MISCELLANEA ON COMPLEX PROJECTIVE MANIFOLDS

Prof. Antonio Lanteri

corso per il Dottorato in Matematica

Il corso si terrà a distanza a partire dal 20 aprile 2020 con le seguenti modalità.

Nell'orario previsto dal calendario per ogni lezione, il docente invierà per email ai partecipanti il pdf degli appunti che avrebbe usato come traccia per tenere la lezione in presenza, qualora fosse stato possibile, scritti in forma abbastanza estesa.

Il giorno successivo a quello di ogni lezione, per un paio d'ore, il docente sarà disponibile per fornire eventuali chiarimenti via Skype a qualunque partecipante voglia contattarlo allo scopo.

Le 8 lezioni hanno ciascuna un tema. Ogni partecipante che ha in programma di sostenere l'esame dovrà preparare un seminario su uno di tali temi, concordato con il docente, per il quale il docente stesso darà appropriate indicazioni bibliografiche.

La lettura degli appunti di tutto il corso fornirà ad ogni candidato un quadro complessivo di riferimento per effettuare consapevolmente la scelta del tema per il seminario.

Di seguito l'orario delle lezioni virtuali ed i contenuti di ciascuna di esse.

Lect. 1. April 20 (Monday, 2 pm: 2 hours)

Background on surfaces. Intersections; examples; topological interpretation; Riemann--Roch; the genus formula; Nakai--Moishezon; the Hodge index theorem; some relevant cones in N(S), including nef divisors and Kleiman's theorem; an example: the cubic scroll in $\mathbf{P}^4$.

Lect. 2. April 23 (Thursday, 2 pm: 2 hours)

The birational point of view. Morphisms and spanned line bundles; rational and birational maps; examples; blowing-ups and their properties; resolution of the indeterminacies of rational maps and the structure of birational morphisms; (-1)-curves (stating Castelnuovo contraction theorem); an example; minimal models in the classical sense.

Lect. 3. April 27 (Monday 2 pm: 2 hours)

Ruled and rational surfaces. Statement of the Noether--Enriques theorem; a comment; numerical characters of rational and ruled surfaces; the Castelnuovo rationality criterion (with proof); the Enriques ruledness criterion (statement and idea of the proof); fibrations in rational curves; minimal models of ruled and rational surfaces. More on $\mathbf{P}^1$-bundles.

Lect. 4. April 30 (Thursday 2 pm: 2 hours)

Nef threshold and the key lemma. Kodaira vanishing theorem; the nef threshold of a polarized surface with non-nef canonical bundle; examples; the key lemma and characterization of ruled surfaces; del Pezzo surfaces, including the cubic surface and the 27 lines.

Lect. 5. May 4 (Monday 2 pm: 2 hours)

Mori's theorem in the setting of surfaces. The cone \overline{NE}(S) and Kleiman's ampleness criterion; digression on the higher dimensional context; The Mori cone theorem and extremal rational curves on surfaces; Polarized surfaces: some special classes; basic lemmas and characterization of polarized surfaces of low sectional genera.

Lect. 6. May 7 (Thursday 2 pm: 2 hours)

Reider's theorem and some applications. Bogomolov instability, Cayley--Bacharach property and sketch of proof of Reider's statement concerning spannedness; application to the pluri-anticanonical maps of del Pezzo surfaces; description of the bi-anticanonical map for the del Pezzo surface of degree 1.

Lect. 7. May 11 (Monday 2 pm: 2 hours)

The adjunction mapping in the very ample setting. Basic properties of the adjunction mapping, including the enumeration of projective surfaces of degree < 5; reductions; the Sommese -- Van de Ven theorem (proof of selected points according to [L]); surfaces with hyperelliptic hyperplane sections; classical vs modern points of view.

Lect. 8. May 14 (Thursday 2 pm: 1 hour)

Fano manifolds. Index and some general properties of Fano manifolds, including the Kobayashi--Ochiai theorem; del Pezzo manifolds and Fujita's classification.

References

[Be] A. Beauville, Complex algebraic surfaces, Cambridge Univ. Press, 1983, transl. of Surfaces algébriques complexes, Astérisque 54 (1978).

[BS] M.C. Beltrametti, A.J. Sommese, The adjunction theory of complex projective varieties, Expositions in Mathematics, vol. 16 , W. de Gruyter, Berlin, New York, 1995.

[Fr] R. Friedman, Algebraic surfaces and holomorphic vector bundles, Springer, 1998.

[Fu] T. Fujita, Classification theories of polarized varieties, London Math. Soc. Lecture Notes Series, 155, Cambridge Univ. Press, 1990.

[LP] A. Lanteri, M. Palleschi, About the adjunction process for polarized algebraic surfaces, J. reine angew. Math. 352 (1984), 15—23.

[L] A. Lanteri, The Sommese -- Van de Ven theorem on the adjunction mapping, notes for a course in Madrid, 1996.