

**Japanese-European Symposium on
Symplectic Varieties and Moduli Spaces –
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Short courses

Laurent Manivel (University de Toulouse)

Title Orbital degeneracy loci and moduli spaces

Abstract Orbital degeneracy loci are generalizations of zero loci of vector bundles and ordinary degeneracy loci of morphisms between vector bundles. I will explain the basics of the general theory, and show how it can be used in a few specific situations. In particular I will explain how it allows to relate geometry of skew-symmetric three-forms or four-forms with abelian varieties, moduli spaces of vector bundles on curves and generalized Kummer manifolds.

Ken-Ichi Yoshikawa (Kyoto University)

Title Enriques $2n$ -folds and analytic torsion

Abstract In my lectures, a non-simply-connected compact Kähler manifold of dimension $2n$, whose universal covering is either a Calabi-Yau manifold or a hyperkahler manifold, is called an Enriques $2n$ -fold. This class of manifolds was introduced and studied by Boissière-Nieper-Wisskirchen-Sarti and Oguiso-Schröer, independently. The goal of my lectures is to introduce an analytic invariant of Enriques $2n$ -folds by using analytic torsion and study the invariant as a function on the moduli space. After explaining the notion of analytic torsion and its basic properties, as well as the holomorphic torsion invariant for $K3$ surfaces

with involution as a model case, I explain the construction of the invariant of Enriques $2n$ -folds, some basic properties of the invariant viewed as a function on the moduli space, and some explicit examples. Then I would like to focus on the case of Enriques $2n$ -folds of hyperkahler type of index 2 satisfying certain conditions. Essentially by Joumaah, once the deformation type is fixed, the Hausdorff reduction of the moduli space of those Enriques $2n$ -folds is a Zariski open subset of a modular variety of orthogonal type. In this case, our invariant is given by the Petersson norm of an automorphic form on the modular variety of Joumaah with singularities on the Heegner type divisors attached to the anti-invariant MBM vectors. This automorphic form is nowhere vanishing on the moduli space, which implies the quasi-affinity of the Hausdorff reduction of the moduli space. I would also like to explain a possible link to the recent non-existence result of reflective modular forms in higher dimensions due to Ma and Wang. Under the condition that the anti-invariant MBM vectors are reflective and consist of a single orbit under the action of a certain natural modular group, the dimension of the Kuranishi space of those Enriques $2n$ -folds is at most 26. If time permits, I would also like to explain some cases of Enriques $2n$ -folds of Calabi-Yau type, whose invariant is related to automorphic forms.

Abstracts

Olivier Debarre (Jussieu)

Title Smooth subvarieties of Jacobians of curves

Abstract Borel and Haefliger asked whether algebraic cohomology classes on smooth complex projective varieties were integral linear combinations of classes of smooth subvarieties. Hartshorne, Rees, and Thomas showed that the answer to this question is negative in general. The aim of this talk is to present new counter-examples, some in dimension 6 (the smallest possible dimension), on Jacobians of curves. This is work in collaboration with Olivier Benoist.

Franco Giovenzana (Paris Saclay)

Title Hodge structure of O'Grady's singular moduli spaces.

Abstract Motivated by a question of Bakker and C. Lehn about the singularities type of O'Grady's singular moduli spaces, we study their Hodge structure. As an application, we deduce that O'Grady's moduli spaces do not have finite quotient singularities. In this talk I will illustrate the method we applied for the computation of the Hodge structure and Betti numbers, then I will explain the consequences on the type of their singularities. This is based on a joint work with Valeria Bertini.

Annalisa Grossi (Paris Saclay)

Title Birational equivalence and deformation equivalence for Hyperkähler manifolds

Abstract Huybrechts proved that if X and Y are birational HK then they have to be deformation equivalent, but this implication is far from being an equivalence. It is then natural to ask which additional assumptions are needed to get that two HK manifolds of the same deformation type are birational. In this talk I will give a gentle introduction on lattice theory for HK manifolds, then I will provide a lattice-theoretic criterion to determine when two HK manifolds of OG10 type are birational (joint work with C.Felisetti and F.Giovenzana), and I will show an application to the Li-Pertusi-Zhao variety. If time permits I will discuss another related topic about sufficient numerical conditions to determine the deformation type of a given HK manifold of dimension 4. This is a first result about a joint work in progress with P.Beri.

Masafumi Hattori (Kyoto University)

Title On boundedness and moduli spaces of K-stable Calabi-Yau fibrations over curves

Abstract K-stability was introduced to detect a constant scalar curvature Kähler metric that is a generalization of a Ricci flat Kähler metric. The characterization of K-stable varieties is well-studied when K_X is ample or X is a Calabi-Yau or Fano variety. However, K-stability of Calabi-Yau fibrations (i.e., K_X is relatively trivial) is not known much in algebraic geometry. We introduce uniform adiabatic K-stability (If $f : (X, H) \rightarrow (B, L)$ is a fibration of polarized varieties, which means that K-stability of $(X, aH + L)$ for sufficiently small $a > 0$) In this talk, I would explain that uniform adiabatic K-stability of a Calabi-Yau fibration over a curve is equivalent to K-stability of the base curve in some sense. Furthermore, we construct separated moduli spaces of polarized uniformly adiabatically K-stable Calabi-Yau fibrations over curves. This talk is based on a joint work with Kenta Hashizume.

Dai Imai (Kyoto University)

Title Analytic torsion for irreducible holomorphic symplectic fourfolds with antisymplectic involution

Abstract Yoshikawa constructed an invariant of 2-elementary K3 surfaces by using equivariant analytic torsion. He also proved that the invariant is expressed as the Petersson norm of a certain automorphic form on a bounded symmetric domain of type IV and a certain Siegel modular form. In this talk, we generalize this result to a class of higher dimensional manifolds. More precisely, we construct an invariant of irreducible holomorphic symplectic manifolds which are deformation equivalent to the Hilbert scheme of two points of a K3 surface with antisymplectic involution by using equivariant analytic torsion. In addition, we show that the invariant is expressed as the Petersson norm of a certain automorphic form on a bounded symmetric domain of type IV and a certain Siegel modular form in some cases.

Akihiro Kanemitsu (Saitama University)

Title $K3$ surfaces of genus 13 and curves of genus 3

Abstract Let C be a hyperelliptic curve of genus 3 and L a line bundle of odd degree. The moduli space N of rank 2 stable bundles with determinant L is embedded into the Grassmann variety $\text{Gr}(8, 2)$ as the zero locus $\text{Gr}(8, 2, P)$ of a pencil P of quadric forms (Desale-Ramanan).

We provide a description of $K3$ surfaces of genus 13 as complete intersection varieties with respect to vector bundles on N . And we discuss some generalizations of this description. This is a joint work with Professor Shigeru Mukai.

Dominique Mattei (Bonn University)

Title Lagrangian subfibrations in Prym varieties in integrable systems.

Abstract Beauville-Mukai systems are well known examples of hyperkähler manifolds X equipped with a map $p : X \rightarrow B$ onto a projective space, the fibers of which are Lagrangian with respect to the symplectic structure on X . Over a dense open of B , the fibers of p are (compactified) Jacobians of curves in a $K3$ surface. In this talk, we will construct subfibrations $p : Y \rightarrow T$ (over a locally closed subspace T of B), the fibers of which are Prym varieties, and such that Y is symplectic and p is Lagrangian. If time permits, we will show that this construction recovers (rationally and up to a cover) the intermediate Jacobian of a cubic fourfold containing a plane.

Shigeru Mukai (RIMS Kyoto)

Title Kummer quartics and associated holomorphic symplectic 6-folds

Abstract Kondo(1998) described the automorphism group of the generic Jacobian Kummer surface $\text{Km}(A)$ using a Conway chamber in the nef cone. As a sample of higher dimensional analogue, we describe

the birational automorphism group of a certain holomorphic symplectic 6-fold associated with $\text{Km}(A)$. It is the semi-direct product of Borchers(2000)'s group of 896 reflections by an extra special group 2^{1+8} whose center is Rapagnetta(2007)'s involution.

Yoshinori Namikawa (Kyoto University)

Title Symplectic singularities in algebraic geometry

Abstract Symplectic varieties play important roles in algebraic geometry and geometric representation theory. They often show up with a good \mathbb{C}^* -action. Such a variety is called a conical symplectic variety. A conical symplectic variety has a close relationship with Poisson geometry and contact geometry. In this talk we first look at conical symplectic varieties from the view point of Poisson geometry and contact geometry and next discuss the following topics: a finiteness theorem for conical symplectic varieties, a characterization of the closures of nilpotent orbit of complex semisimple Lie algebras, Poisson deformation and birational geometry. Finally we will report on recent developments on the explicit constructions of \mathbb{Q} -factorial terminalizations of conical symplectic varieties.

Arvid Perego (Genova University)

Title The monodromy of moduli spaces of sheaves on K3 surfaces

Abstract The monodromy group of an irreducible symplectic manifold X is an important tool for the study of the birational geometry of X . It is calculated for all the known examples of irreducible symplectic manifolds by the work of Markman, Mongardi, Rapagnetta and Onorati. For irreducible symplectic varieties, it is the locally trivial monodromy group that plays the role of the monodromy group. In a joint work with Onorati and Rapagnetta, we calculate the locally trivial monodromy group for moduli spaces of semistable sheaves on K3 surfaces, generalizing the known result of Markman for smooth moduli spaces.

Alexandra Viktorova (KU Leuven)

Title Lagrangian Prym fibrations associated to double covers of rational surfaces

Abstract Let S be a K3 surface with an antisymplectic involution i . After fixing a smooth ample curve on the quotient S/i , one can construct the corresponding relative Prym variety P . By the work of Arbarello, Saccà and Ferretti, we know that if S/i is a general Enriques surface, then P is an irreducible symplectic variety. Inspired by this result, we investigate the situation when S/i is a rational surface and find sufficient conditions to ensure that the normalization of P is an irreducible symplectic variety. This is joint work in progress with E. Brakkee, C. Camere, A. Grossi, L. Pertusi and G. Saccà.

Ryo Yamagishi (University of Bath)

Title The McKay correspondence and moduli of G -constellations

Abstract The McKay correspondence relates the representation theory for a finite group G with the geometry of a resolution of a quotient singularity by G . One natural way to see this correspondence is to consider a moduli space of G -constellations, which is a generalized notion of the G -Hilbert scheme. In this talk I will give an overview of this theory and explain my results about it.