Motivic Galois Group and Anabelian Geometry

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Abstract: The goal of this course is to study the natural action of the motivic Galois group on the motivic completions of the fundamental groups of algebraic varieties. In particular, we will explain a proof of a motivic analog of a theorem of Pop characterising the motivic Galois group as the automorphism group of a certain diagram of pro-algebraic groups.

Lecture 1: **Voevodsky motives, I**
In this lecture, I will recall the construction of the triangulated categories of Voevodsky motives.

Lecture 2: **Voevodsky motives, II**
In this lecture, I will recall (mostly with no proofs) what is known on motives. I'll also state some of the important conjectures in the field.

Lecture 3: **The motivic Galois group, I**
In this lecture, I will introduce the main object of study of these lectures, namely the motivic Galois group of a field embedded into the complex numbers.

Lecture 4: **The motivic Galois group, II**
In this lecture, I will recall (mostly with no proofs) what is known about the motivic Galois group.

Lecture 5: **Preliminaries on ∞-categories, I**
The goal of this lecture is to introduce the language of ∞-categories which will be crucial for stating and proving the main theorem.

Lecture 6: **Preliminaries on ∞-categories, II**
In this lecture, I will discuss some facts from Derived Algebraic Geometry (DAG) that are needed for stating and proving the main theorem.

Lecture 7: **The motivic IOM conjecture**
The goal of this lecture is to state the main theorem of these lectures, which we call the `motivic Ihara-Oda-Matsumoto conjecture'. Roughly speaking the main theorem is an identification of the motivic Galois group as the group of automorphisms of the functor that sends a variety to its topological fundamental group, conveniently completed.

Lecture 8: **Proof of the motivic IOM conjecture**
In this lecture, I will sketch the proof of the main theorem.