Let R be a rational surface admitting an elliptic fibration. The surface R is a blow up of P^2 in 9 points which are the base points of a pencil of plane cubics.
It is known that the Cox ring of R is finitely generated if and only if the elliptic fibration on R is extremal, i.e., its group of sections is finite. Under this condition, we describe a set of generators and the set of relations among these generators, so we give a presentation for the Cox ring. In particular we discuss some specific examples, analyzing both the generators and the relations in terms of the geometry of plane curves related to the pencil of cubics associated to R.
This talk is based on a joint work with M. Artebani and A. Laface