

Irreducible Hyperkähler varieties

Chiara Camere, Giovanni Mongardi

This is the proposal for a graduate course on irreducible hyperkähler varieties, starting from the basics and then focusing on their automorphisms, moduli spaces, ample cones and parallel transport groups. The course will consist in sixteen lessons of one hour each.

- **Known families of hyperkähler varieties - 1 hour.** We will review currently known deformation families of hyperkähler varieties, such as manifolds of $K3^{[n]}$ -type, generalized Kummer varieties and O'Grady manifolds of dimension six and ten.
- **The Beauville–Bogomolov–Fujiki lattice - 1 hour.** We will review some notions of lattice theory and then we will see how to endow the second integral cohomology group of a hyperkähler variety with a lattice structure, using the so-called Beauville–Bogomolov–Fujiki quadratic form.
- **Deformations and flops of hyperkähler varieties - 1 hour.** We will construct twistor families and the versal local deformation space of hyperkähler varieties. Then we will analyse birational transformations of hyperkähler manifolds and compare their deformations.
- **Parallel transport and exceptional divisors - 2 hours.** We introduce the notion of parallel transport operators, then we prove that certain reflections lie in this group and use them to compute these operators in the $K3^{[n]}$ case.
- **Period domains and period maps - 2 hours.** We will introduce the period domains associated to hyperkähler manifolds and construct the moduli spaces of marked hyperkähler manifolds. We will then sketch the proof of Torelli type results.
- **Cones, divisors and rational curves - 2 hours.** We introduce the ample and movable cones on hyperkähler manifolds and the class of wall divisors. We then prove that wall divisors cut out the ample cone and compute wall divisors in the $K3^{[n]}$ case. We will also discuss deformations of rational curves on hyperkähler manifolds.
- **Density results - 1 hour.** We will look at special subsets of hyperkähler manifolds, given by requiring certain classes to be algebraic, and prove that they are dense in the moduli space of marked hyperkähler manifolds.

- **Lagrangian fibrations - 2 hours.** We will see what is currently known about Lagrangian fibrations on hyperkähler varieties, focusing in particular on what is known about their singular fibres. We will see some examples and open questions.
- **General facts about automorphisms - 1 hour.** We will see which constraints can be given on an automorphism of a hyperkähler variety in general. The notions of symplectic and non-symplectic automorphisms will be introduced and illustrated also through examples.
- **Automorphisms of fourfolds of $K3^{[2]}$ -type - 2 hours.** We will see the classification of symplectic and non-symplectic automorphisms of prime order of fourfolds of $K3^{[2]}$ -type, and also discuss questions about the existence of non-natural automorphisms.
- **Complex ball quotients from fourfolds of $K3^{[2]}$ -type - 1 hour.** We will study moduli spaces of fourfolds of $K3^{[2]}$ -type endowed with a non-symplectic automorphism of prime order and see how they relate to complex ball quotients via the period map.