or negative: positive when $|\phi(\mathbf{k}_{sy}, \mathbf{k}_{iy})|^2$ is more narrow than $|\alpha(\mathbf{k}_{sy} + \mathbf{k}_{iy})|^2$, and negative when $|\phi(\mathbf{k}_{sy}, \mathbf{k}_{iy})|^2$ is more narrow than $|\alpha(\mathbf{k}_{sy} + \mathbf{k}_{iy})|^2$. It follows that the momentum correlation approaches zero when the widths of the two functions are similar. In this case, the two-photon state has a low degree of spatial entanglement. This is accomplished in our experiment by increasing the transverse momentum bandwidth of the pump, that is, by focusing the pump.

For the noncritically phase-matched PPKTP under consideration here, the pump and phase-matching functions happen to be oriented orthogonally in the $\mathbf{k}_s/\mathbf{k}_i$ space. This means that spatial entanglement will be minimized when the two functions have similar widths. However, it is not the case in general that the phase-matching function should have this orientation. In particular, the presence of spatial walkoff will lead to a different ‘‘slope’’ in the $\mathbf{k}_s/\mathbf{k}_i$ space. Nevertheless, it should be possible to eliminate nearly all of the spatial entanglement as long as the phase-matching function is not oriented so as to produce photons with negatively correlated momenta. If this

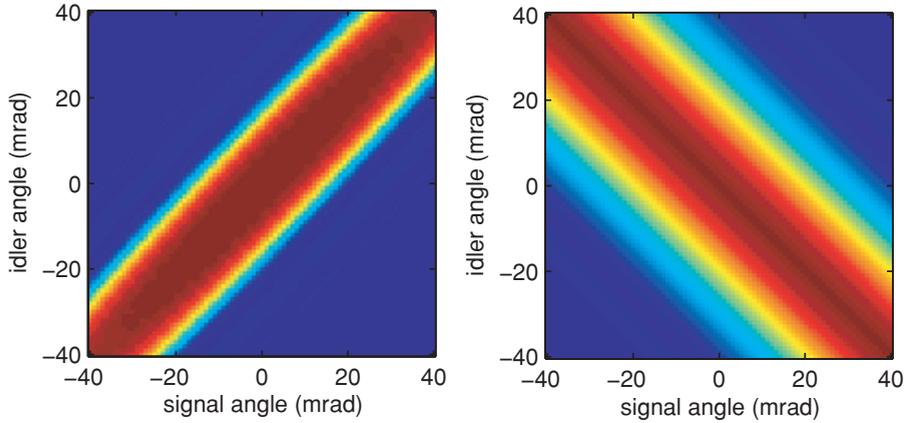


FIG. 3. (Color online) Density plots of the phase-matching function (left panel) and pump function (right panel).

condition is satisfied, the optimal focusing, and therefore the optimal width of the pump function, will depend both on the width and the orientation of the phase-matching function [10]. In addition, the signal and idler photons may require different optics to maximize coupling to their respective single-mode collection systems.

The ideas presented here are similar in many respects to those pertaining to spectral entanglement [10–14]. In both cases, entanglement is minimized when the correlations due to the phase-matching function can be balanced by the correlations due to the pump. Moreover, all of the arguments advocating a spectrally pure state—better heralding efficiency, improved interference, etc.—can just as easily be applied to the spatial domain, with the added benefit of improved single-mode coupling. However, the techniques are generally much easier to implement in the spatial domain. One reason is that the transverse momentum bandwidth can be changed using only simple lenses, while the spectral bandwidth is difficult to change. In addition, the spatial phase-matching function is more favorably oriented than its spectral counterpart. Whereas a spatial solution can be found for almost any SPDC material, there are very few good candidates for the elimination of spectral entanglement, particularly at visible wavelengths.

We have thus far shown that spatial entanglement is minimized with a properly focused pump. The results of Sec. II suggest that this should, in turn, maximize the fraction of photons collected in a pair of single-mode collection systems. This relationship is confirmed in Figs. 4 and 5, which were obtained by numerical modeling of our experiment. The two-photon state $|\Psi\rangle$ from Eq. (4) was calculated over a broad range of $\mathbf{k}_s, \mathbf{k}_i$ using accurate dispersion curves for PPKTP [15], rather than the approximate formulas of Eqs. (6)–(8). From this state, p_{emit} was obtained as the total probability of the two-photon state, while p_{fiber} was obtained as the probability of the projection of the state onto a pair of optimally focused Gaussian spatial modes [6]. Schmidt decomposition of the state yielded the Schmidt number N and largest Schmidt probability p_{max} . Figure 4 shows the single-mode coupling efficiency and spatial entanglement as a function of the dimensionless focusing parameter $L/k_p w_p^2$, where w_p is the radius of the pump beam waist. Here entanglement is measured by the purity $1/N$, where a larger purity corresponds to a lower degree of entanglement. From the plot, it is apparent that single-mode coupling efficiency and spatial purity are maximized for nearly identical focusing conditions. In principle this need not be the case—the purity only puts an upper bound on the coupling efficiency—but in this system the upper bound is fairly tight. As can be seen

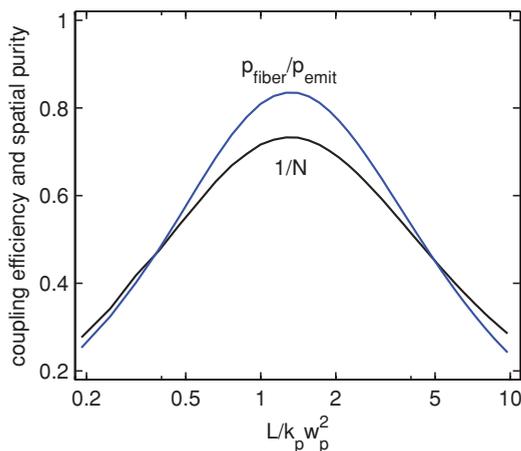


FIG. 4. (Color online) Fiber-coupling efficiency $p_{\text{fiber}}/p_{\text{emit}}$ and inverse spatial Schmidt number $1/N$ as a function of pump focus (focal strength increases from left to right).

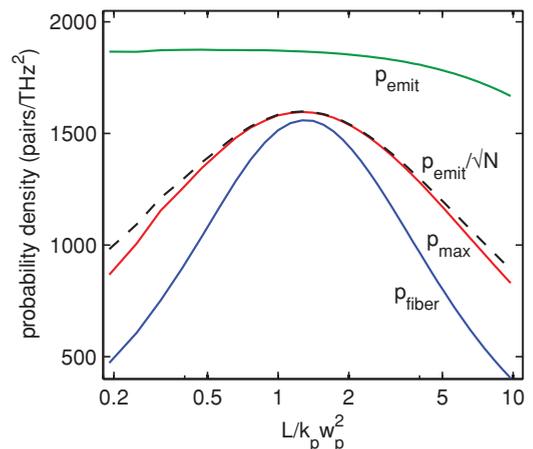


FIG. 5. (Color online) Dependence of the related probabilities $p_{\text{fiber}} \leq p_{\text{max}} \leq p_{\text{emit}}/\sqrt{N} \leq p_{\text{emit}}$ on pump focus.

in Fig. 5, p_{fiber} approaches p_{max} in the regime of interest, which in turn nearly equals p_{emit}/\sqrt{N} . The near-saturation of $p_{\text{fiber}} \leq p_{\text{max}}$ is due to the fact that the predominant Schmidt mode is quasi-Gaussian and couples well to a Gaussian fiber mode. The inequality $p_{\text{max}} \leq p_{\text{emit}}/\sqrt{N}$ is nearly saturated because that Schmidt mode is strongly predominant.

It is also worth pointing out that the total emission probability is (approximately) independent of pump focus. This can be understood by noting that the phase-matching function is (approximately) independent of the pump wave vector. This result is important because it shows that any coupling improvements accrued by focusing are not offset by a reduced pair emission rate. Thus maximizing the collection efficiency also maximizes the absolute fiber coupling rate. It should be noted that all these results are based on the spatial state of the photons at the wavelengths that yield the phase matching condition. At other wavelengths, the spatial state can be significantly different. Thus the results presented here are most applicable to the collection of spectrally filtered photons. A complementary study may be found in Ref. [6], which addresses the optimal collection of broadband SPDC photons into Gaussian spatial modes.

IV. SUMMARY

We have addressed from an entanglement perspective the challenge of optimizing the single-mode coupling of SPDC photons. It was shown that coupling efficiency is better for a

low degree of spatial entanglement, and that perfect coupling can be realized only in the absence of entanglement. It was also shown, both through experimental evidence and through a theoretical approach, that the conditions that lead to better single-mode coupling—a focused pump—also lead to a reduced degree of spatial entanglement. Finally, numerical simulations showed that single-mode coupling and spatial entanglement are optimized with nearly identical pump-focusing parameters.

The results presented here are consistent with earlier works addressing the optimization of single-mode flux from SPDC sources. Those works, although differing in the geometry or materials studied, manage to reach the common conclusion that focusing generally improves single-mode coupling. However, the approach employed here—and the result that single-mode coupling is inextricably linked to spatial entanglement—is more general. Indeed, the requirement of a low degree of spatial entanglement can be applied to any configuration for which a high coupling efficiency is the goal. This suggests that it might be best to approach the single-mode optimization problem in two stages: minimize the entanglement first, and then determine the best way to collect the emitted modes.

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