Japanese-European Symposium on Symplectic Varieties and Moduli Spaces – 8th Edition

Short courses

Justin Sawon(University of North Carolina - Chapel Hill)

Title Dual Lagrangian fibrations and equivalences

Abstract In these talks we will consider Lagrangian fibrations on holomorphic symplectic manifolds, their dual fibrations obtained by replacing smooth fibres by dual abelian varieties, and relative Fourier-Mukai equivalences between them. In this first talk we will describe the Beauville-Mukai integrable system on the Hilbert scheme of points on a K3 surface, whose fibres are auto-dual, as well as the Debarre integrable system on the generalize Kummer variety, whose fibres are non-principally polarized (and therefore not auto-dual). In the second talk we will allow twisted versions of these fibrations, i.e., not admitting global sections, and twisted Fourier-Mukai equivalences. We will also describe geometric constructions of dual fibrations of certain Lagrangian fibrations by Prym varieties.

Akira Ishii (Nagoya University)

Title On the McKay correspondence for some reflection groups

Abstract We consider the McKay correspondence for a complex reflection group by using a maximal $\mathbb{Q}$-factorial terminalization of the pair consisting of the quotient variety and the boundary divisor with standard coefficients, which Kawamata used to study GL(3)-McKay correspondence. For a complex reflection group that satisfies some condition, there is a conjectural semiorthogonal decomposition of the G-equivariant derived category by Polishchuk and Van den Bergh indexed by the conjugacy classes in G. In dimension two, the conjecture follows from a theorem of Kawamata. In this talk, we recall some basics of the derived McKay correspondence, explain the idea of Kawamata, and then describe the cases for some reflection groups in dimension three.

Abstracts

Emma Brakkee (Leiden University)

Title Bounding Brauer groups of K3 surfaces using moduli spaces

Abstract The transcendental Brauer group of a K3 surface over a number field is finite. It was shown by Cadoret-Charles that the size of its p-primary torsion is uniformly bounded for K3 surfaces in one-dimensional families. We give a different proof of this result for one-dimensional families of K3 surfaces with a polarization by a fixed lattice. To be precise, we construct moduli spaces of K3 surfaces with a lattice polarization and a Brauer class, and use the geometry of their complex points to prove boundedness of Brauer groups for the K3 surfaces they parametrize. I will explain the construction and give a sketch of the proof of our boundedness result. This is joint work in progress with D. Bragg and T. Várilly-Alvarado.

Atsushi Ito (Okayama University)

Title Review of some recent results on linear systems on abelian varieties

Abstract M-regularity of coherent sheaves introduced by Pareschi and Popa is very useful in studying linear systems on abelian varieties. Recently, Jiang and Pareschi extend the notion of M-regularity to $\mathbb{Q}$-twisted sheaves. In this talk, I will explain some recent results on linear systems on abelian varieties obtained by using M-regularity of $\mathbb{Q}$-twisted sheaves.

Ljudmila Kamenova (Stony Brook University)

Title On dominability of hyperkähler manifolds

Abstract An n-dimensional complex manifold M is dominable by $\mathbb{C}^n$ if there is a holomorphic map $F : \mathbb{C}^n \to M$ such that the Jacobian determinant of F is not identically zero. In this talk we are going to discuss some cases when a hyperkaehler manifold is dominable by the complex n-space $\mathbb{C}^n$. We are also going to mention connections with Brody non-hyperbolicity. This work is joint with Steven Lu.

Ryushi Goto (Osaka University)

Title Holomorphic Poisson structures and generalized Kähler structures

Abstract We discuss unobstructed deformations of holomorphic Poisson manifolds in terms of Generalized Kähler geometry. We introduce the notion of scalar curvature of generalized Käher manilds derived from the spin geometry. We show that scalar curvature arises as a moment map of generalized Hamiltonian actions on certain generalized Kähler manifolds, which is a generalization of the result of the scalar curvature as a moment map in the ordinary Kähler geometry, due to Fujiki and Donaldson.

Annalisa Grossi (Bologna University)

Title ISV's via relative Prym varieties

Abstract In this talk I will exhibit how to construct examples of irreducible symplectic varieties via relative Prym varieties associated to a K3 surface with an anti-symplectic involution. This construction has been studied before for the anti-canonical linear system on low degree del Pezzo surfaces, and for Enriques surfaces. We generalize this by considering arbitrary surfaces with a K3 double cover. I will explain the construction and discuss numerical criteria for when the relative Prym variety is a primitive symplectic variety or an irreducible symplectic variety. This is joint work with E. Brakkee, C. Camere, L. Pertusi, G. Saccà and A. Viktorova.

Wahei Hara (IPMU)

Title Partial crepant resolution of surface ADE singularities and spherical twists.

Abstract n this talk we discuss the classification of spherical objects in the derived category of a partial crepant resolution of a surface ADE singularity. The notion of spherical object is an algebra-geometric analog of Lagrangian submanifolds in symplectic geometry, and associates autoequivalences of the derived category. The structure sheaf of a (-2)-curve

on a smooth surface is an example of spherical objects. If one considers singular situations like partial crepant resolutions, slightly more general objects than spherical objects also provide autoequivalences. The case of partial crepant resolutions are related to 3-fold Gorenstein terminal flops. The aim of this talk is to explain the classification theorem of (generalised) spherical objects that works in the minimal and a partial crepant resolutions of ADE singularities and 3-fold flopping contractions. This is all joint work with Michael Wemyss.

Francesco Denisi (Jussieu)

Title MMP for Enriques pairs and singular Enriques varieties.

Abstract An Enriques manifold is a connected complex manifold that is not simply connected and whose universal covering is an irreducible holomorphic symplectic (IHS) manifold. Given a projective IHS manifold X and an effective R-divisor D such that the pair (X,D) is log canonical, Matsushita and Zhang showed that any sequence of flops starting from (X,D) terminates. Lehn and Pacienza proved, more generally, that any MMP starting from (X,D) terminates. The goal of this talk is twofold. First, we discuss an analogous result for Enriques pairs, which we define as log canonical pairs (Y, D), where Y is an Enriques manifold and D is an effective R-divisor. Second, we characterize the underlying variety of the resulting minimal model (Y', D') of (Y,D). This leads naturally to the definition of primitive Enriques varieties, for which we provide examples and explore some of their properties. The talk is based on joint work with Á. D. Ríos Ortiz, N. Tsakanikas and Z. Xie.

Kohei Kikuta (Osaka University)

Title Geometrical finiteness for automorphism groups via cone conjecture

Abstract Geometrical finiteness is one of the central notions in the study of Kleinian groups. In this talk, we explain the geometrical finiteness for the natural isometric actions of automorphism groups on the hyperbolic spaces for K3 surfaces, Enriques surfaces, Coble surfaces, and irreducible symplectic varieties. Then the cone conjecture is a key to the proof. If time permits, some applications for K3 surfaces will be discussed.

Enrico Fatighenti (Bologna University)

Title Examples of modular vector bundles with and without moduli

Abstract We exhibit examples of slope-stable and modular vector bundles on a hyperkähler manifold of $K3^{[2]}$-type. These are obtained by performing standard linear algebra constructions on the examples studied by O'Grady of (rigid) modular bundles on the Fano varieties of lines of a general cubic 4-fold and the Debarre-Voisin hyperkähler. Interestingly enough, these constructions are almost never infinitesimally rigid, and more precisely we show how to get (infinitely many) 20 and 40 dimensional families. This is a joint work with Claudio Onorati.

Yuya Matsumoto (Tokyo University of Science)

Title Inseparable Kummer surfaces

Abstract Kummer surfaces are the minimal resolutions of the quotients of abelian surfaces by the inversion map. They are K3 surfaces in most cases but not for supersingular abelian surfaces in characteristic 2. We propose a certain type of supersingular K3 surfaces, which we call inseparable Kummer surfaces, as a replacement of such non-K3 Kummer surfaces. We are trying to show that, from a family of abelian surfaces, we can construct a family of (possibly inseparable) Kummer K3 surfaces after replacing the base scheme with a suitable modification. Related preprints are https://arxiv.org/abs/2302.09535 and https://arxiv.org/abs/2403.02770 .

Pietro Beri (Université de Lorraine)

Title Cones of divisors for moduli spaces of polarized hyper-Kahler manifolds and uniruledness

Abstract Many informations on hyper-Kähler manifolds can be deduced from the study of the associated period domains, which are orthogonal modular varieties. The Picard group of these latter is in many cases generated by the so-called Noether-Lefschetz divisors. In the last years,

the work of many authors unveiled the relation of these divisors with some vector-valued modular forms with respect to Weil representations.

In this talk, I will present a joint work with I. Barros, L. Flapan and B. Williams, in which we give a general formula for generators of the NL-cone, the cone of effective linear combinations of Noether-Lefschetz divisors. This is done by bounding explicitly the growth of the coefficients of the relevant vector-valued modular forms.

Building from that, we obtain new uniruledness results for moduli spaces of polarized manifolds of OG6-type in low degree as well as for some moduli spaces of polarized manifolds of generalized Kummer type.

Genki Ouchi (Nagoya University)

Title Spectra of derived categories of elliptic curves and $K3$ surfaces

Abstract Matsui introduced the notion of spectra of triangulated categories as ringed spaces. Since spectra of derived categories of smooth projective varieties contain all Fourier-Mukai partners, it is interesting to describe them. In this talk, we study spectra of derived categories of coherent sheaves on elliptic curves and K3 surfaces in terms of moduli spaces of stable sheaves on them. This talk is based on the joint work with Yuki Hirano.