The derived categories of coherent sheaves on algebraic varieties are closely related to the birational geometry.  
I will explain the parallelism between the minimal model program in the birational geometry and the semi-orthogonal decompositions derived categories　in the case of toric varieties.  
This parallelism is realized when one replace the singular varieties to the associated smooth Deligne-Mumford stacks.  
In particular, the weighted projective spaces have especially nice structure, the existence of exceptional collections, when one replaces them to their associated stacks.  
I will also consider the derived category of the singular varieties themselves in the case of weighted projective spaces, and explain that they have still nice structures when one consider deformations of objects which are not necessarily commutative.