Evolution of the Giant Pulse in a Laser

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(Received 19 December 1962)

The differential equations governing inversion and photon density in a laser are solved for giant pulse operation. The simplifying assumptions which permit solution involve homogeneous excitation of the laser and the neglecting of changes produced by pumping and fluorescence during the formation of the giant pulse. Energy, peak output power, pulse delay, and pulse width are calculated.

INTRODUCTION

IT has been shown by Hellwarth and McClung[1–3] that a single pulse of very high intensity may be obtained from a laser if the onset of stimulated emission is delayed by means of a shutter until excitation reaches a level far above the threshold with the shutter open. Then the shutter is suddenly opened; that is, the threshold level is lowered, whereupon the radiation intensity builds up to a sharp peak and the excess excitation or population inversion is quickly exhausted. Under ordinary experimental conditions the process takes place so rapidly that the replenishment of the population inversion through absorption of the pumping radiation may be neglected.

The following quantities are of interest in connection with the giant pulse: The total energy radiated, the peak power radiated, the delay or the time of formation of the giant pulse, and the rates of its rise and of its fall.

Hellwarth[1,2] published several estimates for these parameters in terms of the parameters of the laser and the variables determining its physical state, as well as the rate of switching. Here we are concerned only with the fast switching case which we regard as physically and technically the most important. Fast switching is characterized by a switching of the shutter or cavity Q in time so short that no significant change of population inversion takes place during the switching process.

In formulating the equations which govern the process of stimulated emission in the period following the switching, we neglect the effects of processes which are slow in comparison to the formation of the giant pulse. In particular, we neglect the effects of continued pumping and of spontaneous emission on the population inversion. Under the described circumstances we obtain two nonlinear differential equations which are simpler than the general Statz-DeMars equations and which may be solved completely. The first integral of the equations permits calculation of the total energy obtainable from the pulse, the population inversion remaining and the peak power. Delay time and pulse width require further computations.

FORMULATION OF THE PROBLEM

The laser material is characterized by the following parameters: N_0, the number of active ions in the volume element, τ_L, the lifetime of spontaneous (fluorescent) decay, and α_0, the absorption coefficient of the unexcited laser material. The parameter α_0 is a function of the frequency, we use its peak value at the center of the fluorescent line.

The laser geometry is characterized by the following variables: V, the volume of the laser material, ℓ, the length of the laser material, and L, the optical distance between the reflectors calculated with due regard for the refractive indices of the materials situated between reflectors.

The physical state of the laser is characterized by the following variables: Φ, the photon density at the laser frequency ν, and $N=N_2-N_1$, the population inversion per unit volume.

An important device parameter is the loss coefficient γ, which is the fractional photon loss in a single passage. It may be subdivided into $\gamma=\gamma_1+\gamma_2$, where γ_1 represents the fraction of photons emitted as useful output of the device and γ_2 represents incidental losses.

The time of a single passage is $t_1=L/c$, therefore, the lifetime of a photon within the Fabry–Perot interferometer is $T=t_1/\gamma$. This is a fundamental unit of time characteristic of the laser.

The initial state for the formation of the pulse is achieved by pumping the laser with an optical source and keeping the loss coefficient at a value γ' much higher than γ. During this period of excitation, the population inversion rises from $-N_0$ to a positive value N_i; the photon density also rises to a value Φ_i. The subscript i indicates that the values are "initial" values for the giant pulse. At time $t=0$ the loss coefficient is reduced to γ and the formation of the pulse begins.

Photons are amplified in the laser at the rate of $\Phi\alpha x$ on traversing a distance x in the active material. Here α, the coefficient of amplification, satisfies the equation

$$\alpha=\alpha_0 N/N_0. \tag{1}$$

The full length of the laser is traversed $1/t_1$ times per second. Photons are lost at the rate of Φ/T, therefore

[1] R. W. Hellwarth, *Advances in Quantum Electronics*, edited by J. R. Singer (Columbia University Press, New York, 1961), pp. 334–341.
[2] F. J. McClung and R. W. Hellwarth, J. Appl. Phys. **33**, 828 (1962).
[3] F. J. McClung and R. W. Hellwarth, Proc. IRE **51**, 46 (1963).

neglecting photons created by spontaneous emission, the variation of photon density is described by the equation:

$$\frac{d\Phi}{d}=\left(\frac{\alpha\ell}{t_1}-\frac{1}{T}\right)\Phi. \tag{2}$$

If the contribution of continued pumping is neglected the density of population inversion varies at the following rate:

$$\frac{dN}{dt}=-\frac{2\alpha\ell}{t_1}\Phi, \tag{3}$$

because the stimulated emission of a photon causes N to decrease by 2.

We eliminate α by means of (1) and introduce the normalized variables

$$n=N/N_0, \quad \varphi=\Phi/N_0, \tag{4}$$

$$\frac{d\varphi}{dt}=\left(\frac{\alpha_0\ell}{t_1}n-\frac{1}{T}\right)\varphi, \tag{5}$$

$$\frac{dn}{dt}=-\frac{2\alpha_0\ell}{t_1}n\varphi. \tag{6}$$

Now we change the timescale to make T the unit of time. This process is equivalent to the substitution $t'=t/T$ and subsequent dropping of the primes. In this manner we obtain

$$d\varphi/dt=[(\alpha_0\ell/\gamma)n-1]\varphi, \tag{7}$$

$$dn/dt=-(2\alpha_0\ell/\gamma)n\varphi. \tag{8}$$

We further introduce the constant n_p defined by the equation

$$\alpha_0\ell n_p=\gamma. \tag{9}$$

Clearly n_p is the population inversion which corresponds to threshold for the given laser. The final form of the differential equations is:

$$d\varphi/dt=[(n/n_p)-1]\varphi, \tag{10}$$

$$dn/dt=-(2n/n_p)\varphi. \tag{11}$$

SOLUTION OF THE EQUATIONS

At the start of the process the photon density φ is very low. It rises from φ_i, reaches a peak φ_p generally many orders of magnitude higher than φ_i. Then φ declines to zero because of the exhaustion of its source of supply. The population inversion is a monotone decreasing function of time starting at n_i and ending at n_f. Figure 1 illustrates the typical curves traced out by these variables.

The total energy obtainable from the pulse is proportional to n_i-n_f, the peak power radiated is proportional to the peak photon density φ_p. These are the quantities of prime interest; they must be calculated in terms of the initial values n_i and φ_i. The laser itself is characterized by the parameters T and n_p to which one might add N_0 and ν if a variation of materials is also contemplated.

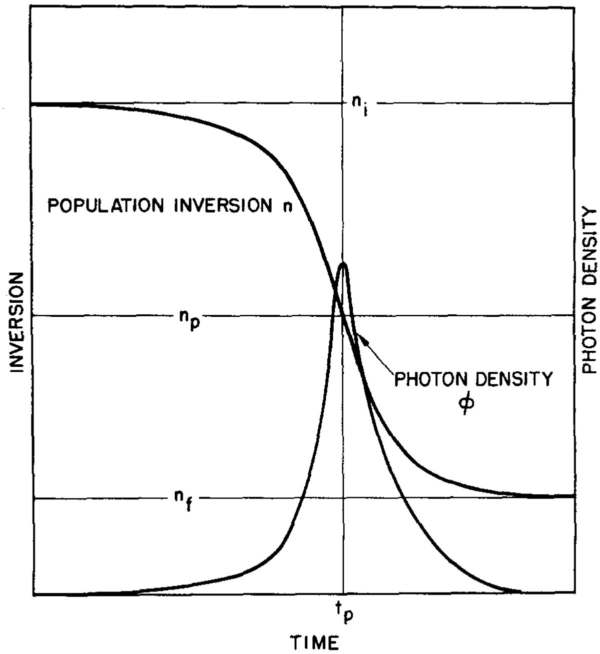

FIG. 1. Inversion and photon density in the giant pulse.

Equations (10) and (11) may be combined in the form:

$$d(n+2\varphi)/dt=-2\varphi. \tag{12}$$

Integrating and neglecting φ_i and φ_f we obtain

$$n_i-n_f=2\int_0^\infty \varphi dt, \tag{13}$$

which is the statement of the elementary fact that the time integral of the photon loss rate is equal to the number of stimulated transitions. In our units of time, the rate of loss of photons is equal to the photon density.

To solve the differential equations we divide (10) by (11).

$$d\varphi/dn=(n_p/2n)-\tfrac{1}{2}. \tag{14}$$

Therefore,

$$\varphi-\varphi_i=\tfrac{1}{2}[n_p\log(n/n_i)-(n-n_i)]. \tag{15}$$

Since $\varphi_i\approx 0$ and $\varphi_f\approx 0$, we can determine the final population inversion from

$$n_p\log(n_f/n_i)=n_f-n_i. \tag{16}$$

Equation (16) may be put in the form

$$n_f/n_i=\exp\{(n_i/n_p)[(n_f/n_i)-1]\}. \tag{17}$$

Here $x=n_f/n_i$ is unknown; $\beta=n_i/n_p$ is given. The equation $x=\exp\beta(x-1)$ may be resolved in the form

$$\beta=(\log x)/(x-1). \tag{18}$$

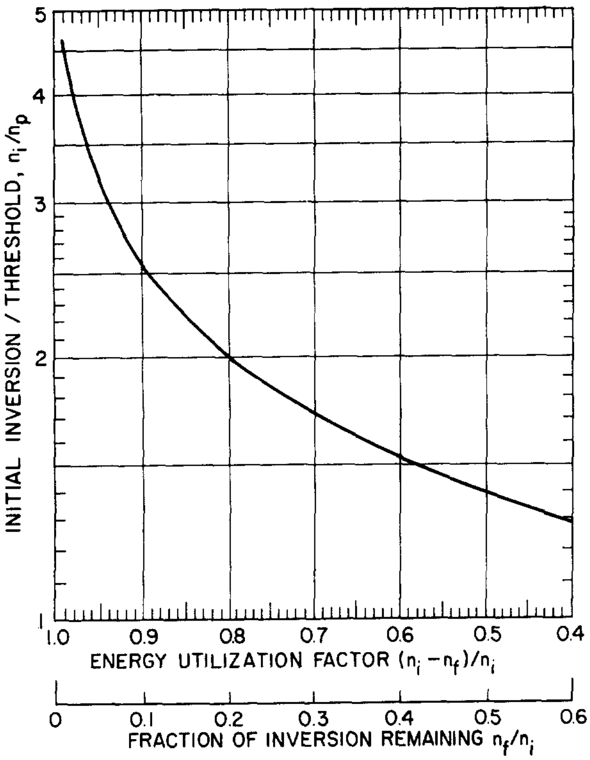

FIG. 2. Energy utilization factor and inversion remaining after pulse.

Figure 2 shows the relationship between n_f/n_i and n_i/n_p as determined from (18). The graph enables one to determine the fraction of inversion remaining from the initial conditions. The energy utilization factor is $(n_f-n_i)/n_i$. The total energy output of the pulse is

$$E=\tfrac{1}{2}(n_i-n_f)VN_0h\nu. \tag{19}$$

Two limiting cases are of particular interest. When the inversion is high, the energy utilization factor is nearly one. In fact, for $n_i/n_p>4$, $n_f/n_i<0.02$. In this case n_f may be neglected and we have

$$E_h=\tfrac{1}{2}n_iVN_0h\nu. \tag{20}$$

When the inversion is slight, $x=n_f/n_i$ is nearly one, and we may expand the logarithm in (18) in powers of $x-1$ with the result that

$$(n_i-n_p)/n_p\approx\tfrac{1}{2}[(n_i-n_f)/n_i],$$

which leads to

$$E_l\approx(n_i/n_p)(n_i-n_p)VN_0h\nu. \tag{21}$$

Since n_i and n_p are nearly equal, (21) does not differ substantially from the estimate of McClung and Hellwarth,[3] who obtained $(n_i-n_p)VN_0h\nu$ for this case.

The peak power is calculated from (15) noting that the peak is reached when $n=n_p$. Therefore, neglecting φ_i,

$$\varphi_p=\tfrac{1}{2}[n_p\log(n_p/n_i)-(n_p-n_i)]. \tag{22}$$

By means of Taylor series expansion, we obtain

$$\varphi_p=(n_i-n_p)^2/4n_p, \tag{23}$$

an approximation good only when $(n_i-n_p)/n_i\ll1$, which is frequently not the case. Hellwarth's recent estimate of the peak power [Eq. (14) in Ref. 3] is identical to (23) except for the omission of the 4 in the denominator. Equation (22) determines the peak photon density. Since photons decay with a lifetime T, the peak power radiated from the laser is

$$W_p=\tfrac{1}{2}[n_p\log(n_p/n_i)+n_i-n_p](VN_0h\nu/T). \tag{24}$$

When not all escaping photons contribute to the useful output of the laser then the peak effective output is $W_{pe}=W_p\gamma_1/\gamma$.

ENERGY AND POWER IN A TYPICAL RUBY LASER[4]

In the case of pink ruby $N_0h\nu=4.65$ J/cm^3. At room temperature $\alpha_0=0.4$ cm^{-1}. In a typical situation the loss coefficient with the shutter open varies between 0.05 and 0.20. For an optical length of 30 cm, the time of passage is 10^{-9} sec. For $\gamma=0.1$ the lifetime of the photon becomes $T=10^{-8}$ sec.

The parameter n_p is now determined by the length ℓ of the laser rod since (9) must be satisfied. For $\ell=5$ cm, and the assumed $\gamma=0.1$, we have $n_p=0.05$. An initial inversion between 0.1 and 0.25 may be regarded as typical.

If we take for example $n_i=0.15$ and $n_p=0.05$, we obtain from the graph of Fig. 2 $n_f/n_i=0.06$, the energy utilization factor is 0.94. The energy of the pulse is calculated from Eq. (19) as 0.321 joules per cc of ruby, or of the order of one joule for a ruby of convenient thickness.

The peak power output as calculated from Eq. (24) is 10.5×10^6 watts per cc of ruby. When the length of the ruby is increased, the value of n_p is proportionately reduced. In the example under consideration, this affects the energy utilization only slightly, the output energy per cc remains practically unchanged, provided n_i remains the same. The peak power, however, is affected as can be seen by examining (24). Therefore, in this case, the peak power per unit volume increases with the length of the laser. Consider, however, the effects of shortening the laser. If the ℓ is reduced from 5 to 2.5 cm, n_p increases from 0.05 to 0.10. If the inversion at the start of the pulse remains unchanged, the energy utilization factor is reduced from 0.94 to 0.58, therefore, the energy output per unit volume is drastically reduced. This unsymmetrical response to a change in length is due to the fact that the data chosen for illustration place the laser in an operating region where the energy utilization factor can hardly be increased.

However, the value of n_i is frequently determined by the value of γ prevailing prior to switching. In such a

[4] Bela A. Lengyel, *Lasers* (John Wiley & Sons, Inc., New York, 1962), p. 54 ff.

case the reduction of laser length alters n_i and n_p in the same proportion. Hence the energy utilization factor and the product n_iV remain unchanged. Therefore, under such circumstances the peak power and energy output are not affected by the length of the ruby.

EVOLUTION OF THE PULSE IN TIME

The energy and power calculations of the preceding sections were carried out by eliminating time from the system of differential equations governing the physical process. They resemble the calculations pertaining to the geometrical parameters of orbits in mechanics. We turn now to the dynamics of the situation. This has already been studied by Hellwarth,[1,3] who noted that initially the photon density starts at a low value and rises at an approximately exponential rate with a time constant (in physical units)

$$\tau = t_1/(\alpha_i l - \gamma) = n_p T/(n_i - n_p). \qquad (25)$$

This can be seen readily by substituting the fixed value of n_i for n in (5). We are able not only to predict the rate of initial rise, but to derive a complete solution by giving t in a closed form as a function of n. Since φ, as a function of n, has already been obtained in Eq. (15), we have formally solved the mathematical problem.

The formal solution is obtained by using the fact that $\varphi(n)$ is given by (15), therefore (11) may be integrated with the result

$$t = -\int_{n_i}^{n} \frac{n_p dn'}{2n'\varphi(n')}, \qquad (26)$$

or in greater detail

$$t = -\int_{n_i}^{n} \frac{n_p dn'}{2n'(2\varphi_i + n_p \log n'/n_i + n_i - n')}. \qquad (27)$$

In view of the shape of $\varphi(n)$ as shown in Fig. 1, we recognize three regions of integration: the central region B where φ is large, and the regions A and C preceding and following it, respectively. In handling the integral (27), it proves convenient to treat the regions A and C analytically and the central region B by machine computation. In the latter interval the properties of the solution are independent of the initial photon density φ_i (to a very high degree of accuracy) and, therefore, the single parameter n_i/n_p characterizes the giant pulse.

The substitution

$$z = -\log n/n_p \qquad (28)$$

places the origin of the new variable at the peak of the pulse. To separate the three regions let us introduce the quantities $z_1 = z_i + 0.01 < 0$, and $z_2 = z_f - 0.01 > 0$. The regions A, B, and C are then characterized by $z_i < z < z_1$, $z_1 < z < z_2$, and $z_2 < z < z_f$, respectively. The expression for time becomes

$$t = \int_{z_i}^{z} \frac{dz'}{F(z')}, \qquad (29)$$

where

$$F(z) = 2\varphi(n)/n_p = (2\varphi_i/n_p) + z_i - z + e^{-z_i} - e^{-z}. \qquad (30)$$

It is convenient to arrange calculations so that the instants of passage from region A to B and from region B to C serve as reference points in regions A and C, respectively. We introduce

$$T_1 = \int_{z_i}^{z_1} \frac{dz'}{F(z')} \quad \text{and} \quad T_2 = \int_{z_i}^{z_2} \frac{dz'}{F(z')}. \qquad (31)$$

In interval A the function $F(z)$ may be approximated by

$$(2\varphi_i/n_p) + (e^{-z_i} - 1)(z - z_i),$$

because $z_i \le z \le z_i + 0.01$. We note that

$$e^{-z_i} - 1 = (n_i - n_p)/n_p. \qquad (32)$$

Thus in evaluating integrals of the type (29) in the region A the integrand may be written as

$$n_p[2\varphi_i + (n_i - n_p)(z - z_i)]^{-1}.$$

Therefore, noting that $z_1 = z_i + 0.01$, we obtain

$$T_1 = \frac{n_p}{n_i - n_p} \log\left[1 + \frac{n_i - n_p}{200\varphi_i}\right], \qquad (33)$$

and

$$t - T_1 = \frac{n_p}{n_i - n_p} \log\left[\frac{2\varphi_i + (z - z_i)(n_i - n_p)}{2\varphi_i + 0.01(n_i - n_p)}\right]. \qquad (34)$$

To treat region C we transform $F(z)$ by making use of (16) which in terms of the new variables takes the form

$$z_f - z_i = e^{-z_i} - e^{-z_f}. \qquad (35)$$

Then

$$F(z) = (2\varphi_i/n_p) + z_f - z + e^{-z_f} - e^{-z}. \qquad (36)$$

In region C this may be approximated by the expression

$$(z_f - z)(1 - e^{-z_f}),$$

where the last factor is readily recognized as $(n_p - n_f)/n_p$. We obtain

$$t - T_2 = \int_{z_2}^{z} \frac{dz'}{F(z')} = \frac{n_p}{n_p - n_f} \log \frac{z_f - z_2}{z_f - z}$$

$$= -\frac{n_p}{n_p - n_f} \log 100(z_f - z). \qquad (37)$$

Computer solutions have been obtained for the interval containing the pulse, $z_1 < z < z_2$, where

$$F(z) = e^{-z_i} - e^{-z} + z_i - z. \qquad (38)$$

These solutions are plotted in Fig. 3 for $z_i = -0.5$, -1.0, -1.5 and -2.0. These values correspond to the following values of $n_i/n_p = 1.649$, 2.718, 4.482, and 7.389.

In the interval B it is convenient to calculate time

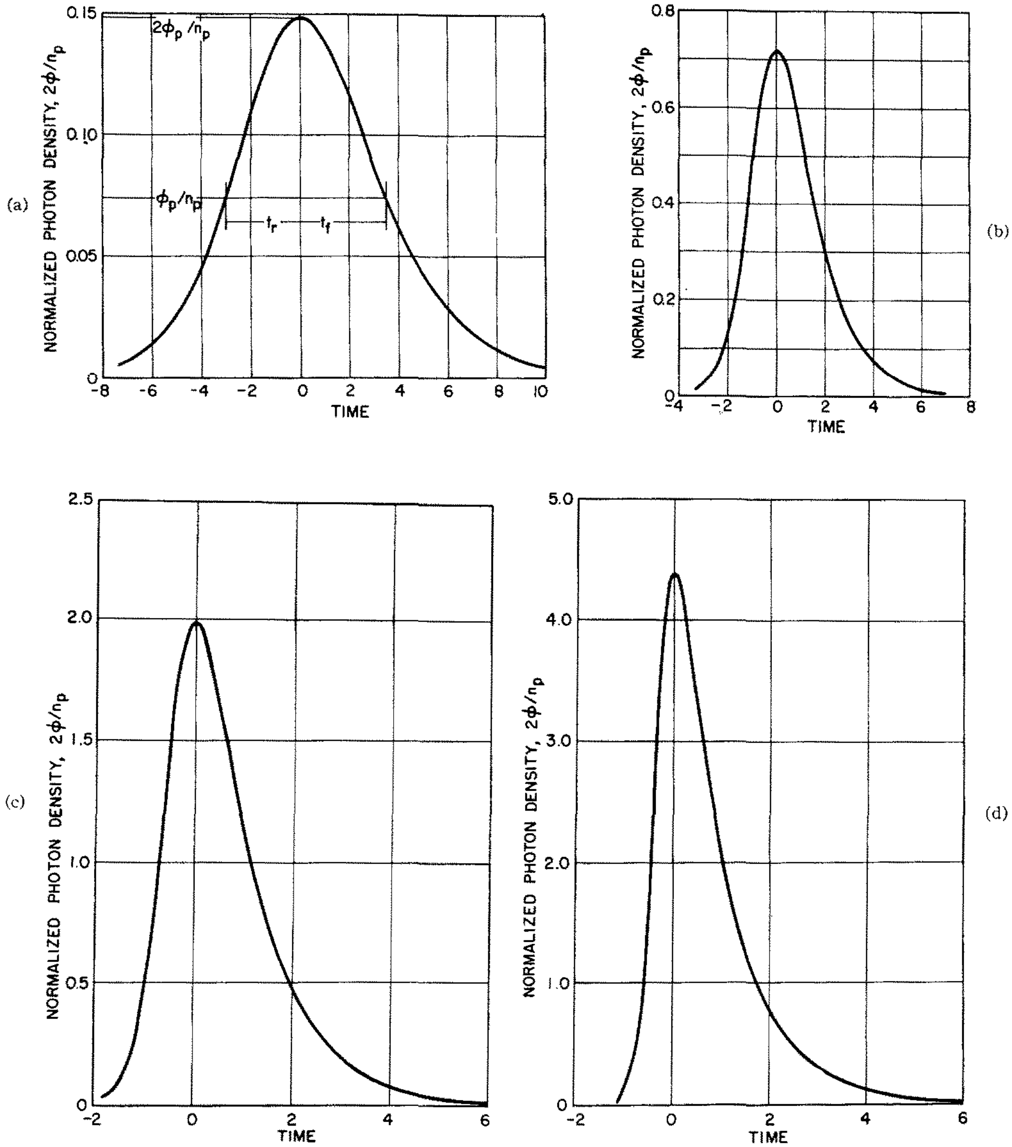

FIG. 3. Photon density vs time in the central region of a giant pulse. (Time is measured in units of photon lifetime T in the Fabry–Perot interferometer. Origin at peak.) (a) $\log n_i/n_p=0.5$, $n_i/n_p=1.649$; (b) $\log n_i/n_p=1.0$, $n_i/n_p=2.718$; (c) $\log n_i/n_p=1.5$, $n_i/n_p=4.482$; (d) $\log n_i/n_p=2.0$, $n_i/n_p=7.389$.

from the instant of the attainment of the peak, therefore the time coordinate is $t'=t-t_p$. Then one calculates t_1' and t_2' by extending the numerical integration to the appropriate points. Since the instant when $t'=t_1'$ coincides with the instant when $t=T_1$, it follows that in terms of the original time reckoned from switching, the peak is reached at $t_p=T_1-t_1'$, where T_1 is given by (33). The important characteristics of the giant pulse in the central region are: $2\varphi_p/n_p$, a normalized measure of peak power; t_r, the time in units of T for the photon density to rise from $\frac{1}{2}\varphi_p$ to φ_p; t_f, the time for the fall of φ from φ_p to $\frac{1}{2}\varphi_p$. These are presented in Table I together with data which specify the boundaries of the central region, namely t_1', t_2', $2\varphi(t_1')/n_p$, and $2\varphi(t_2')/n_p$.

In addition to these "central" characteristics, it is necessary to know the energy utilization factor $(n_i-n_f)/n_i$, which we already discussed and the time required to reach the peak from the time of switching. The last quantity depends on the time it takes the laser to pass through the initial phase which corresponds to region A. This time in turn depends on the initial photon density which may be estimated as follows:

During the period of excitation, inversion and photon density rise toward the initial values n_i and φ_i, respectively. The principal processes governing the rise of φ are spontaneous emission with a lifetime τ_L, stimulated emission, and the escape of photons from the laser at the rate corresponding to the preswitching value of

γ. We count only those photons which propagate in the proper direction; i.e., which are included in the laser beam. It is immaterial what happens to photons which are spontaneously emitted in a lateral direction. Then, assuming N_2 ions in the excited state, the number of photons per second emitted spontaneously in the proper direction is $N_2\Omega/4\pi\tau_L$. We note that

$$2N_2=N_2-N_1+N_2+N_1=N_0(1+n),$$

therefore the differential equation which takes the place of (5) during the excitation period is

$$\frac{d\varphi}{dt}=\frac{\Omega}{4\pi}\frac{1+n}{2\tau_L}+\left(\frac{\alpha_0\ell n}{t_1}-\frac{\gamma'}{\gamma}\frac{1}{T}\right)\varphi, \tag{39}$$

where $\gamma'>\gamma$ is the loss rate prior to switching. Again introducing T as the unit of time we get

$$\frac{d\varphi}{dt}=\frac{\Omega(1+n)T}{8\pi\tau_L}+\left(\frac{n}{n_p}-\frac{\gamma'}{\gamma}\right)\varphi>0. \tag{40}$$

Only a small error is introduced by replacing the $>$ sign in (40) by the $=$ sign, because on the scale of T the rise of φ is slow. The factor of φ in (40) is negative during the excitation period, because if n/n_p were to exceed γ'/γ the laser would fire. How close n/n_p is permitted to get to the limit γ'/γ depends on experimental conditions. The assumption that it rises to one-half of this limit may be approximately true in a typical situation. This means

$$n_i/n_p\approx\tfrac{1}{2}(\gamma'/\gamma). \tag{41}$$

Then

$$\varphi_i\approx(n_p/n_i)[\Omega(1+n_i)T/8\pi\tau_L]. \tag{42}$$

For ruby $\tau_L=3\times10^{-3}$ sec. The value 10^{-6} is typical for $\Omega/4\pi$. Let us consider again the figures $n_i=0.15$, $n_p=0.05$ and $T=10^{-8}$ sec for illustration. Then $\varphi_i\approx0.65\times10^{-12}$, which corresponds to 4×10^6 photons per cc traveling in the correct direction. With these numerical data we obtain from (33) the figure $T_1=10.2$ for the starting time of the central region. Admittedly, the estimate of φ_i is a coarse one. However, because of the logarithmic character of (33) a change in φ_i by a factor of ten would change T_1 only by 1.1.

Now we turn to Table I, where we find for $n_i/n_p=3.00$ the value $t_1'=-2.88$. Therefore, the time from switching to the attainment of the peak is $T_1+t_1'=13.1\times10^{-8}$ sec. This is of the order of observed data.[3] The duration for which the photon density remains above its half-peak level is $t_r+t_f=1.70\times10^{-8}$ sec.

TABLE I. Characteristics of a giant pulse as functions of n_i/n_p.

$\log\frac{n_i}{n_p}$	$\frac{n_i}{n_p}$	$\frac{2\varphi_p}{n_p}$	t_r	t_f	t_1'	t_2'	$\frac{2\varphi(t_1')}{n_p}$	$\frac{2\varphi(t_2')}{n_p}$
0.1	1.105	0.0052	12.291	12.632	−28.26	22.93	0.0001	0.0017
0.2	1.221	0.0214	7.960	8.437	−16.81	18.67	0.0022	0.0020
0.3	1.350	0.0499	5.335	5.803	−11.91	14.43	0.0034	0.0028
0.4	1.492	0.0918	3.892	4.356	−9.14	11.88	0.0048	0.0037
0.5	1.649	0.149	3.016	3.480	−7.33	10.25	0.0064	0.0045^-
0.6	1.822	0.222	2.432	2.896	-6.05^-	9.12	0.0081	0.0053
0.7	2.014	0.314	2.016	2.481	−5.09	8.31	0.0100	0.0060
0.8	2.226	0.426	1.704	2.171	−4.35	7.71	0.0121	0.0066
0.9	2.460	0.560	1.463	1.931	−3.76	7.26	0.0145^-	0.0072
1.0	2.718	0.718	1.271	1.741	−3.28	6.91	0.0170	0.0077
1.1	3.004	0.904	1.114	1.586	−2.88	6.65	0.0199	0.0082
1.2	3.320	1.120	0.984	1.459	−2.54	6.47	0.0230	0.0086
1.3	3.669	1.369	0.875	1.352	−2.26	6.33	0.0265	0.0090
1.4	4.055	1.655	0.782	1.263	−2.01	6.25	0.0304	0.0092
1.5	4.482	1.982	0.702	1.186	−1.80	6.21	0.0346	0.0095
1.6	4.953	2.353	0.633	1.120	−1.61	6.20	0.0393	0.0096
1.7	5.474	2.774	0.572	1.064	−1.45	6.23	0.0445^-	0.0098
1.8	6.050	3.250	0.518	1.015	−1.30	6.29	0.0502	0.0099
1.9	6.686	3.786	0.471	0.973	−1.18	6.37	0.0565	0.0099
2.0	7.389	4.389	0.429	0.936	−1.06	6.48	0.0635	0.0100
2.1	8.166	5.066	0.391	0.905^-	−0.96	6.61	0.0713	0.0100
2.2	9.025	5.825	0.357	0.877	−0.87	6.75	0.0798	0.0100
2.3	9.974	6.674	0.327	0.854	−0.79	6.92	0.0893	0.0100
2.4	11.023	7.623	0.300	0.833	−0.72	7.10	0.0997	0.0100
2.5	12.182	8.683	0.275^-	0.816	−0.65	7.30	0.1112	0.0100

THE EFFECTS OF TEMPERATURE

Temperature enters into giant pulse calculations because α_0 and τ_L are temperature-dependent parameters of the material.[4] The change of α_0 from 0.4 cm^{-1} at room temperature to 10 cm^{-1} at 77°K has a profound effect on n_p because n_p changes in inverse proportion to α_0. This decrease of n_p affects the energy output of the laser to an appreciable degree only when the laser is operated with a medium or low inversion; i.e., $n_i/n_p<3$. Then a decrease of n_p causes n_f to decrease also, hence, if n_i is kept constant the output energy increases as required by (19). For high inversion, $n_i/n_p>4$, the quantity n_f is already nearly zero and no significant change can take place.

Peak power always increases if n_p is decreased and n_i is kept constant, but the rate of increase is generally slow. This is seen by differentiating Eq. (22) with respect to n_p. Then

$$\partial\varphi_p/\partial n_p=\tfrac{1}{2}\log(n_p/n_i)<0. \tag{43}$$

What happens in an actual case is determined to a larger extent on the effect of temperature change on n_i. This depends on the experimental circumstances.

EVALUATION OF THE RESULTS

Although formally simple mathematical results were obtained, care must be exercised in applying these to the physical situation. Some of the assumptions made in arriving at the simple formalism do not correctly reflect the detailed properties of the physical system.

The first basic assumption subject to doubt is the uniform excitation of the laser rod. This is impossible to achieve in practice, the density of the exciting radiation varies with distance from the cylinder axis. The initial photon density also varies. Therefore, the process of pulse formation proceeds not at a constant rate for the entire ruby, but as a function of the radius. Then what is observed as pulse delay time is a time average over the entire laser. So far, our formula for t_p may still correctly represent this average delay. The calculated pulse width is in serious error because the observed pulse results from the superposition of pulses occurring at slightly different times. The same reasoning explains why the observed peak output is lower than what we calculate for a homogeneously excited laser. An additional factor here is that total observable energy is also lower than the calculated one which does not take into account incidental losses in the system.

When the initial inversion n_i is determined by an experimenter by relating it as a threshold to a measured loss coefficient then n_i represents the highest excitation within the laser, and not the average one over the entire cylinder. The fact that the experimenter usually measures $\gamma=\alpha_0 n\ell$ and not n creates an added complication, because α_0 is a function of the temperature and the temperature of the ruby is variable during the process of pumping.

A fundamental limitation of our work is that it neglects the spectral distribution of the fluorescent line and the selective properties of the Fabry–Perot interferometer with regard to frequency. When a laser is excited barely above threshold, the operation in a single Fabry–Perot mode is fairly possible. This is definitely not the case, however, in giant pulse operation. Here we have to deal with the simultaneous evolution of a large number of modes over a range of frequencies. An adequate treatment of the giant pulse phenomenon should take into account the interaction of a number of oscillations all feeding from the same reservoir of excited ions. Such work may have to follow the approach indicated in the paper of Wagner and Birnbaum.[5]

ACKNOWLEDGMENTS

We express our thanks to Ira M. Green for assistance in numerical computations, and to R. W. Hellwarth for constructive criticism and suggestions.

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JOURNAL OF APPLIED PHYSICS VOLUME 34, NUMBER 7 JULY 1963

Equation of State of 6061-T6 Aluminum at Low Pressures*

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(Received 15 October 1962)

The Hugoniot equation of state of 6061-T6 aluminum was determined in the pressure range from 0 to 31 kbars from measurements of velocities of plane waves induced by plate impact. Below the Hugoniot yield point of 6.4±0.7 kbars, only an elastic wave was observed, while above both elastic and plastic waves were observed. The measured wave velocities corresponded closely to values predicted from elastic parameters for the material determined in static tests. Above the elastic limit, the free-surface velocity was less than the interface velocity by an amount slightly greater than predicted by theory, and appeared to be a function of target thickness.

The measured Hugoniot was found to correspond well with elementary elastic–plastic theory, in which the yield stress is assumed to be a constant, and the change in compressibility with compression is predicted by second order elastic theory. It was found that the measured Hugoniot yield stress corresponded to the static yield stress obtained in simple tension tests, and that the Hugoniot joined smoothly onto an extrapolation of the high pressure Hugoniot data of Walsh and Al'Tshuler.

INTRODUCTION

THE method of using explosively generated one-dimensional shock waves to determine the equation of state of a medium in a 150- to 500-kbar (1 kilobar $=10^9$ dyn/cm^2) pressure range was first discussed by Walsh and Christian.[1,2] A modification of this explosive technique was later used to extend the pressure range up to a few megabars.[3–5] The results of the experiments discussed extend the equation of state of 6061-T6 aluminum into the low-pressure region from 0 to 31 kbars. The purpose of measuring the low-pressure equation of state was to investigate the validity of extending the elastic–plastic theory into the region of high rates of loading. Also, various problems, such as the description of the waveform propagated

* This work was supported by the U. S. Atomic Energy Commission and, under contract AF 33(657)-8427, by the Flight Dynamics Laboratory, Aeronautical Systems Division, USAF.

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