

EXPERIMENTAL FUNCTION ESTIMATION FROM QUANTUM PHASE MEASUREMENTS

DR. ILARIA GIANANI

QSPRING SEMINAR - 20/04/2021



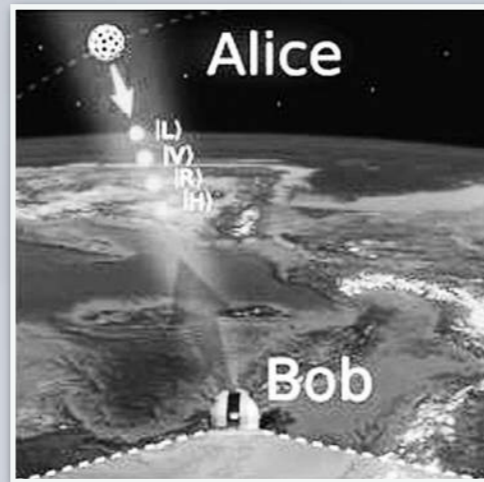
OVERVIEW



QUANTUM COMPUTERS

TRAPPED IONS AND SUPERCONDUCTING CIRCUITS

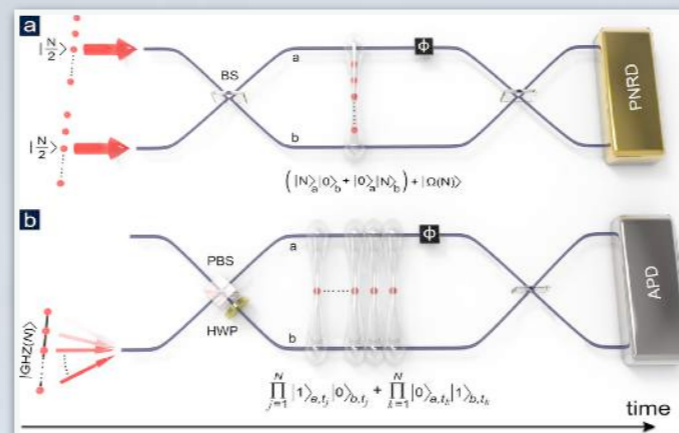
QUANTUM SIMULATORS



QUANTUM COMMUNICATIONS

QUANTUM-SAFE CRYPTOGRAPHY

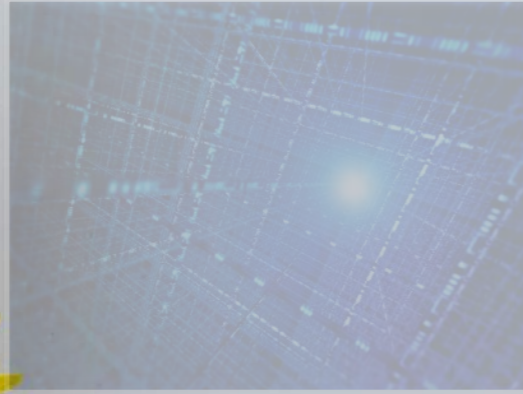
QUANTUM REPEATERS



QUANTUM SENSING

PARAMETER ESTIMATION

OVERVIEW



QUANTUM COMPUTERS

TRAPPED IONS AND SUPERCONDUCTING CIRCUITS

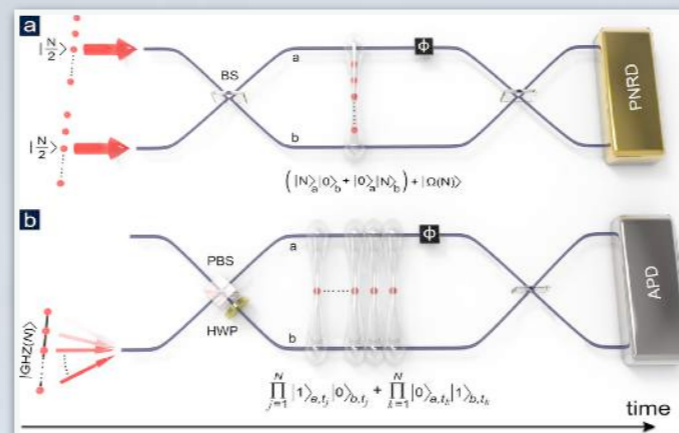
QUANTUM SIMULATORS



QUANTUM COMMUNICATIONS

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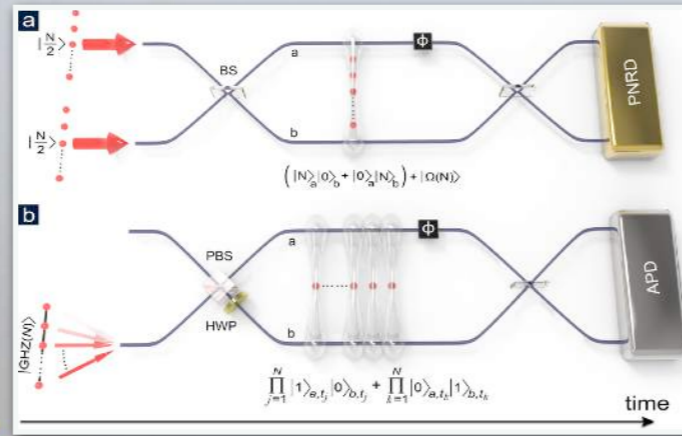
QUANTUM REPEATERS



QUANTUM SENSING

PARAMETER ESTIMATION

OVERVIEW

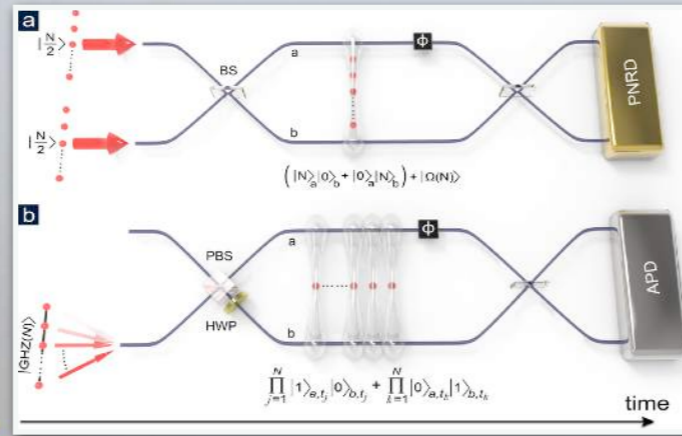


QUANTUM SENSING

PARAMETER ESTIMATION



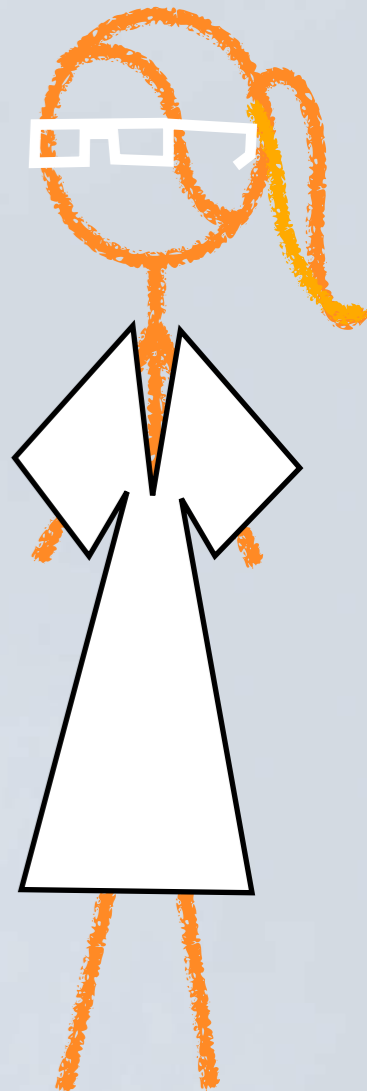
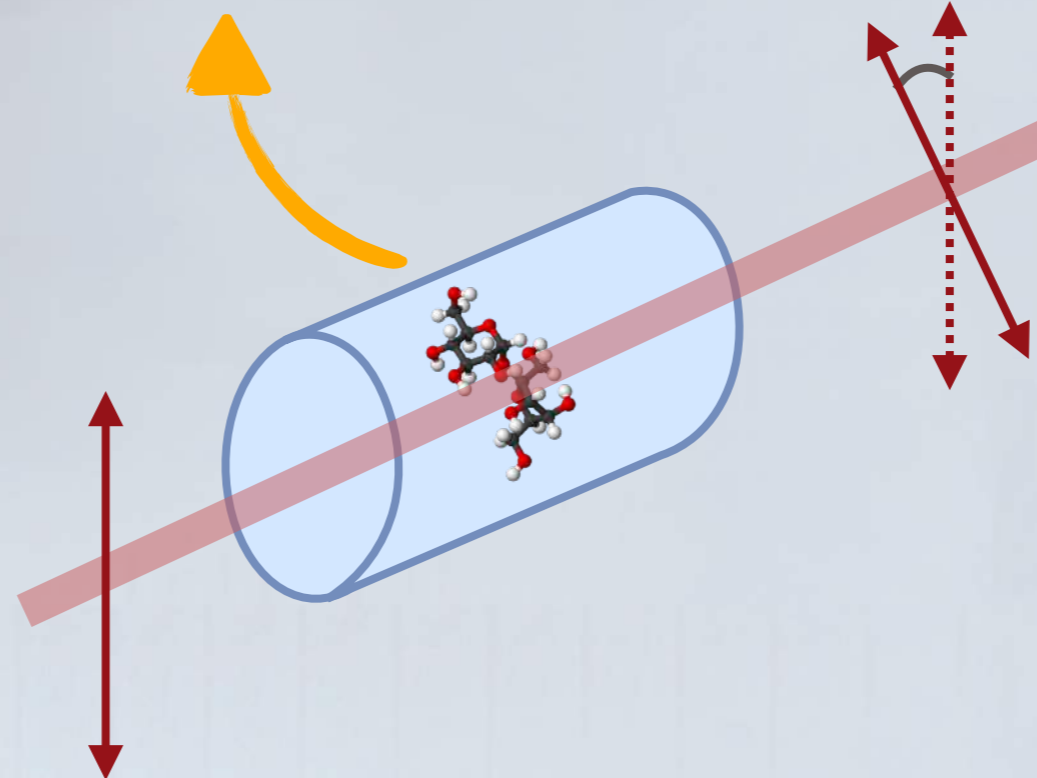
OVERVIEW



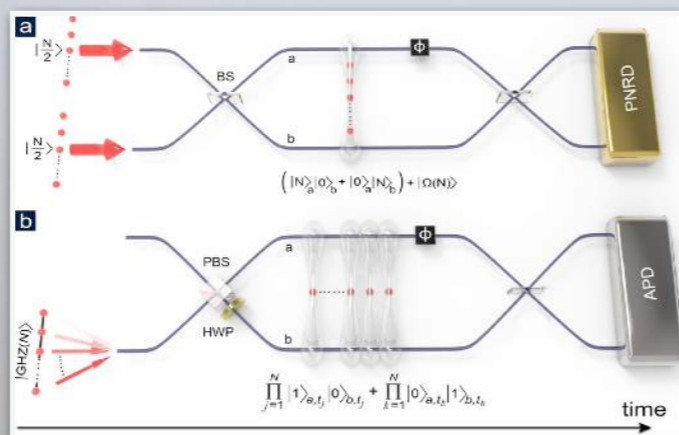
QUANTUM SENSING

PARAMETER ESTIMATION

CHIRAL STUFF



OVERVIEW



QUANTUM SENSING

PARAMETER ESTIMATION

COOL CAN I BUY IT ON AMAZON?

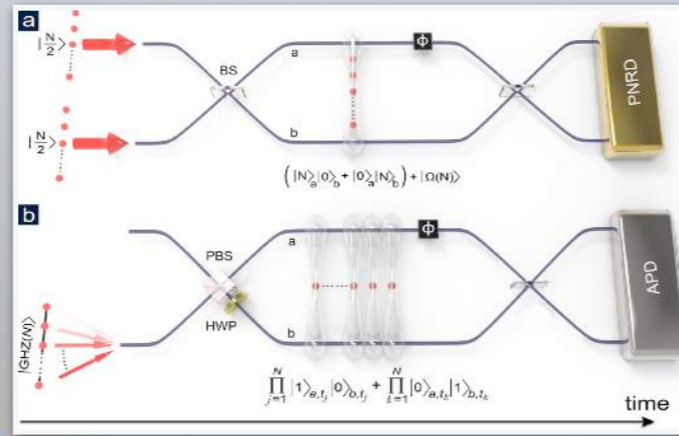
Amazon.co.uk search results for "quantum sensor". The search bar shows "quantum sensor" and the results list includes:

- Quantani Enzyme Tracker**
More buying choices
£662.61 (1 new offer)
- QInvertaser**
More buying choices
£1,054.57
- QuanTech BioModule**

Filters on the left include Amazon Prime, Department (Physics, Quantum Physics), Avg. Customer Review, Book Language, and Book Format.



OVERVIEW



QUANTUM SENSING

PARAMETER ESTIMATION

amazon.co.uk
quantum sensor

1-16 of 187 results for "quantum sensor"

Amazon Prime
 Prime
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Department
PC & Video Games
PC Gaming Mice
Books
Popular Science Physics
Engineering & Technology
Quantum Physics
Physical States of Matter
Physics
[See more](#)
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Avg. Customer Review
★★★★☆ & Up
★★★★☆ & Up
★★★★☆ & Up
★★★★☆ & Up

Book Language
 English

Book Format

Quantani Enzyme Tracker
More buying choices
£662.61 (1 new offer)

QInvertaser
More buying choices
£1,054.57

QuanTech BioModule

TECHNOLOGY

(SOURCE, EFFICIENCY, DETECTORS, ETC)

FUNDAMENTALS

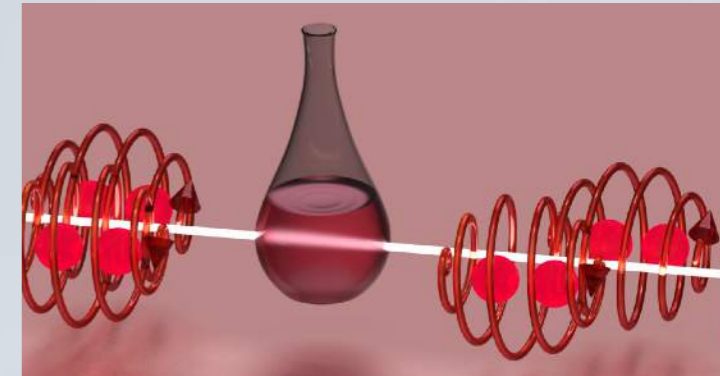
FROM LAB TO REAL OPERATIONS

OUTLINE

MULTIPARAMETER ESTIMATION

PARAMETER ESTIMATION

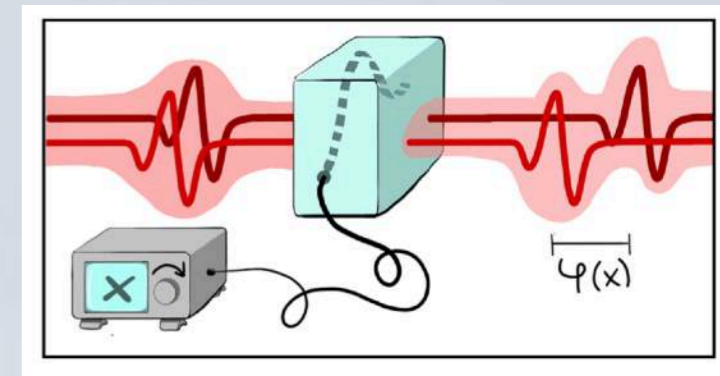
MULTIPARAMETER ESTIMATION OF A CHIRAL SOLUTION



FUNCTION ESTIMATION

ESTIMATION OF THE RESPONSE FUNCTION OF A LIQUID CRYSTAL

SIMULATIONS



PARAMETER ESTIMATION

A vertical yellow ruler is placed on the left side of the slide, showing measurements from 1 to 21 centimeters. To the left of the ruler is a bouquet of flowers, including large yellow daisies, smaller white and yellow daisies, and blue and white roses. A single yellow daisy is also placed on the surface to the right of the ruler.

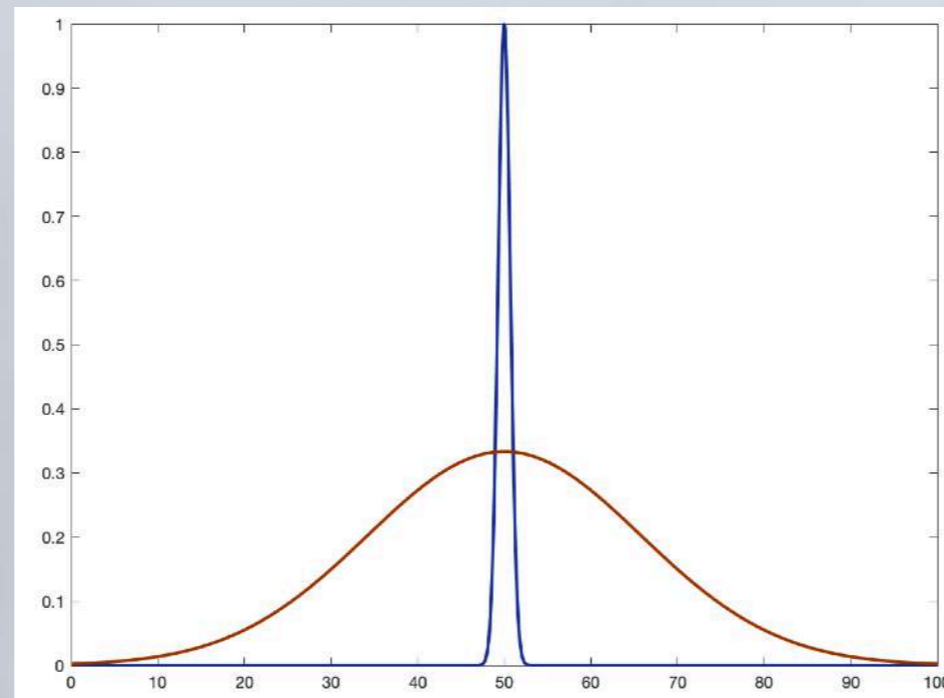
PROBE PREPARATION

PROBE INTERACTION
WITH SYSTEM

PROBE MEASUREMENT

PARAMETER ESTIMATION

PROBABILITY DISTRIBUTION $P(K|\lambda)$



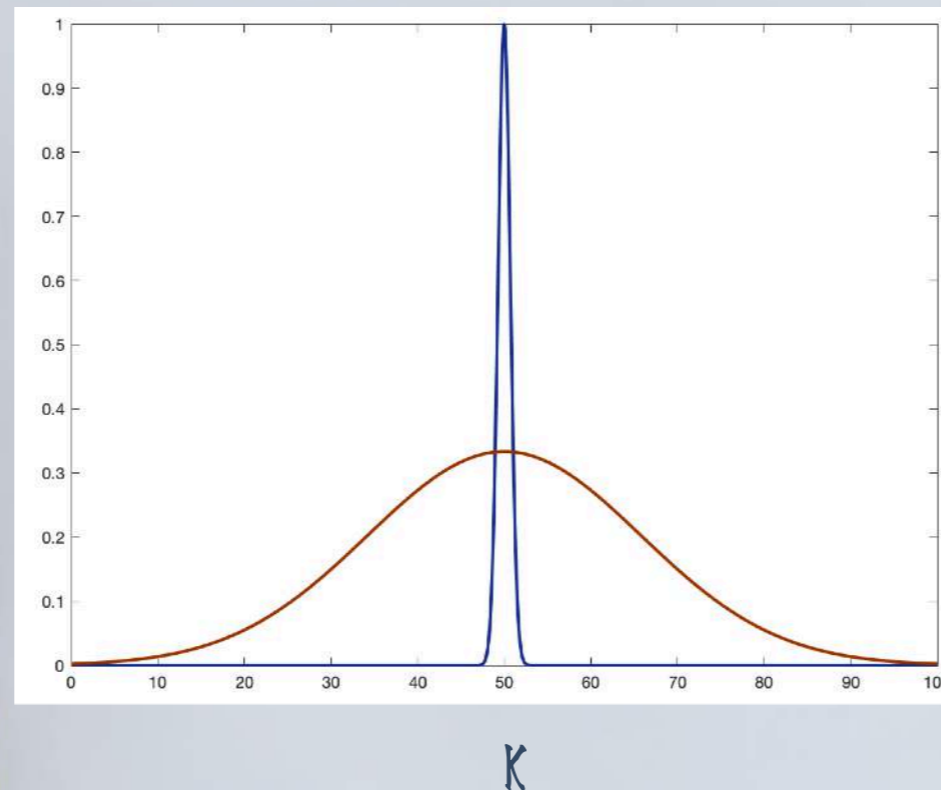
K

λ PARAMETER
K MEASUREMENT OUTCOME

$\chi = k_1, k_2 \dots k_M$
 $\hat{\lambda}(\chi)$ ESTIMATOR

PARAMETER ESTIMATION

PROBABILITY DISTRIBUTION $P(K|\lambda)$



λ PARAMETER
K MEASUREMENT OUTCOME

$\chi = k_1, k_2 \dots k_M$
 $\hat{\lambda}(\chi)$ ESTIMATOR

FISHER INFORMATION

$$\mathcal{F}[p(k|\lambda)] = \sum_k p(k|\lambda) (\partial_\lambda \log p(k|\lambda))^2$$

CRAMER-RAO BOUND (CRB)

$$\mathbb{E} [(\hat{\lambda} - \lambda)^2] \geq \frac{1}{M\mathcal{F}[p(k|\lambda)]}$$

PARAMETER ESTIMATION

MEASUREMENTS

$$\chi = k_1, k_2 \dots k_M$$

ESTIMATOR

$$\hat{\lambda}(\chi)$$



???

BAYESIAN ESTIMATOR

$$p(\chi|\lambda) = \prod_{j=1}^M p(k_j|\lambda)$$

$$P(\lambda|\chi) = \frac{p(\chi|\lambda)p(\lambda)}{p(\chi)}$$

A-POSTERIORI PROBABILITY

PARAMETER ESTIMATION

MEASUREMENTS

$$\chi = k_1, k_2 \dots k_M$$

ESTIMATOR

$$\hat{\lambda}(\chi)$$



BAYESIAN ESTIMATOR

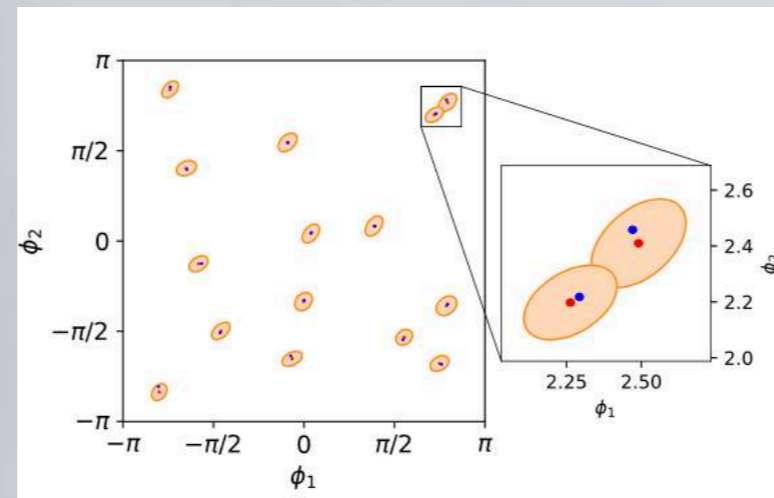
$$\hat{\lambda}_B(\chi) = \int \lambda P(\lambda|\chi) d\lambda$$

$$Var(\lambda) = \int (\lambda - \hat{\lambda}_B(\chi))^2 P(\lambda|\chi) d\lambda$$

MULTIPARAMETER ESTIMATION

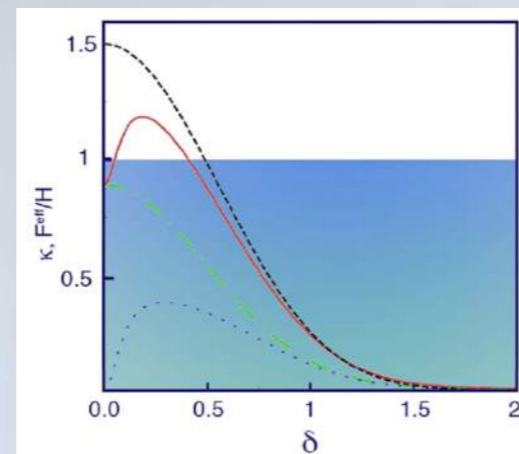


INTERESTING PARAMETERS MULTIPLE PHASES



M VALERI ET AL, NPJQI 6,96 (2020)

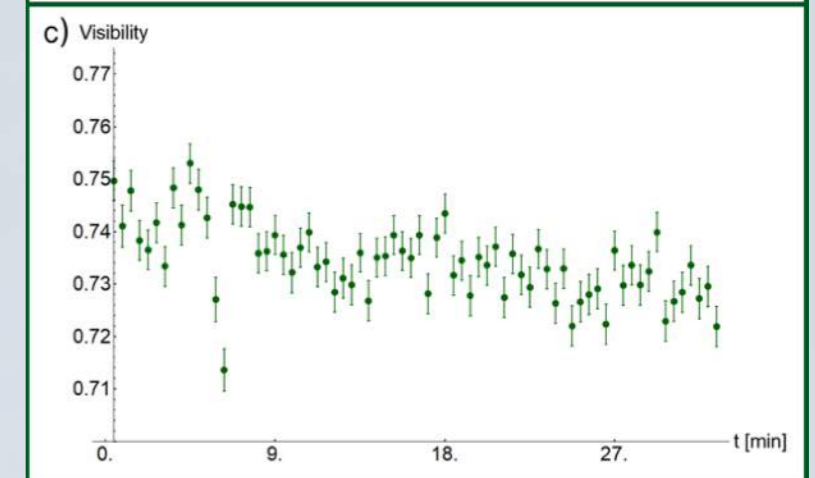
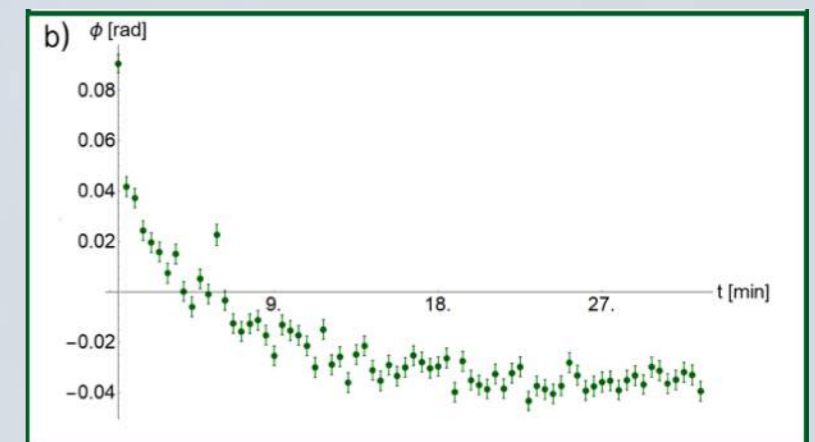
PHASE - DEPHASING



E. ROCCIA ET AL, QST 3 (2018)

M. VIDRIGHIN ET AL, NAT COM 5, 3532 (2014)

NUISANCE PARAMETERS THINGS WE DO NOT CARE ABOUT BUT IF WE DON'T THEY AFFECT OUR ESTIMATION



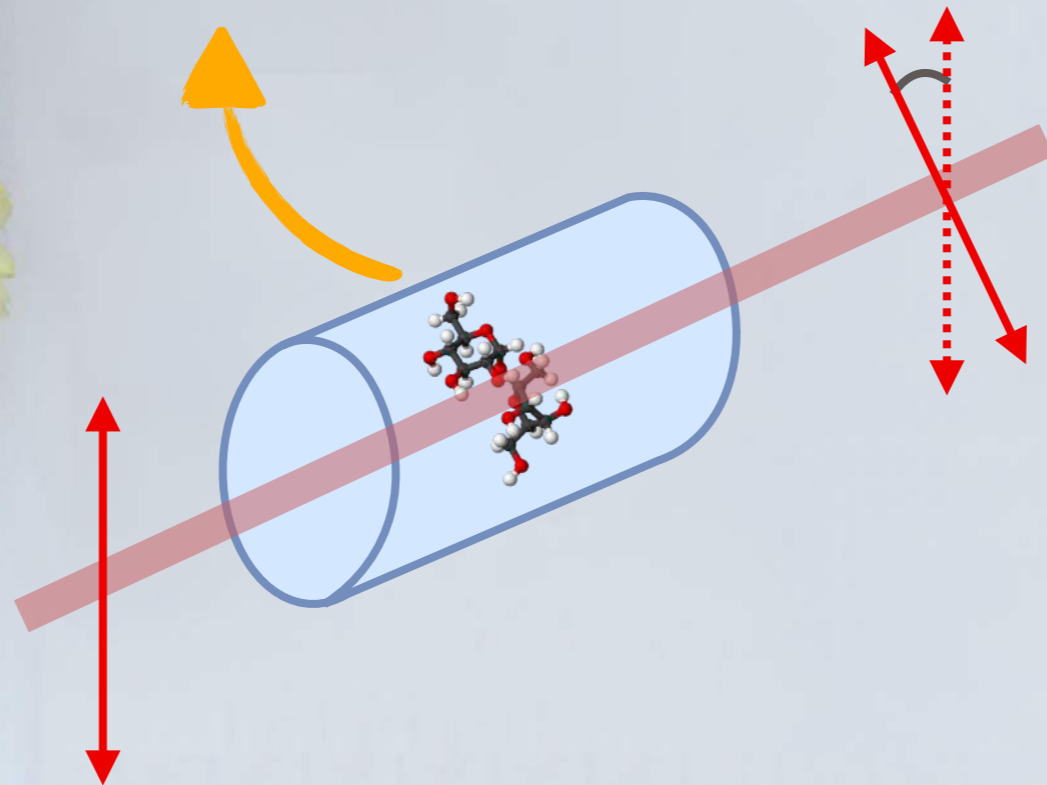
E. ROCCIA ET AL, OPTICA 5,10 (2018)

V. CIMINI ET AL, OP. EX., 27, 24 (2019)

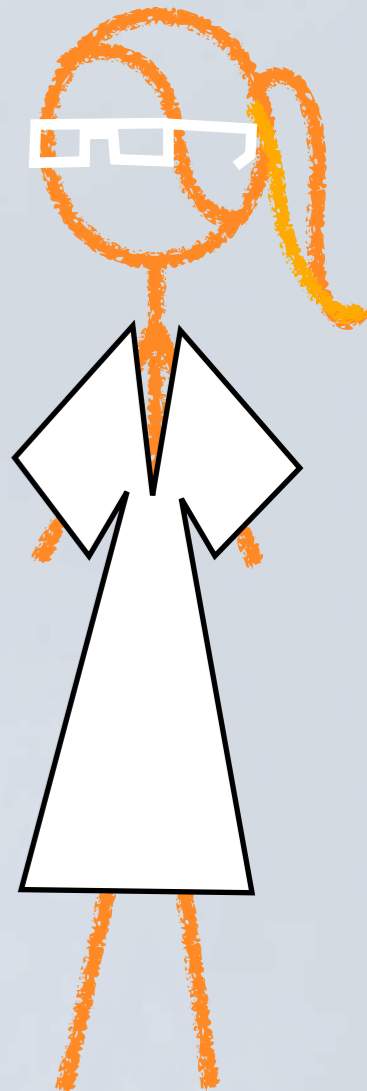
MULTIPARAMETER ESTIMATION

NUISANCE PARAMETERS

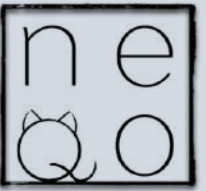
CHIRAL STUFF
PHASE BETWEEN
CIRCULAR POLARIZATION



$\phi?$

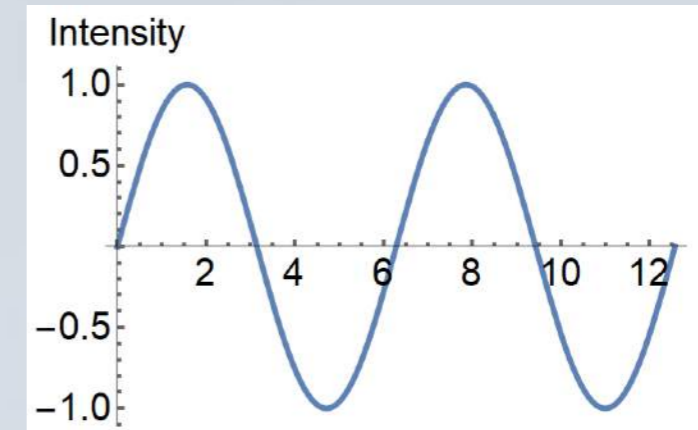
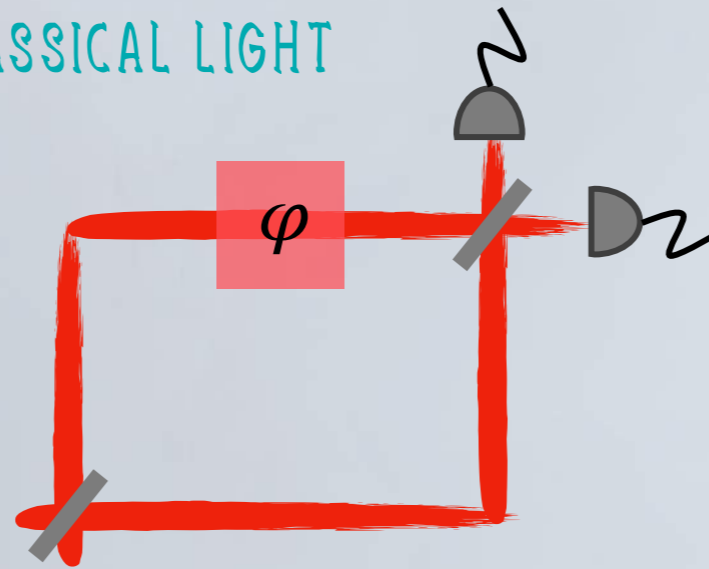


MULTIPARAMETER ESTIMATION



NUISANCE PARAMETERS

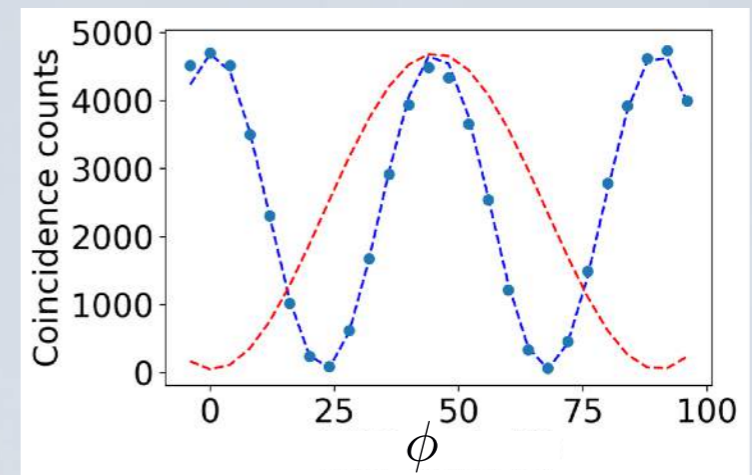
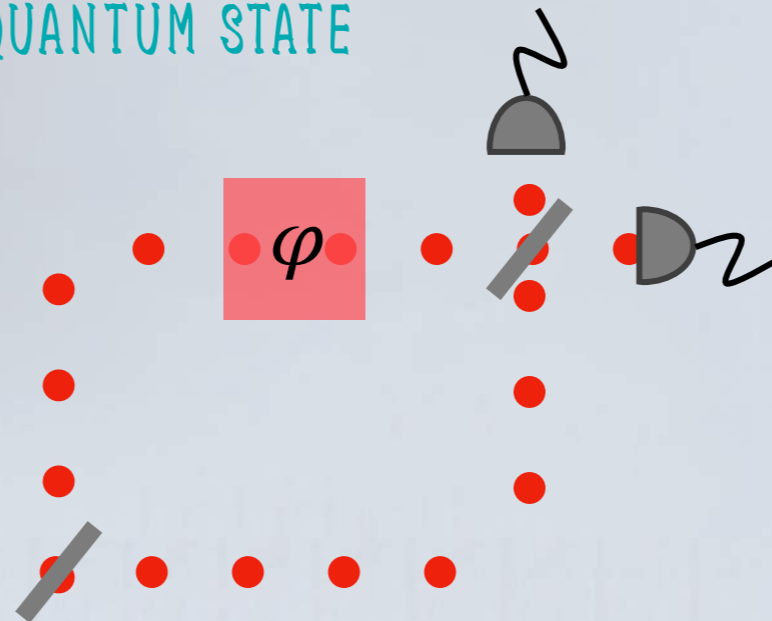
CLASSICAL LIGHT



SHOT NOISE LIMIT

$$\Delta \varphi \geq \frac{1}{\sqrt{N}}$$

QUANTUM STATE

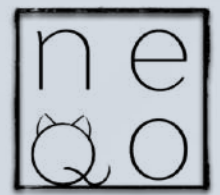


HEISENBERG LIMIT

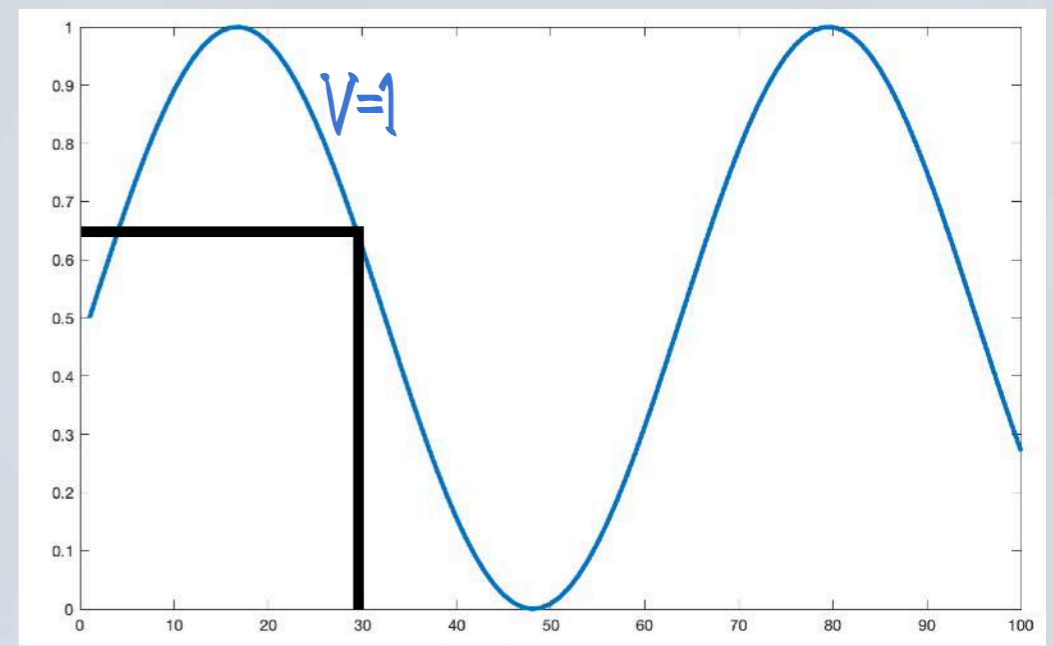
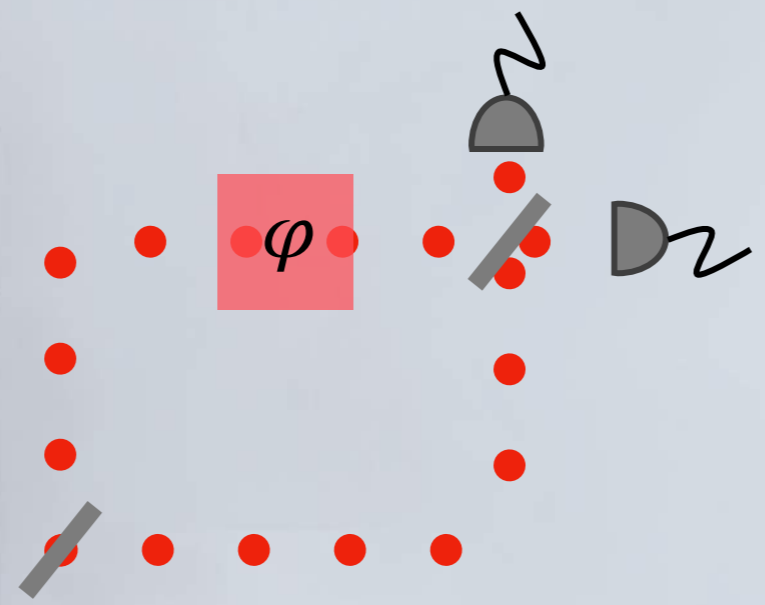
$$\Delta \varphi \geq \frac{1}{N}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|N0\rangle + |0N\rangle)$$

MULTIPARAMETER ESTIMATION



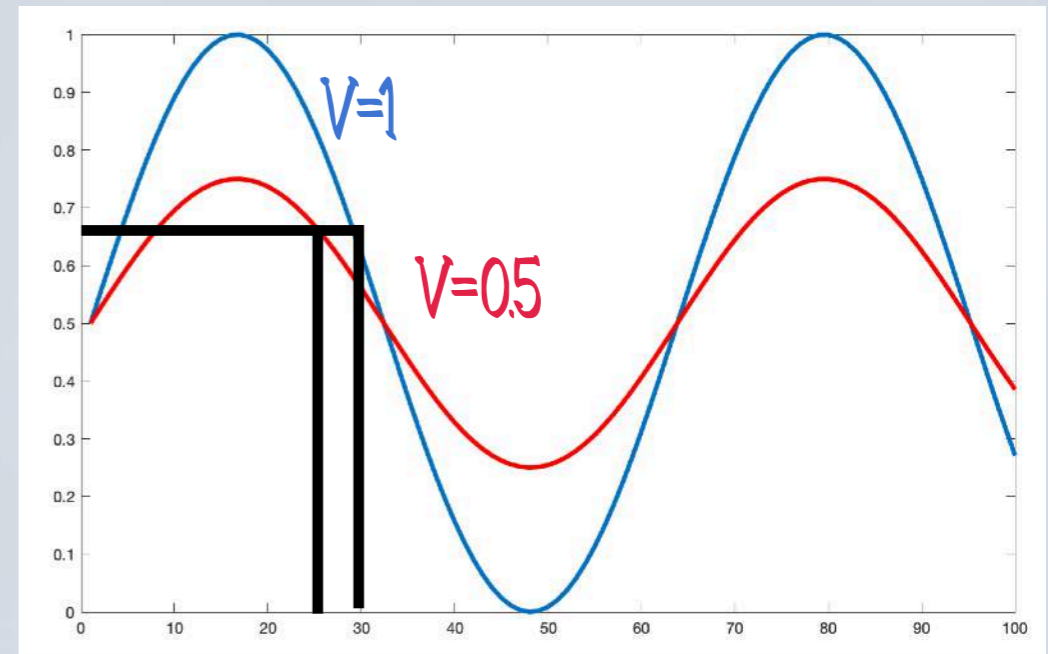
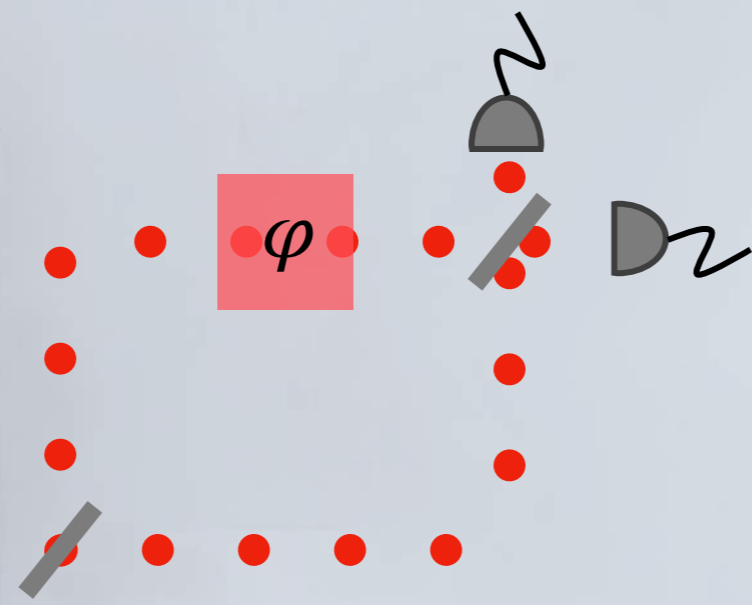
NUISANCE PARAMETERS



MULTIPARAMETER ESTIMATION

NUISANCE PARAMETERS

BIAS!

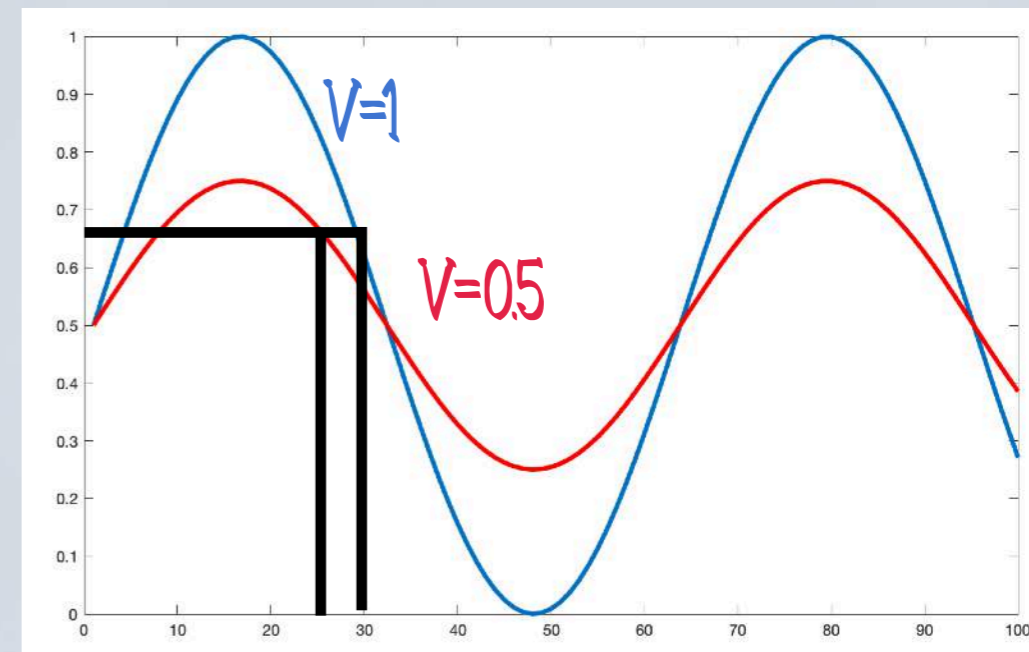
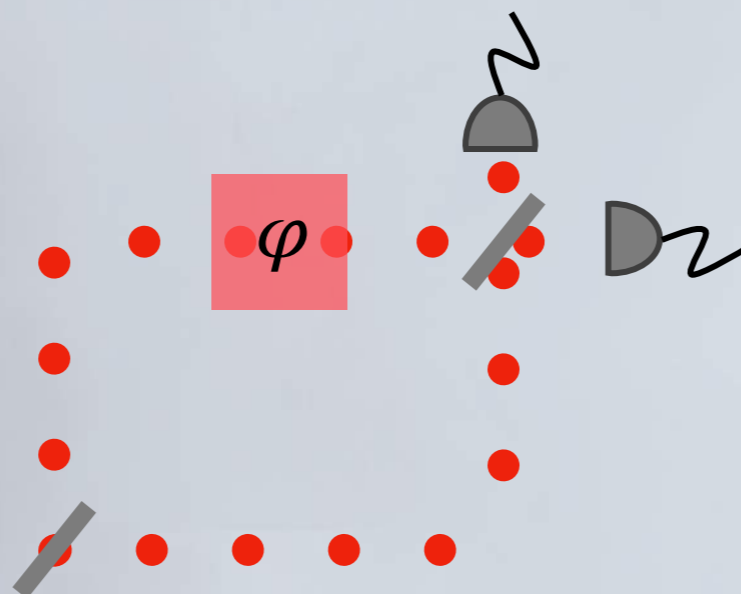


MEASURE PHASE AND VISIBILITY AT THE SAME TIME

MULTIPARAMETER ESTIMATION

NUISANCE PARAMETERS

BIAS!



MEASURE PHASE AND VISIBILITY AT THE SAME TIME

FISHER INFORMATION

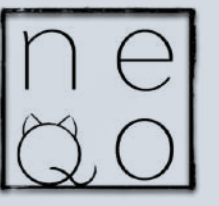
$$\mathcal{F}_{\mu\nu} = \sum_k p(k|\varphi, v) (\partial_\mu \log p(k|\varphi, v)) (\partial_\nu \log p(k|\varphi, v))$$

CRAMER-RAO BOUND (CRB)

$$\text{Var}[\varphi] \geq \frac{\mathcal{F}_{\varphi, \varphi}^{-1}}{M}$$

$$\text{Var}[v] \geq \frac{\mathcal{F}_{v, v}^{-1}}{M}$$

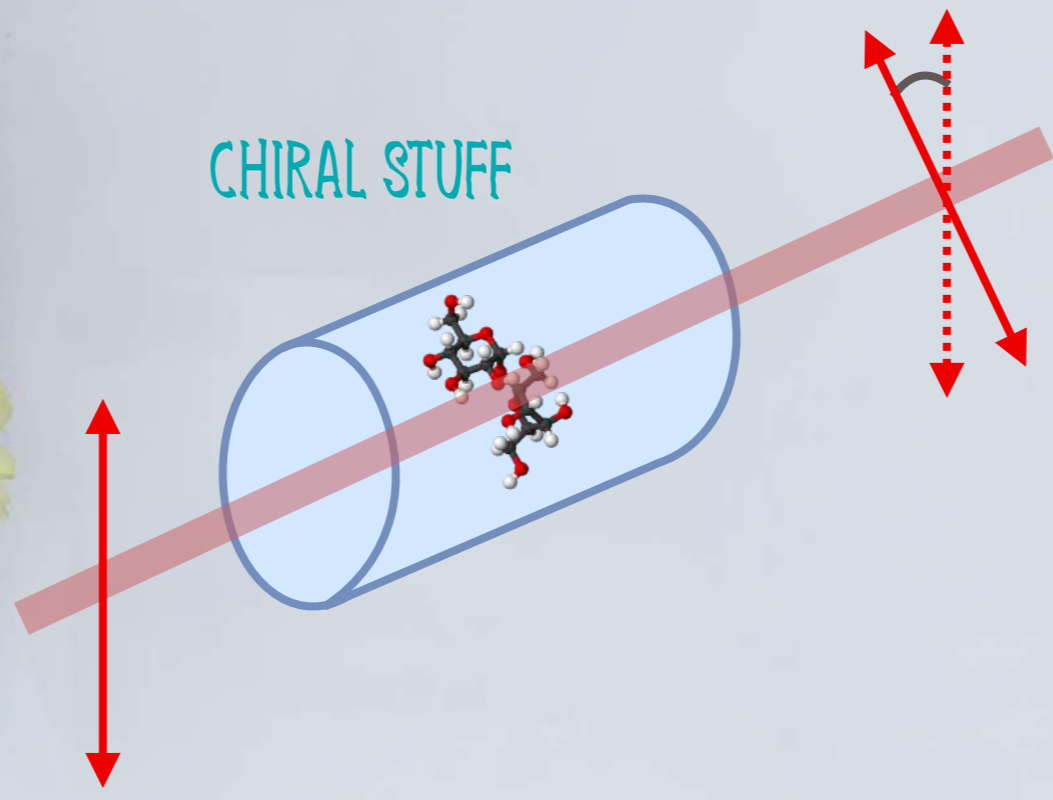
MULTIPARAMETER ESTIMATION



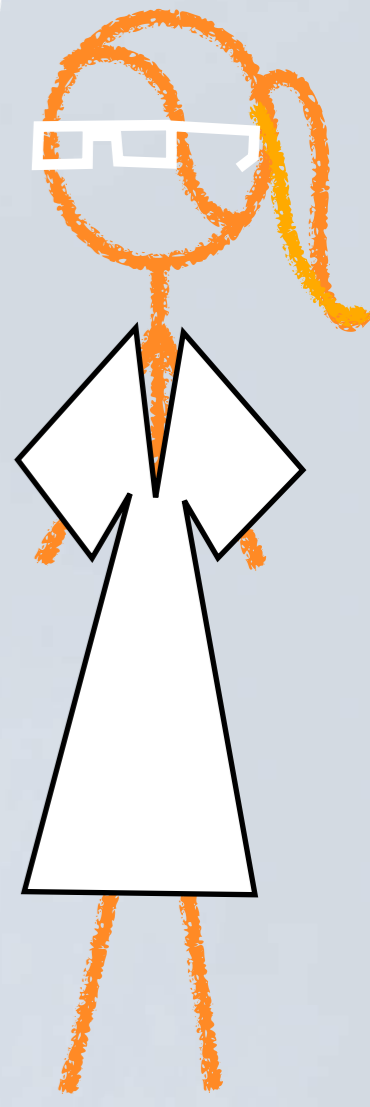
NUISANCE PARAMETERS



CHIRAL STUFF



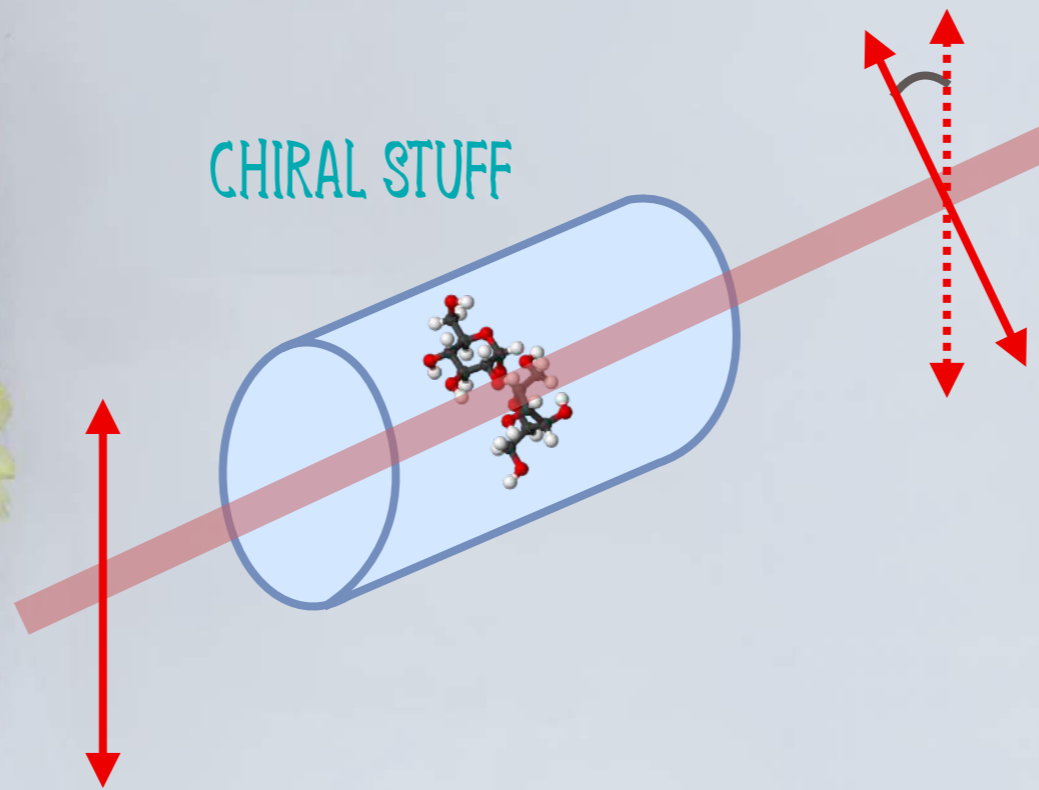
$\phi?$



MULTIPARAMETER ESTIMATION

NUISANCE PARAMETERS

POLARIZATION INTERFEROMETER

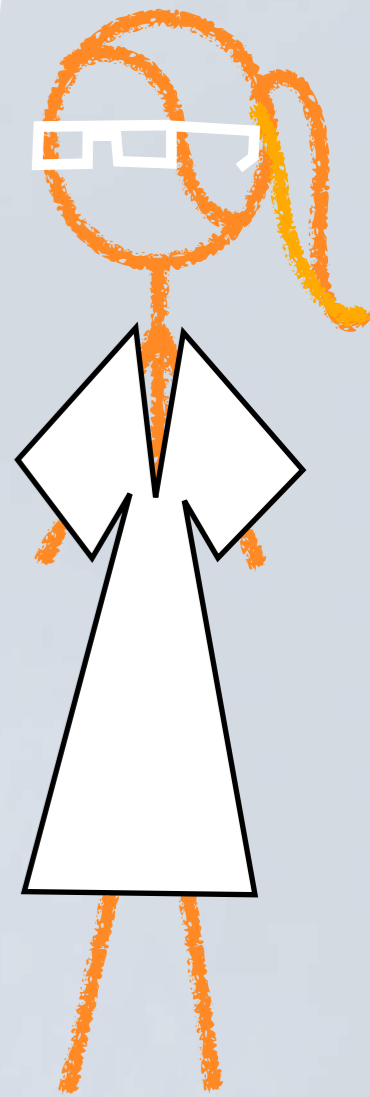


PREPARE
POL R+L

ADD A PHASE BETWEEN R
AND L

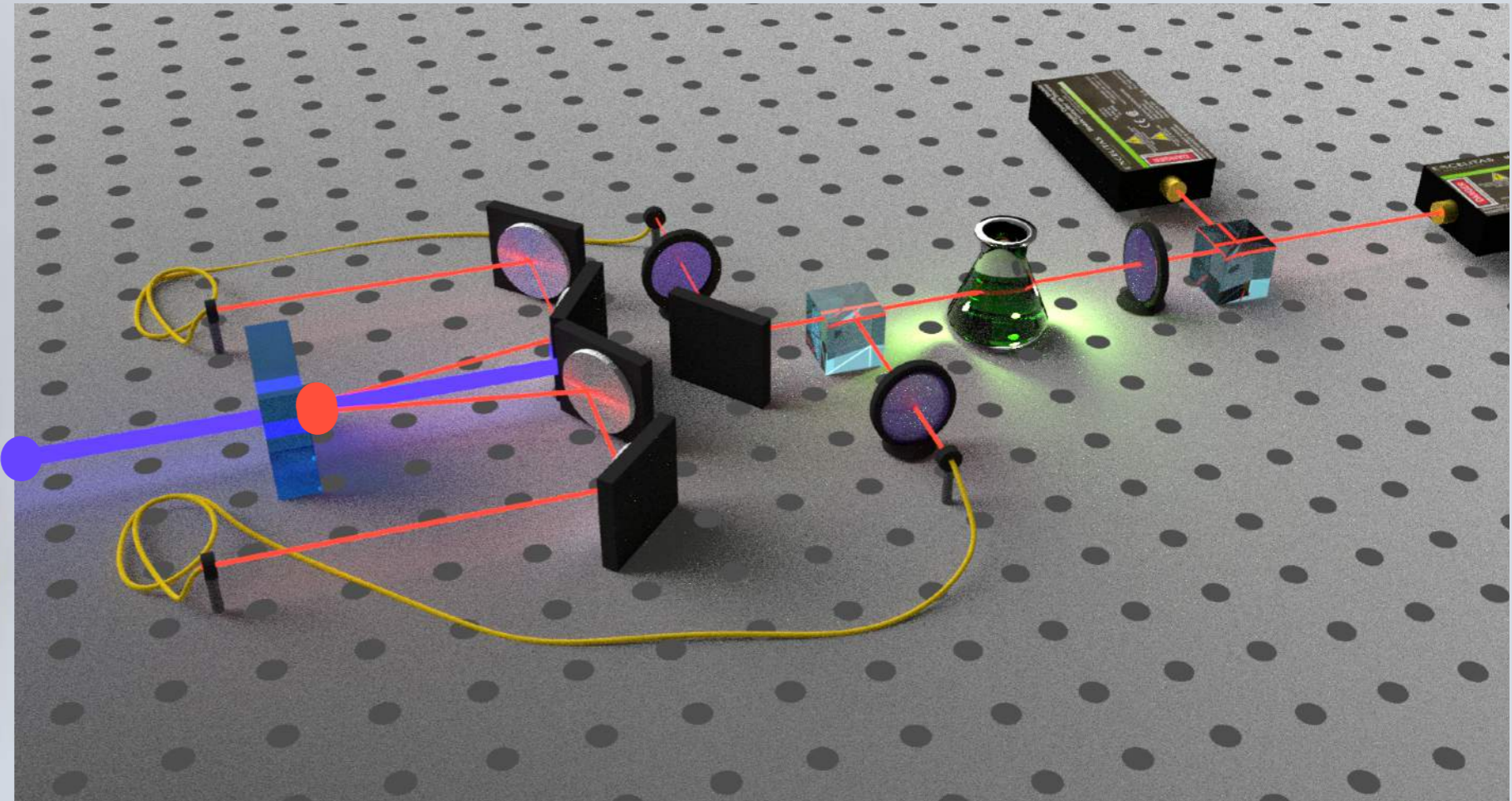
MEASURE POL

$\phi?$



MULTIPARAMETER ESTIMATION

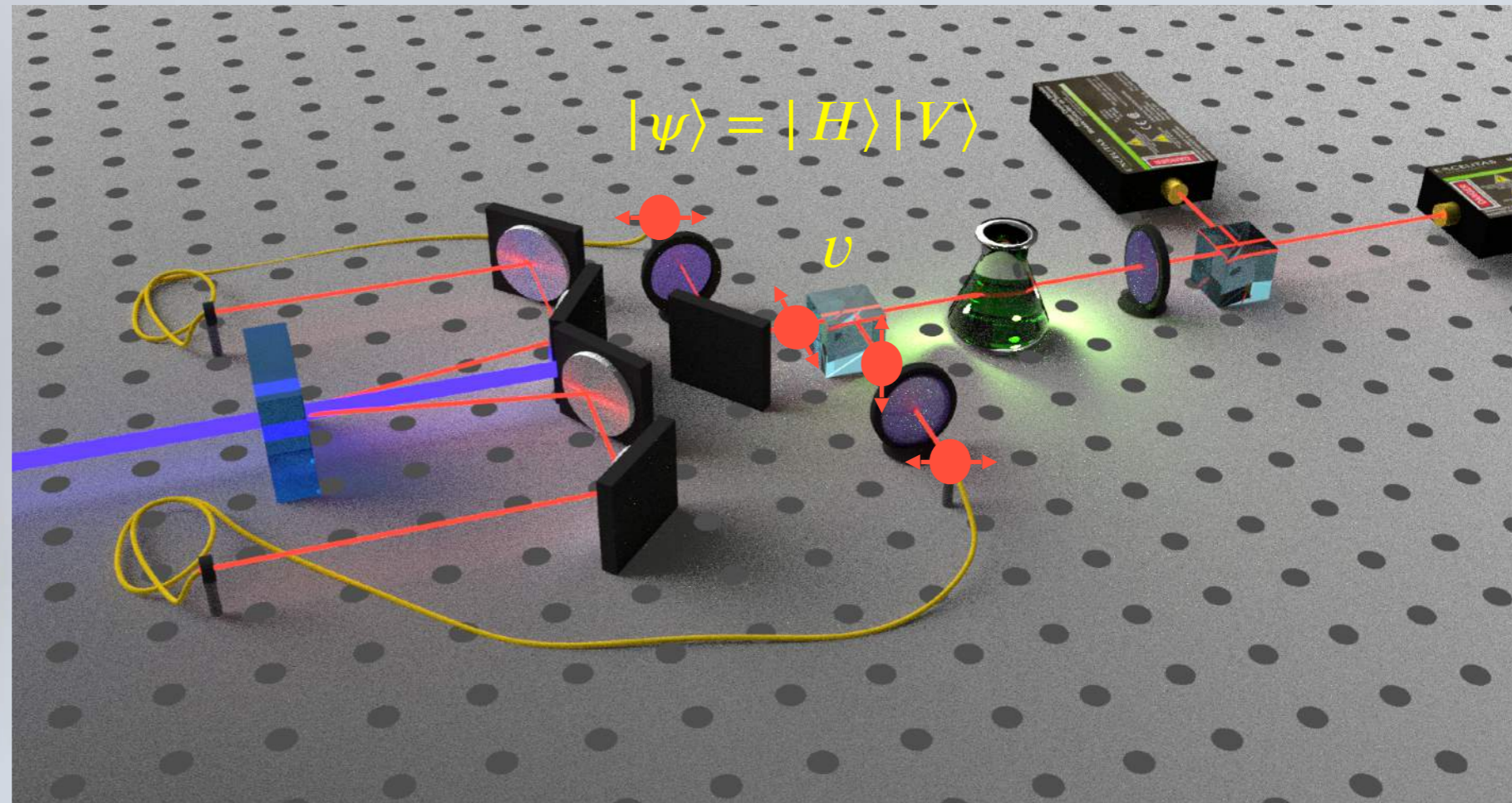
NUISANCE PARAMETERS



PROBE PREPARATION

MULTIPARAMETER ESTIMATION

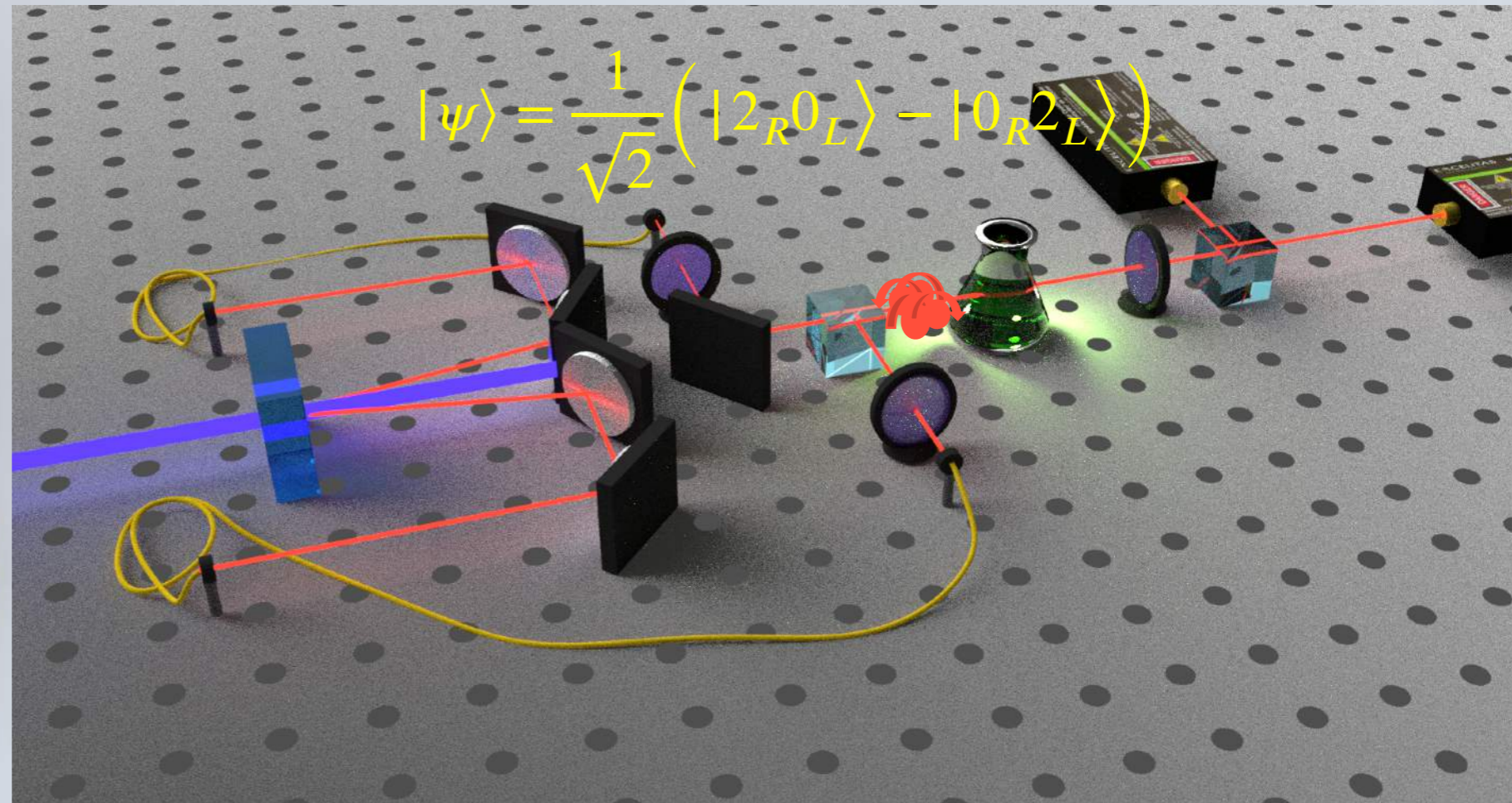
NUISANCE PARAMETERS



PROBE PREPARATION

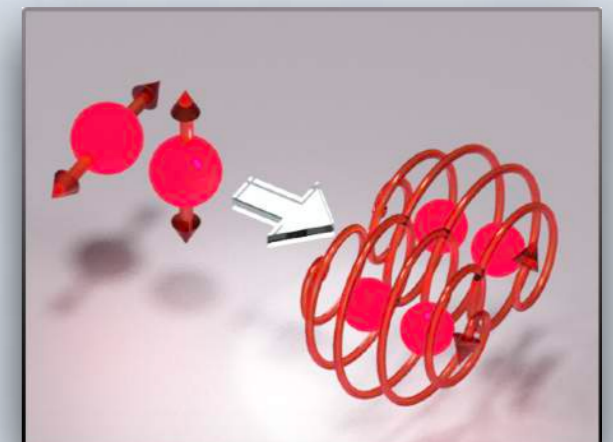
MULTIPARAMETER ESTIMATION

NUISANCE PARAMETERS



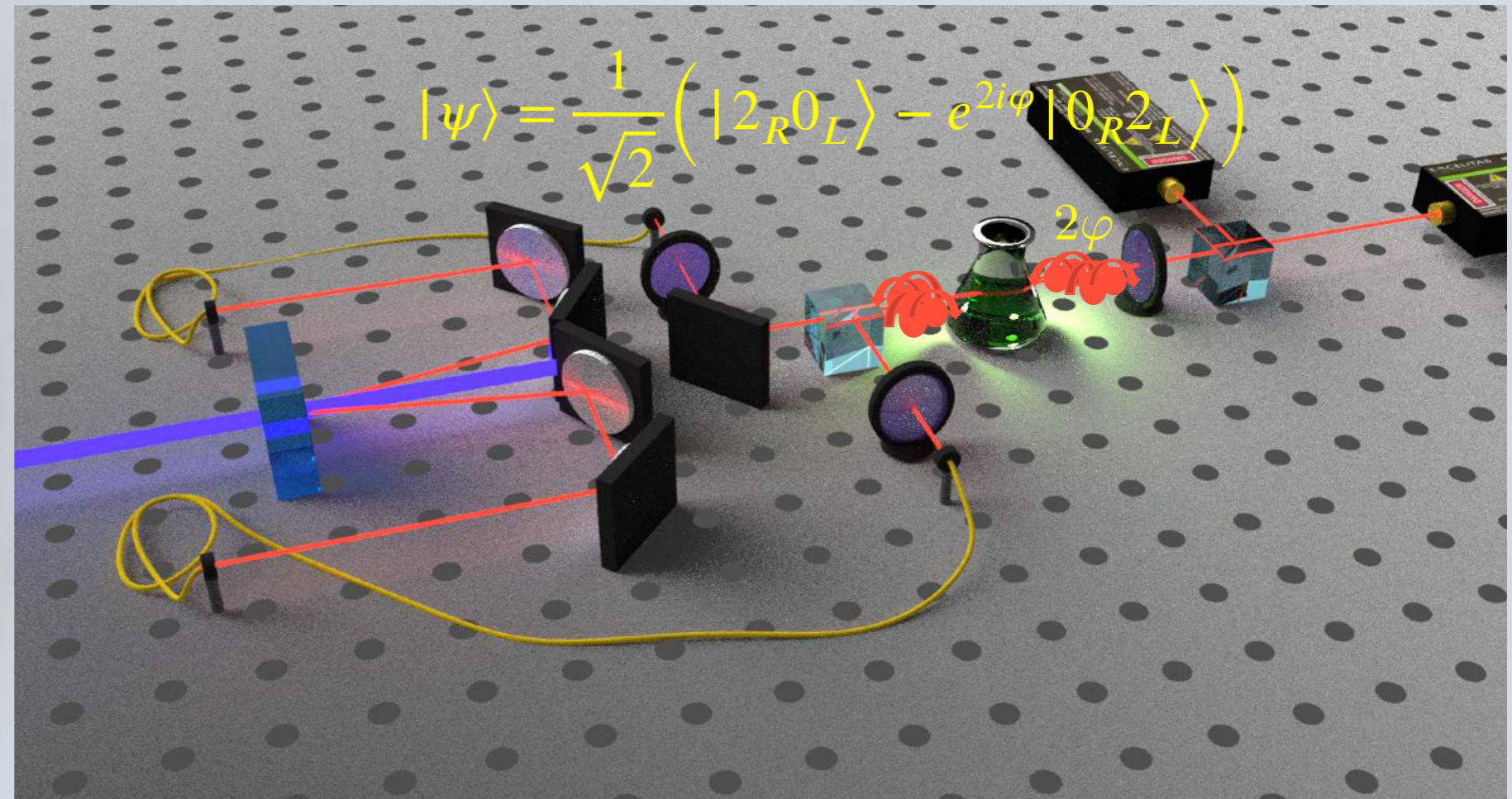
PROBE PREPARATION

NOON STATE IN THE
CIRCULAR
POLARISATION



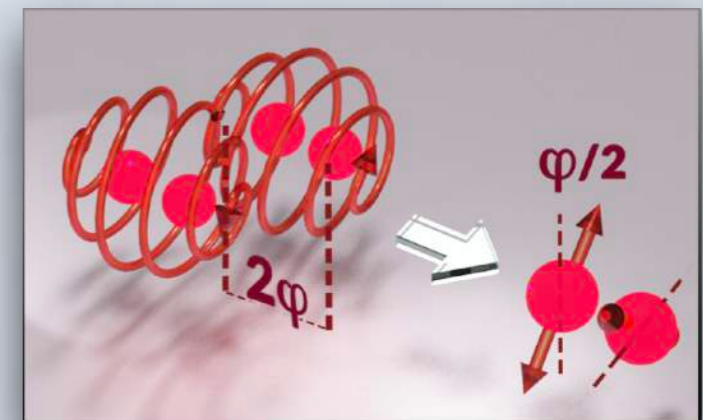
MULTIPARAMETER ESTIMATION

NUISANCE PARAMETERS



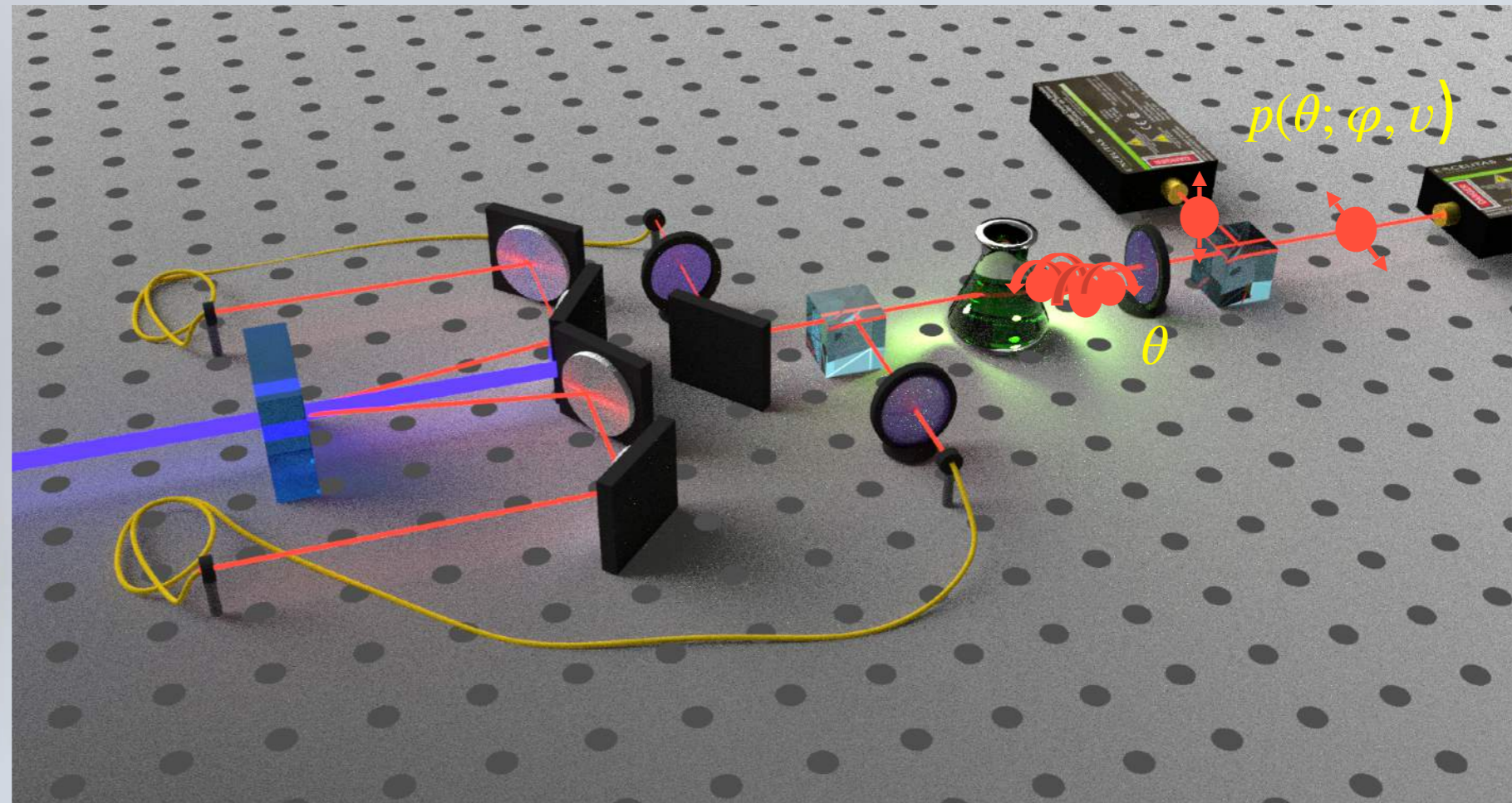
PROBE INTERACTION
WITH SYSTEM

SAMPLE IMPARTS
PHASE BETWEEN R
AND L POLARISATION



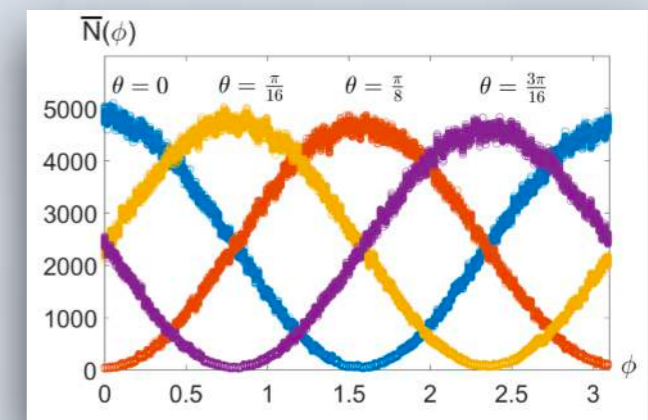
MULTIPARAMETER ESTIMATION

NUISANCE PARAMETERS



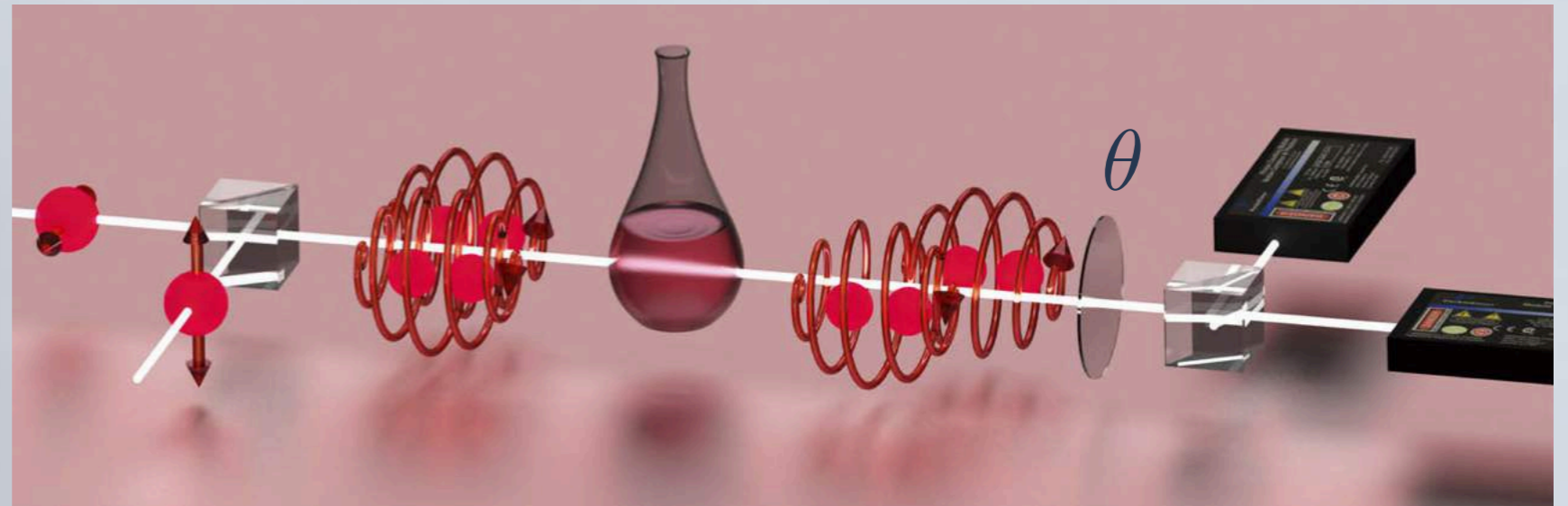
PROBE MEASUREMENT

PERFORM
MEASUREMENTS FOR
DIFFERENT VALUES
OF θ



MULTIPARAMETER ESTIMATION

NUISANCE PARAMETERS



$$p(\theta|\varphi, v) = \frac{1}{4} [1 + v \cos(8\theta + 2\varphi)]$$

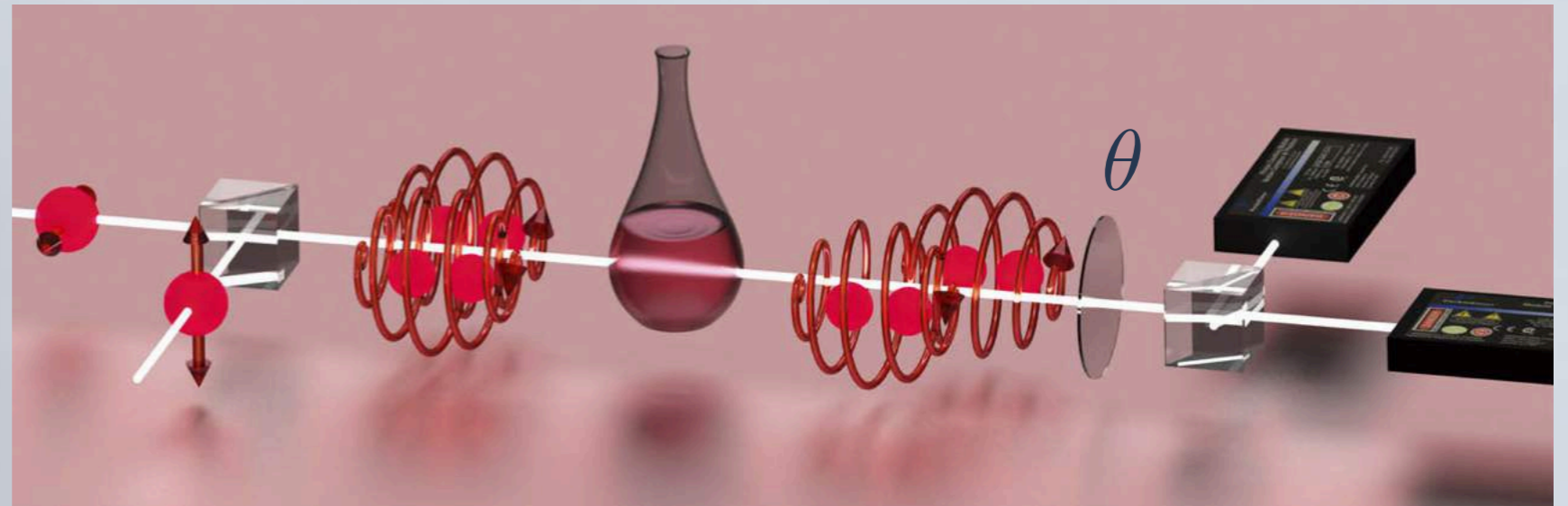
FROM EACH: n_θ

$$P_B(\varphi, v|n) = \mathcal{N} \prod_{\theta} p(\theta|\varphi, v)^{n_\theta} p(\varphi, v)$$

BAYESIAN PROBABILITY

MULTIPARAMETER ESTIMATION

NUISANCE PARAMETERS



$$p(\theta|\varphi, v) = \frac{1}{4} [1 + v \cos(8\theta + 2\varphi)]$$

FROM EACH: n_θ

$$P_B(\varphi, v|n) = \mathcal{N} \prod_{\theta} p(\theta|\varphi, v)^{n_\theta} p(\varphi, v)$$

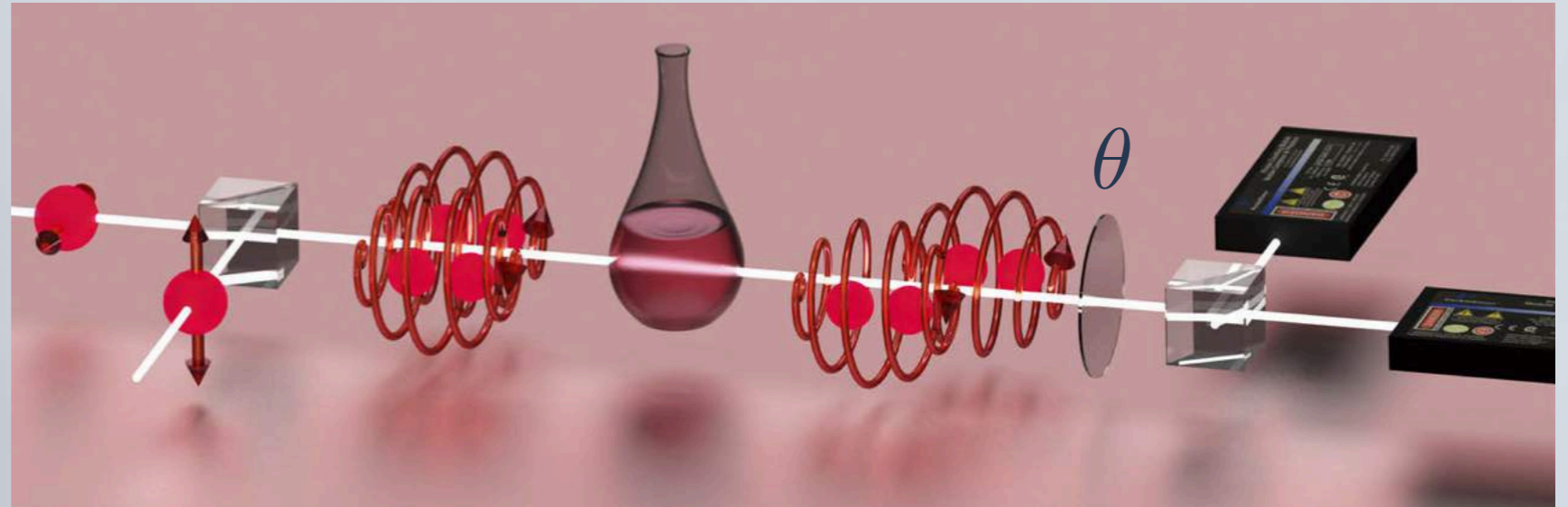
BAYESIAN PROBABILITY

$$\varphi_B = \int \varphi P_B(\varphi, v|n) d\varphi dv \quad \Delta^2 \varphi_B = \int (\varphi - \varphi_B)^2 P_B(\varphi, v|n) d\varphi dv$$

$$v_B = \int v P_B(\varphi, v|n) d\varphi dv \quad \Delta^2 v_B = \int (v - v_B)^2 P_B(\varphi, v|n) d\varphi dv$$

MULTIPARAMETER ESTIMATION

NUISANCE PARAMETERS



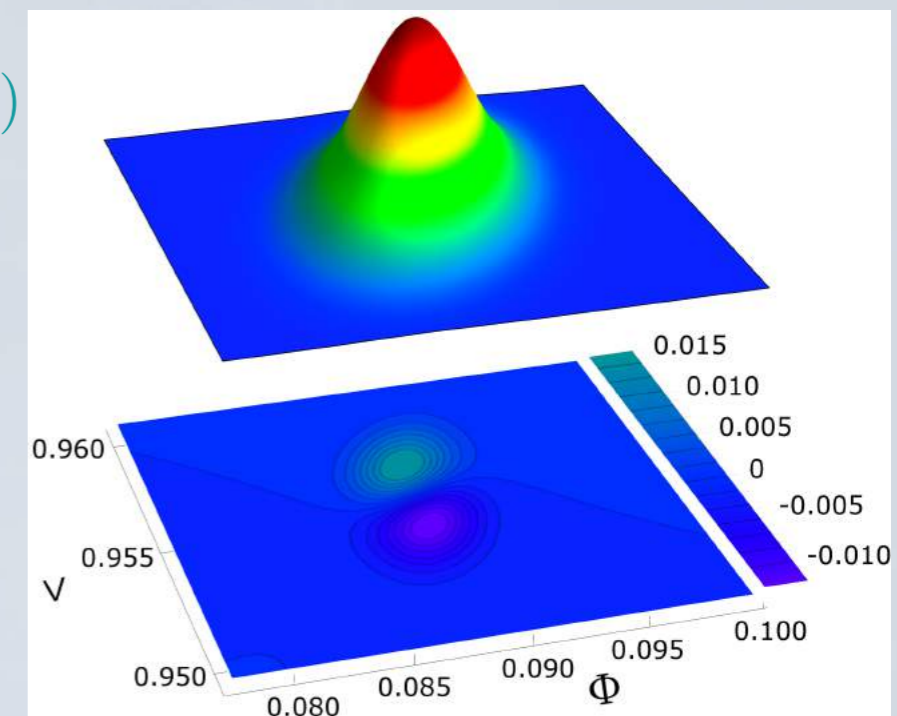
$$p(\theta|\varphi, v) = \frac{1}{4} [1 + v \cos(8\theta + 2\varphi)]$$

$$P_B(\varphi, v|n) = \mathcal{N} \prod_{\theta} p(\theta|\varphi, v)^{n_{\theta}} p(\varphi, v)$$

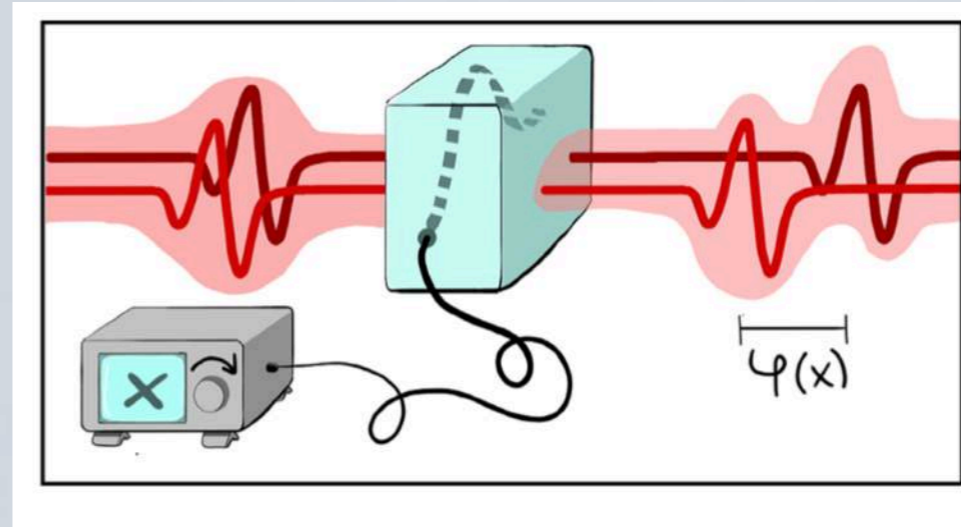
$$\varphi_B = \int \varphi P_B(\varphi, v|n) d\varphi dv$$

$$v_B = \int v P_B(\varphi, v|n) d\varphi dv$$

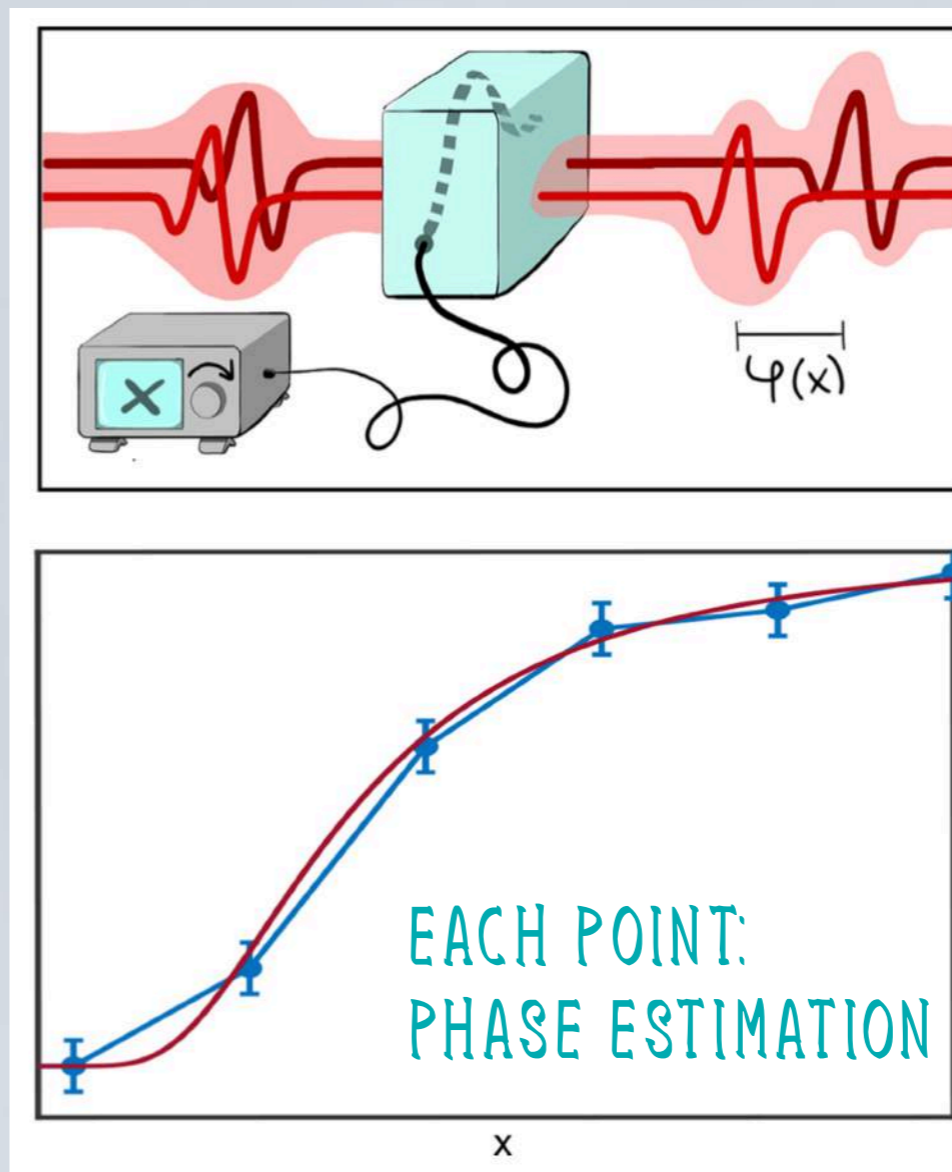
GLUCOSE SOLUTION



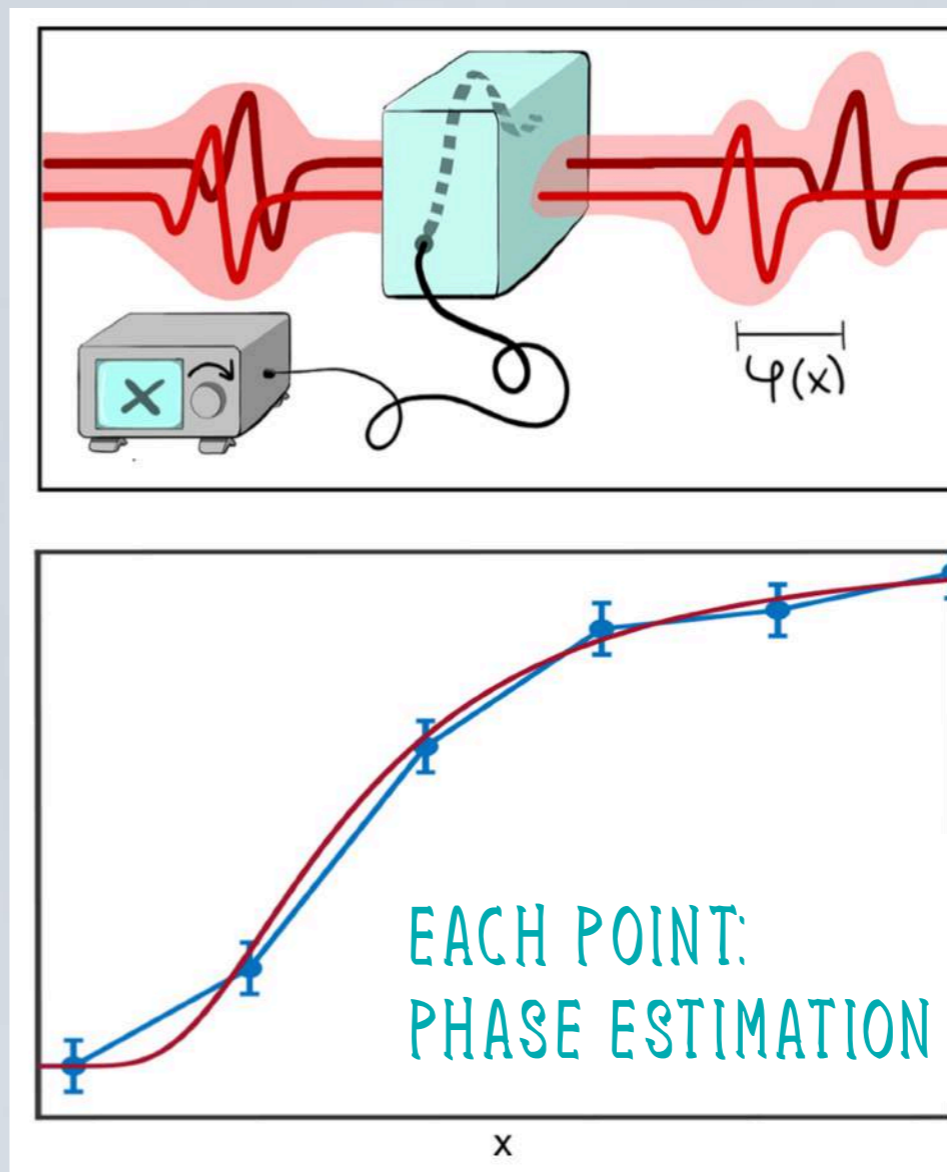
FUNCTION ESTIMATION



FUNCTION ESTIMATION

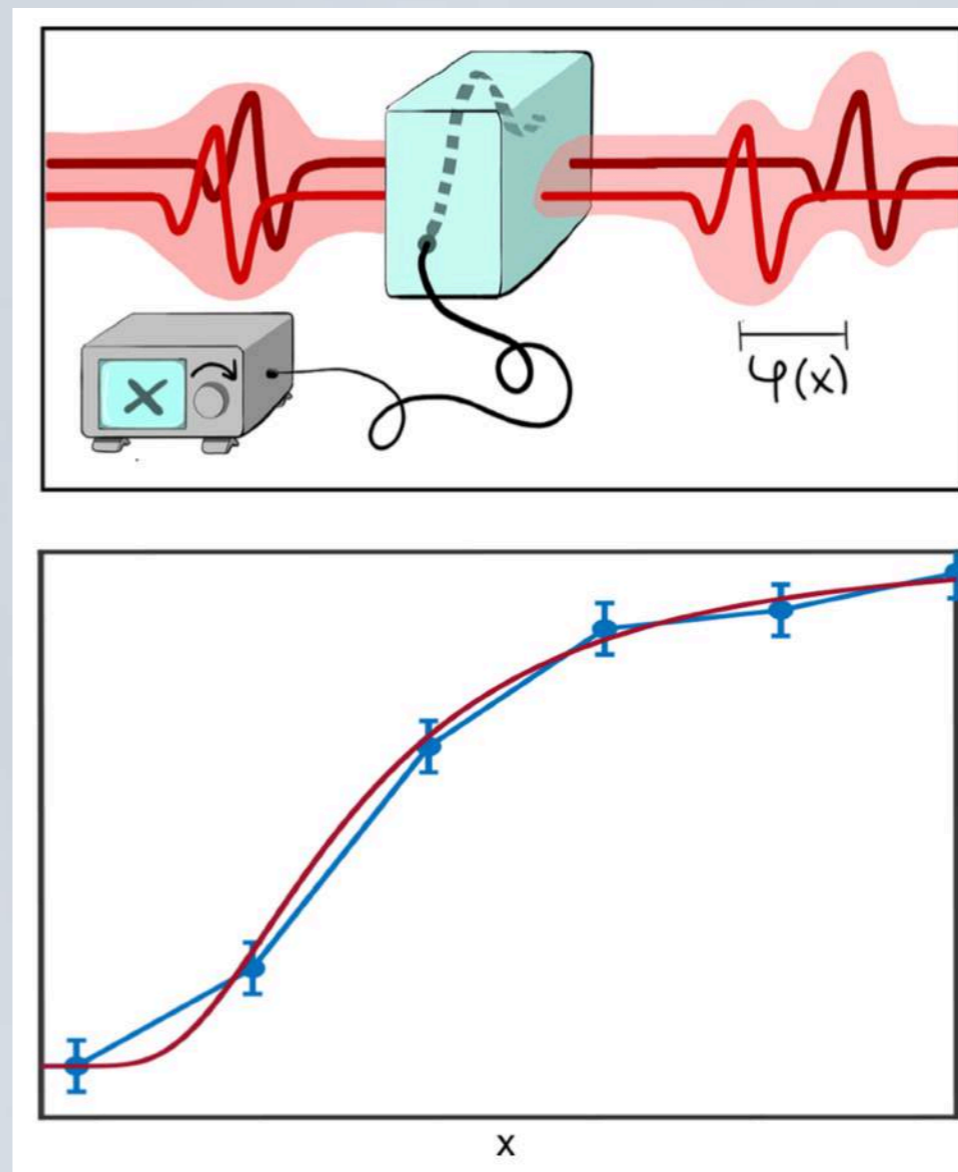


FUNCTION ESTIMATION



$$\delta_0^2 = \frac{1}{L} \mathbb{E} \left[\int |\varphi(x) - \tilde{\varphi}(x)|^2 dx \right]$$

FUNCTION ESTIMATION



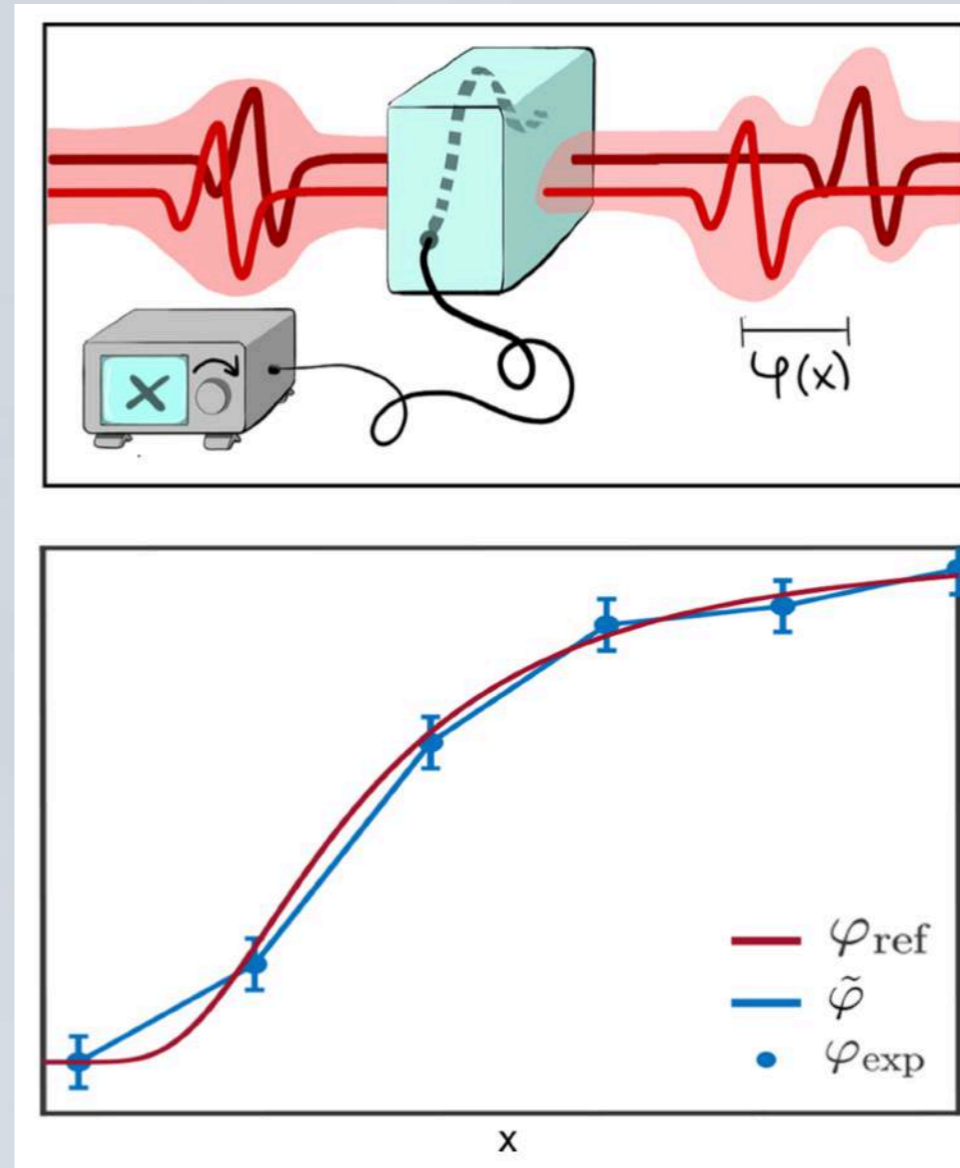
N_R = RES PER POINT

$\rightarrow \delta_0^2$

N_S = # OF POINTS

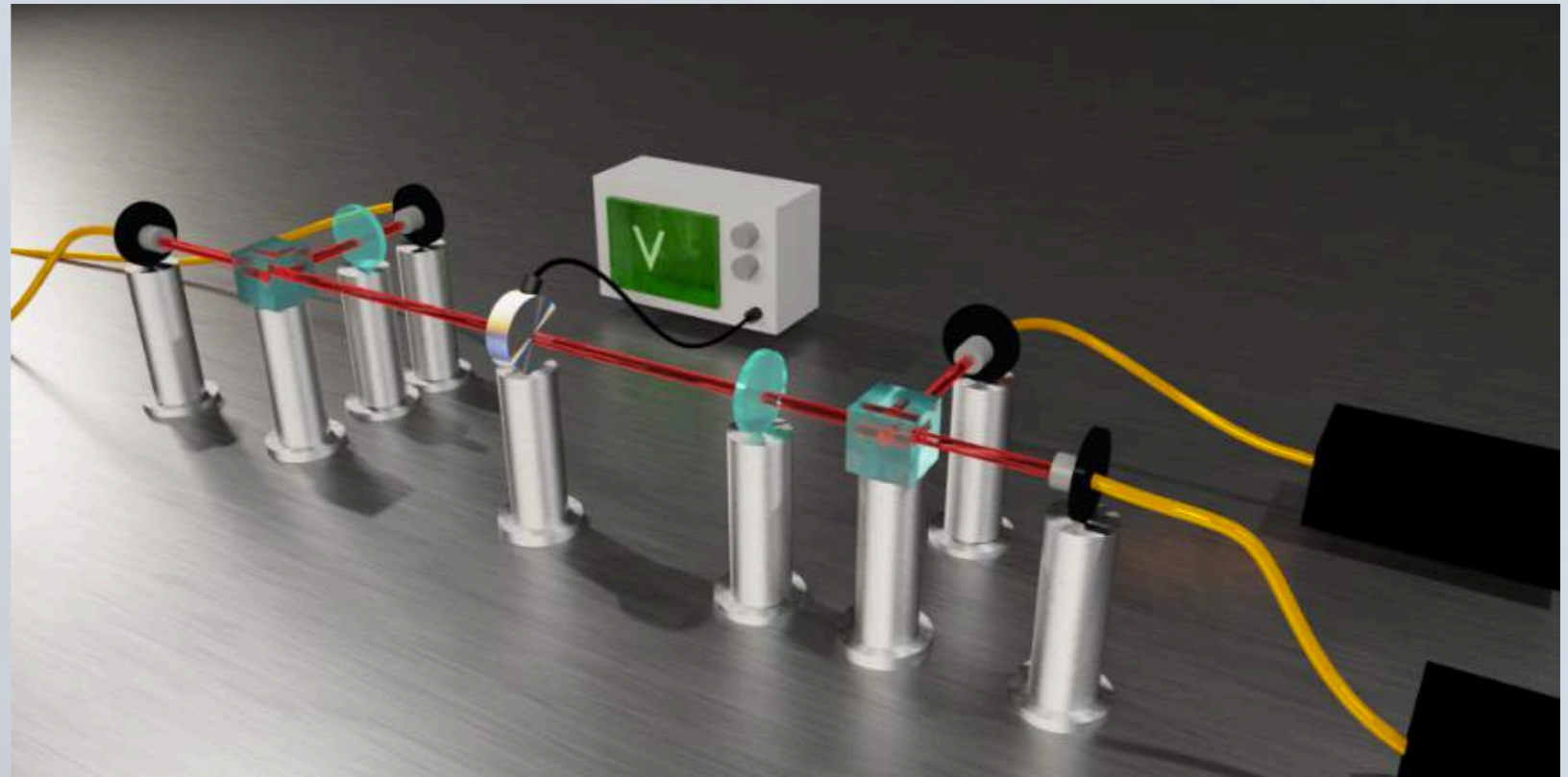
CLASSICAL VS QUANTUM

FUNCTION ESTIMATION

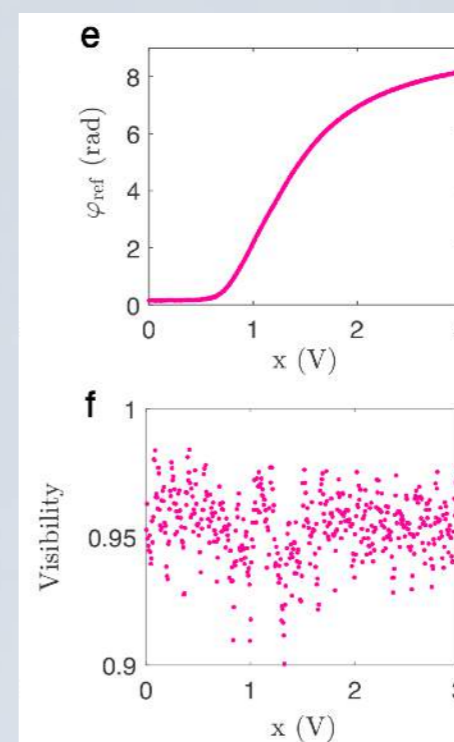
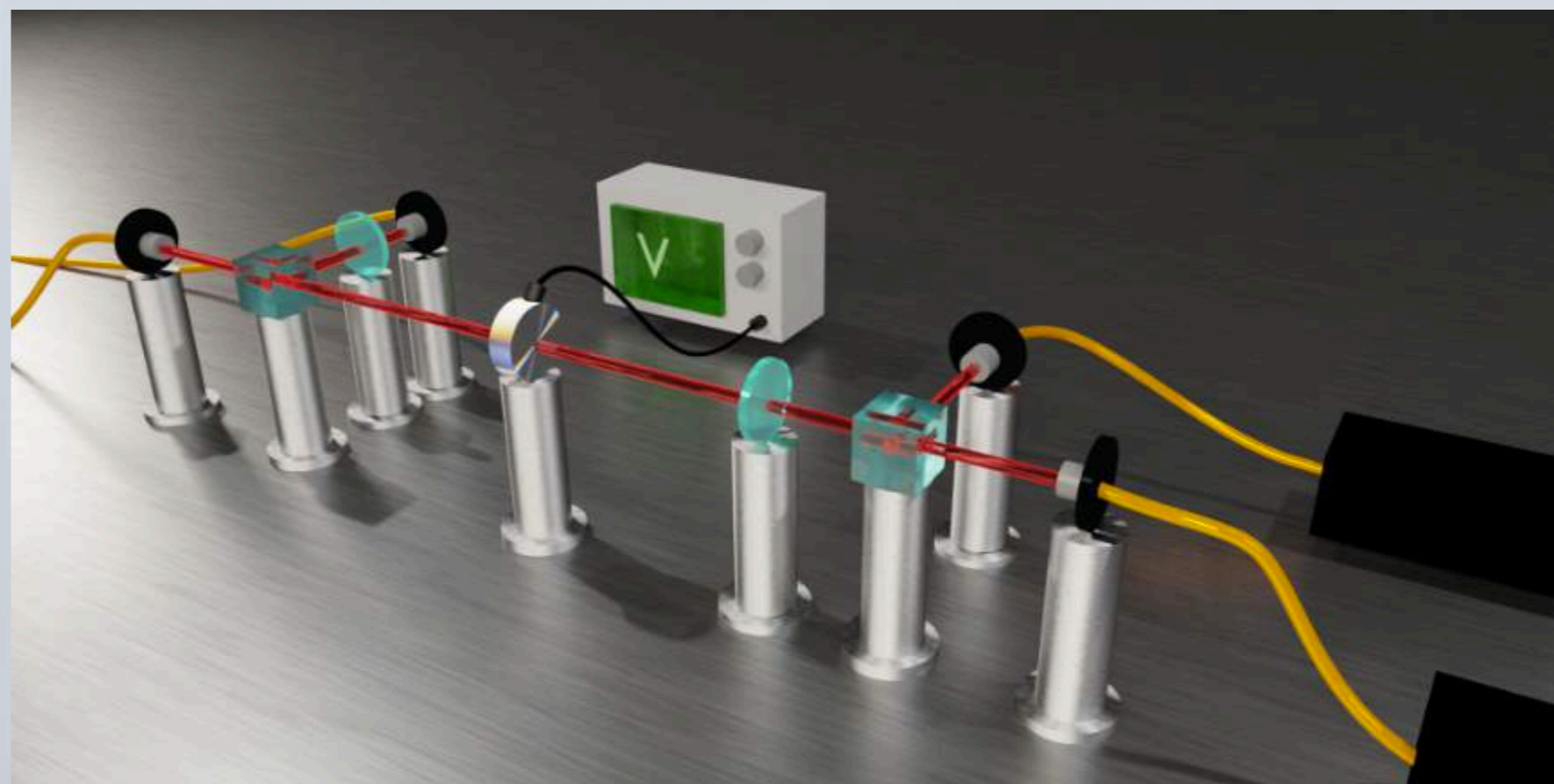


$$\delta_0^2 = \frac{1}{L} \sum_{x=0}^L \mathbb{E} [|\varphi_{ref}(x) - \tilde{\varphi}(x)|^2] \Delta x_{ref}$$

FUNCTION ESTIMATION



FUNCTION ESTIMATION



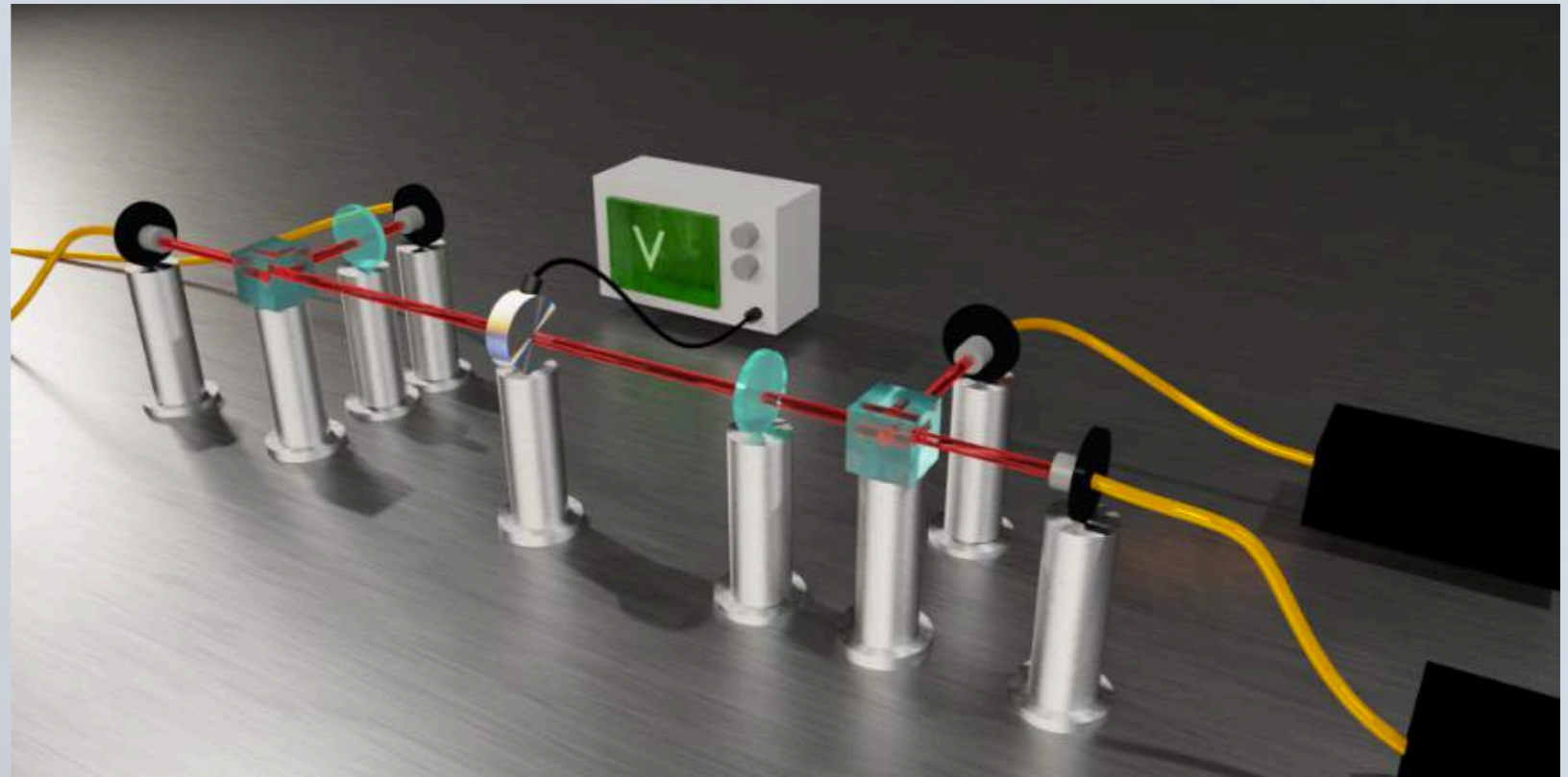
REFERENCE

$N_R = 60K$

$N_S = 500$

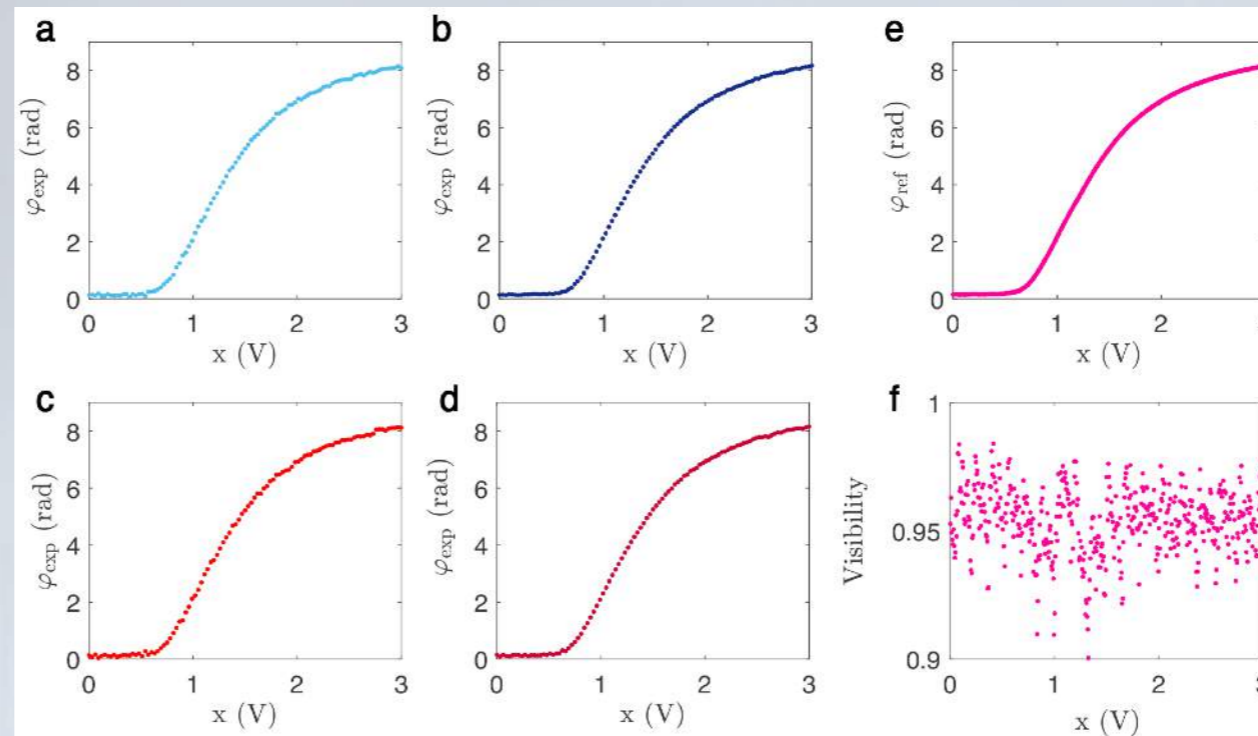
EACH POINT:
MULTIPARAMETER
PHASE ESTIMATION

FUNCTION ESTIMATION



$N_R = 800$

$N_R = 3800$



NOON
100 POINTS

SP
100 POINTS

REFERENCE

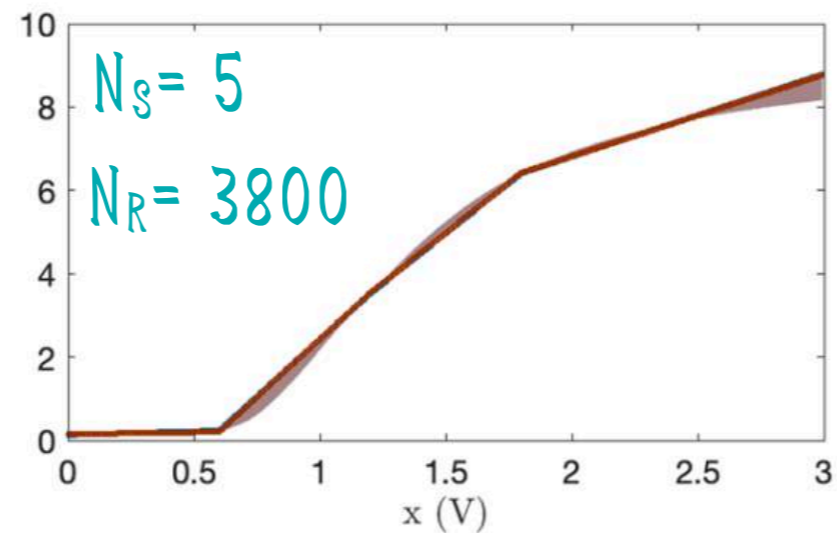
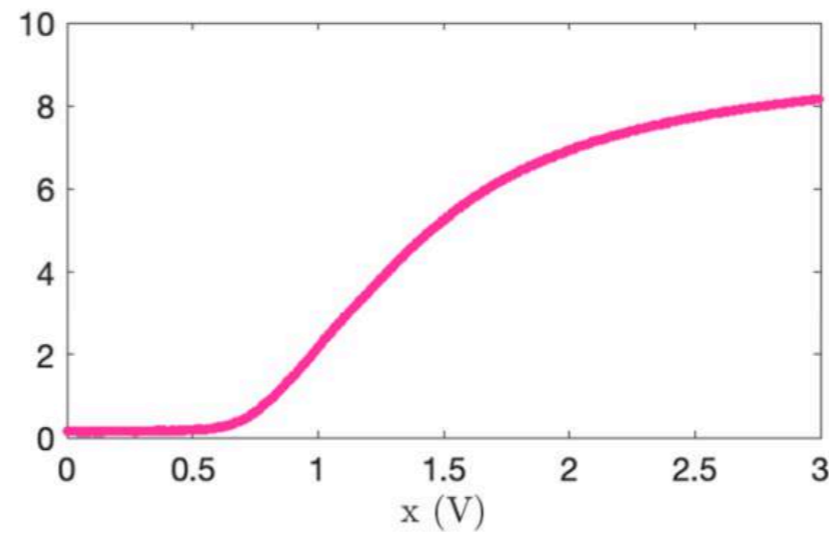
$N_R = 60K$

$N_S = 500$

EACH POINT:
MULTIPARAMETER
PHASE ESTIMATION

FUNCTION ESTIMATION

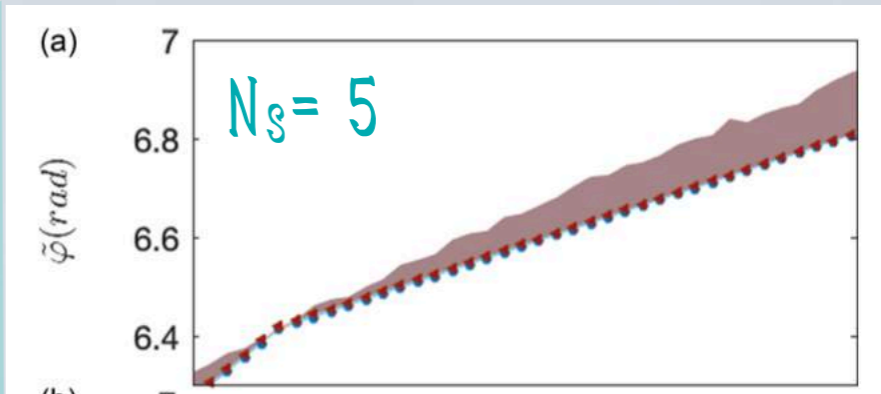
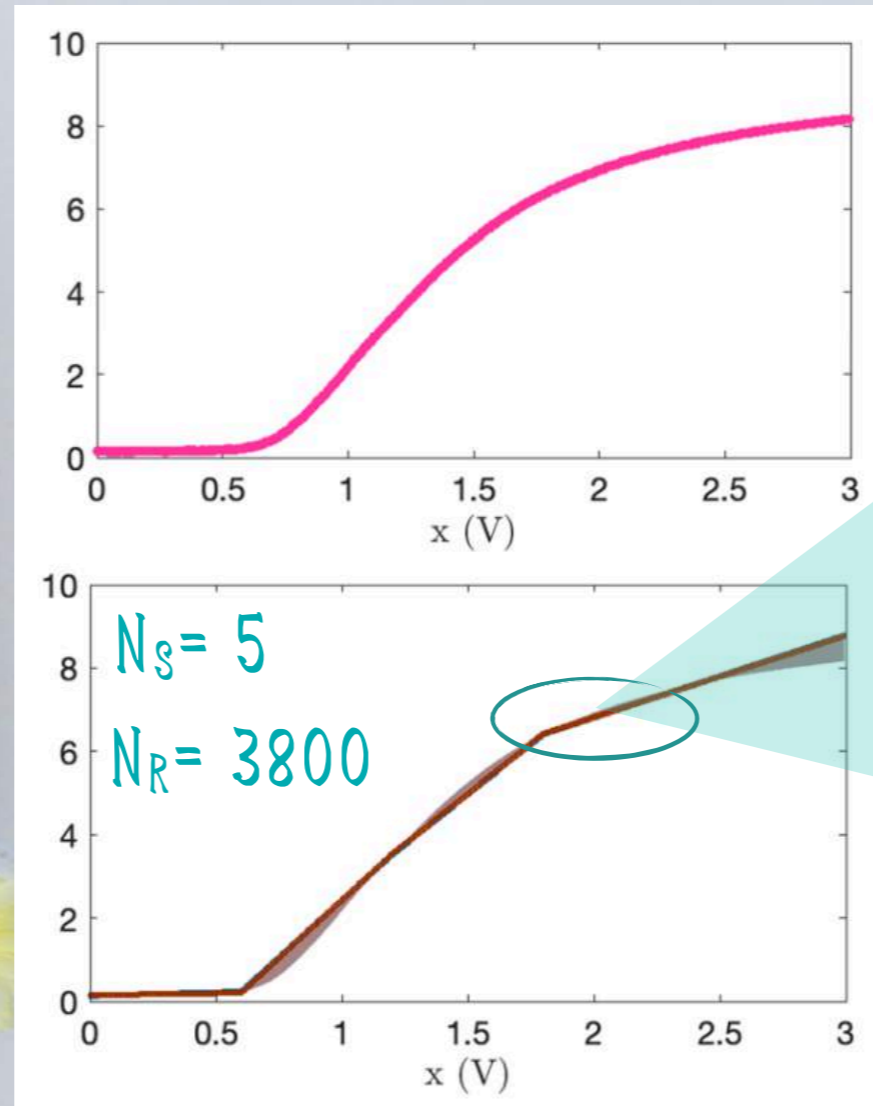
VARYING $N_s=5-100$
INTERPOLATE TO $N_s=500$ (REF)
(LINEAR INTERPOLATION)



$$\Delta \tilde{\varphi}(x) = \varphi_{ref}(x) - \tilde{\varphi}(x)$$

FUNCTION ESTIMATION

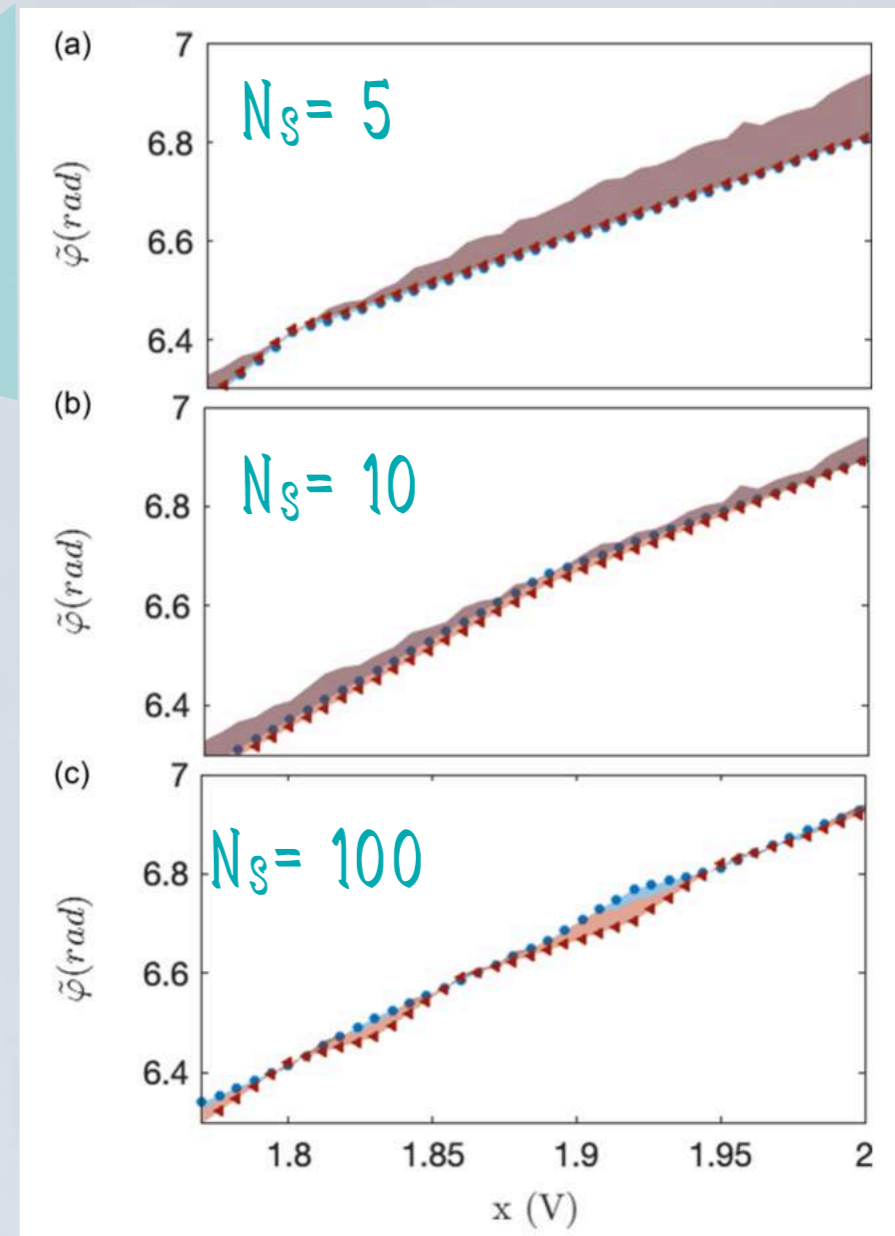
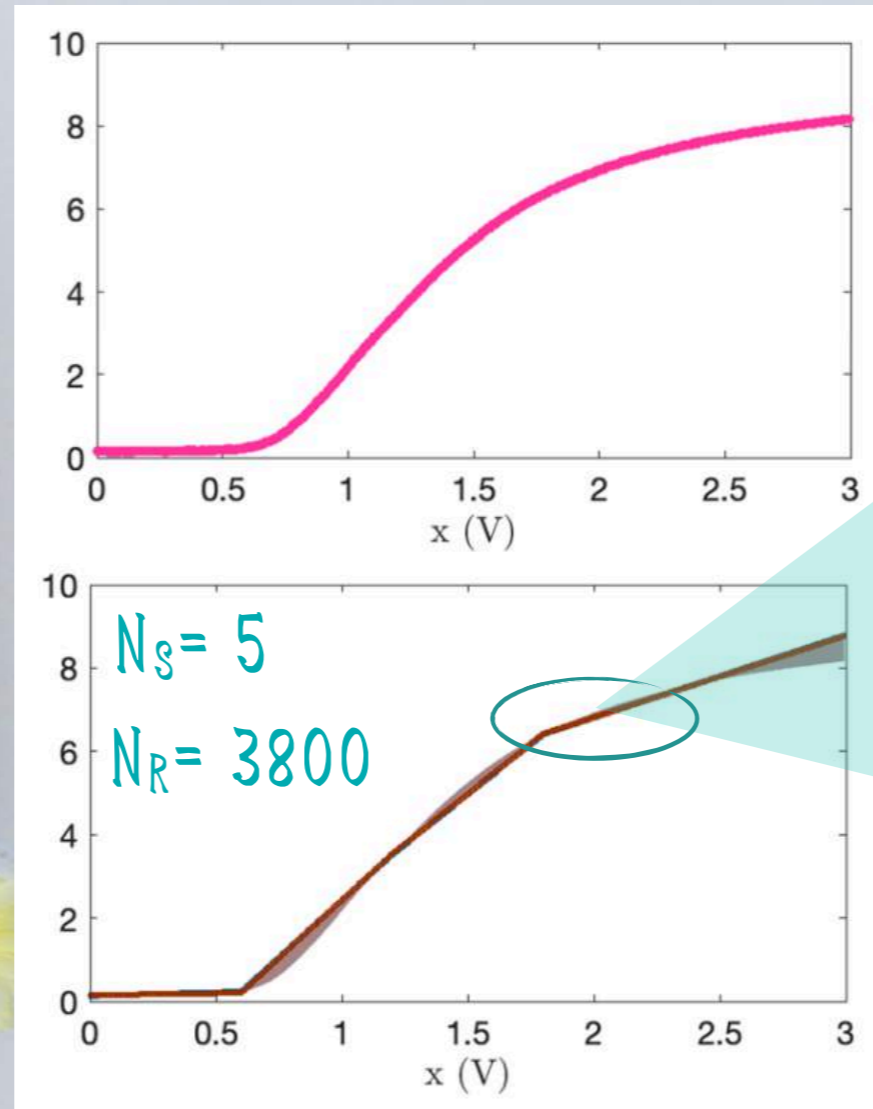
VARYING $N_s=5-100$
INTERPOLATE TO $N_s=500$ (REF)
(LINEAR INTERPOLATION)



$$\Delta\tilde{\varphi}(x) = \varphi_{ref}(x) - \tilde{\varphi}(x)$$

FUNCTION ESTIMATION

VARYING $N_s=5-100$
INTERPOLATE TO $N_s=500$ (REF)
(LINEAR INTERPOLATION)

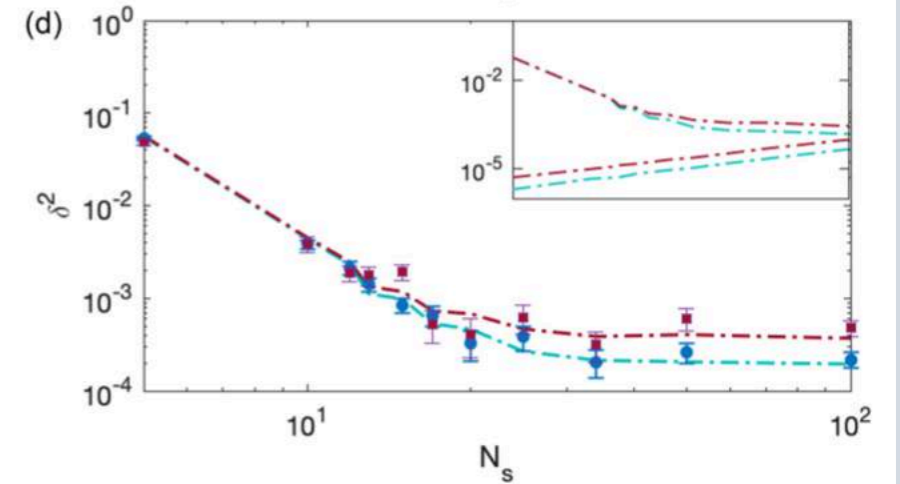
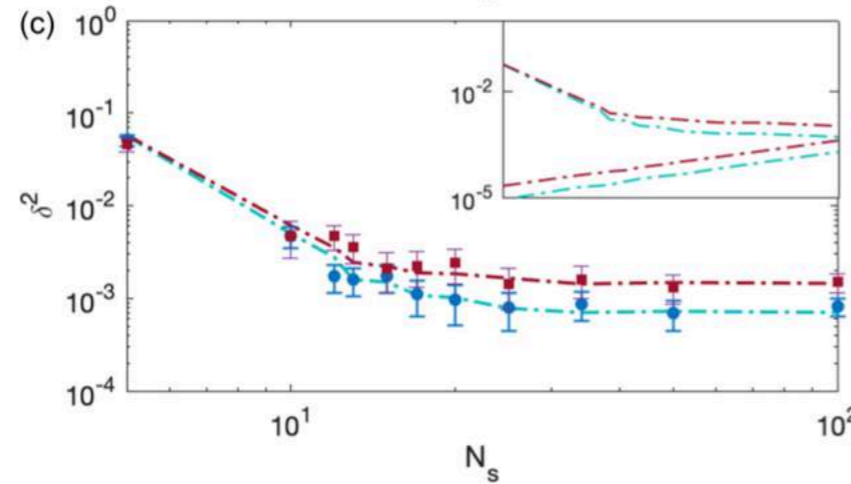
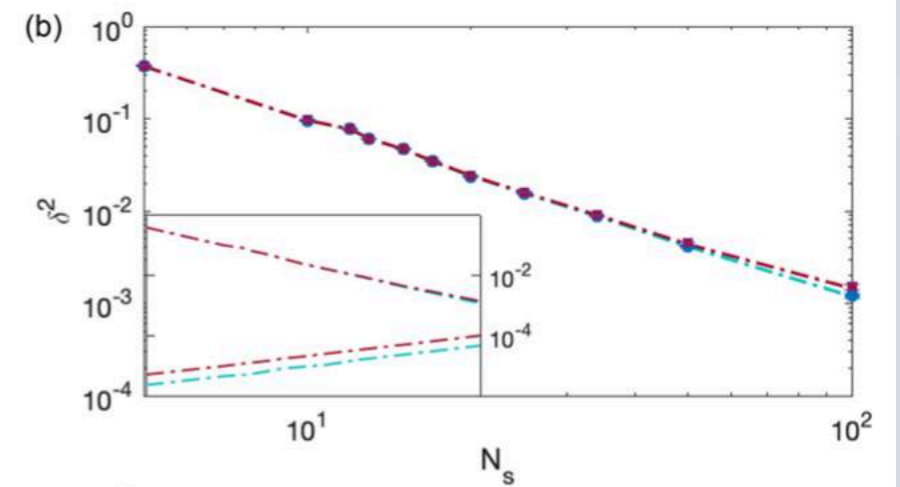
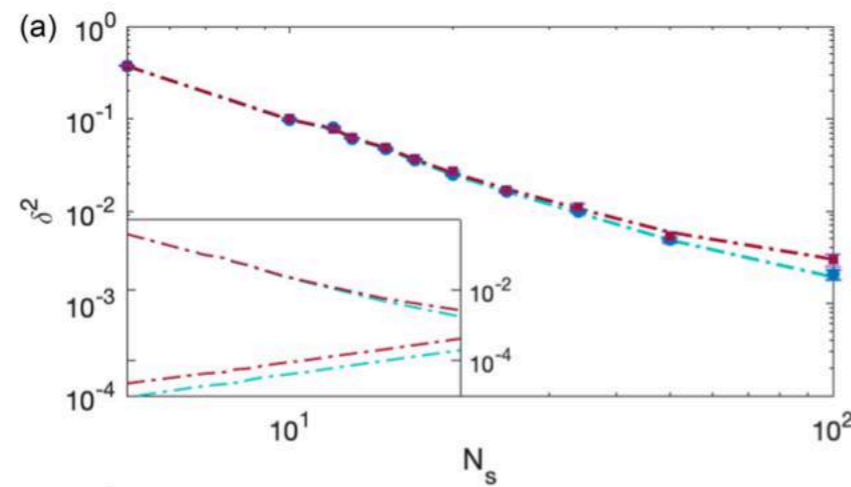


$$\Delta\tilde{\varphi}(x) = \varphi_{ref}(x) - \tilde{\varphi}(x)$$

FUNCTION ESTIMATION



$$\delta_0^2 = \frac{1}{L} \sum_{x=0}^L \mathbb{E} [|\varphi_{ref}(x) - \tilde{\varphi}(x)|^2] \Delta x_{ref}$$



NEAREST NEIGHBOUR

LINEAR

FIXED $N_R = 800$

FIXED $N_R = 3800$

FUNCTION ESTIMATION

TWO SOURCES OF ERROR

STATISTICAL

INCREASE RESOURCES PER POINT

N_R

INTERPOLATION

INCREASE NUMBER OF POINTS

N_S

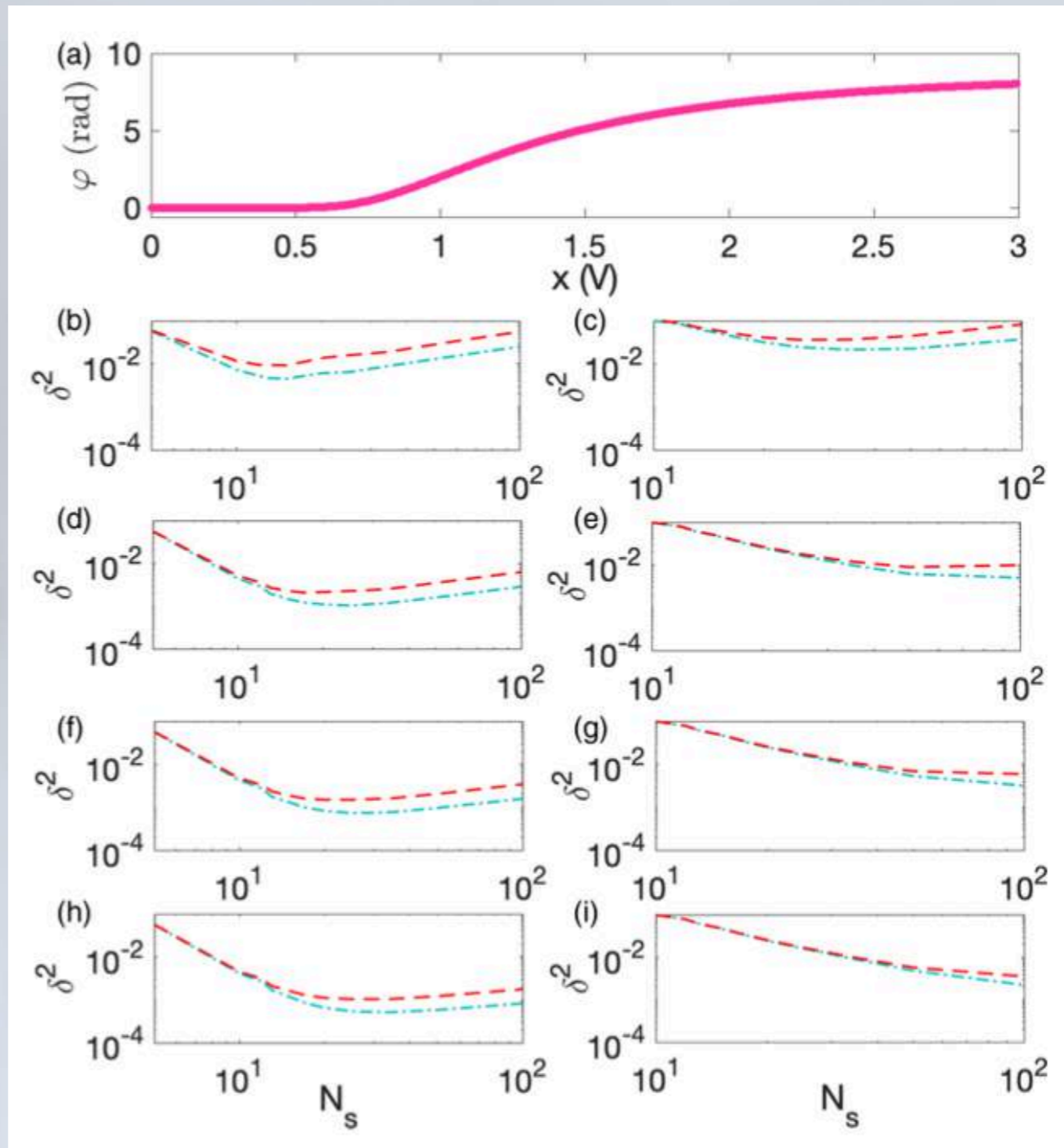
TOTAL NUMBER OF RESOURCES

$$N = N_R N_S$$

FUNCTION ESTIMATION



FIXED TOTAL NUMBER OF RESOURCES $N = N_R N_s$



$N = 1K$

$N = 10K$

$N = 20K$

$N = 50K$

LINEAR

NEAREST NEIGHBOUR

FUNCTION ESTIMATION

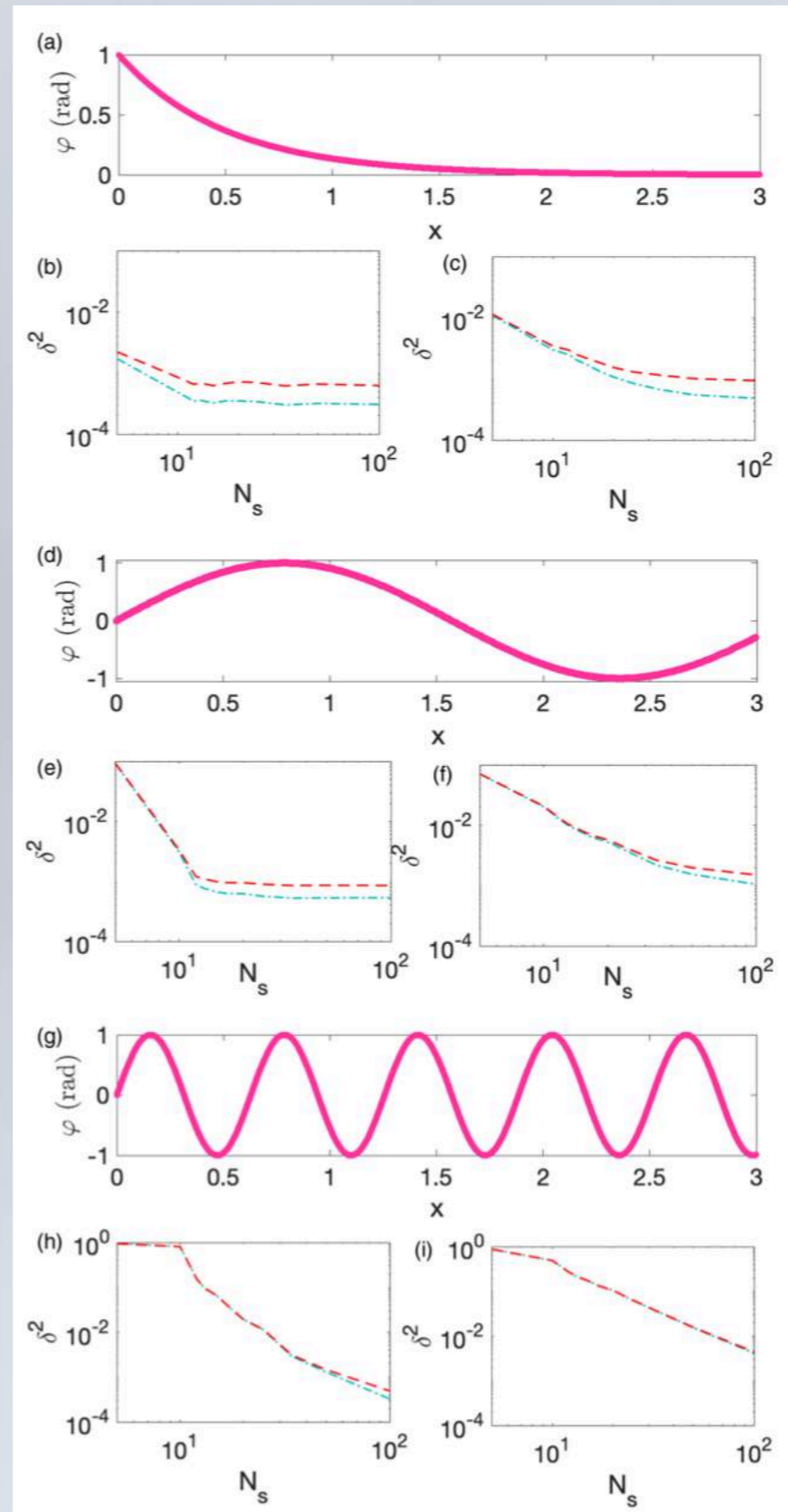


FIXED $N_R=3000$ $N_S=5-100$

YAY ADVANTAGE

SOME ADVANTAGE

NO ADVANTAGE



LINEAR

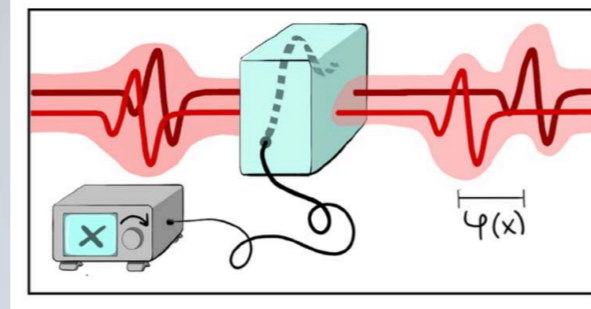
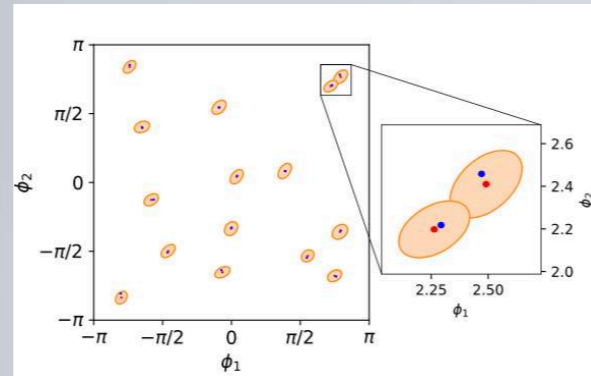
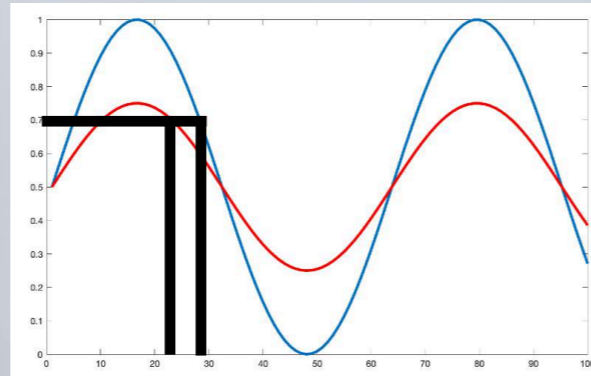
NEAREST NEIGHBOUR



CONCLUSIONS



ESTIMATION OF MULTIPLE PARAMETERS



NECESSARY STEP

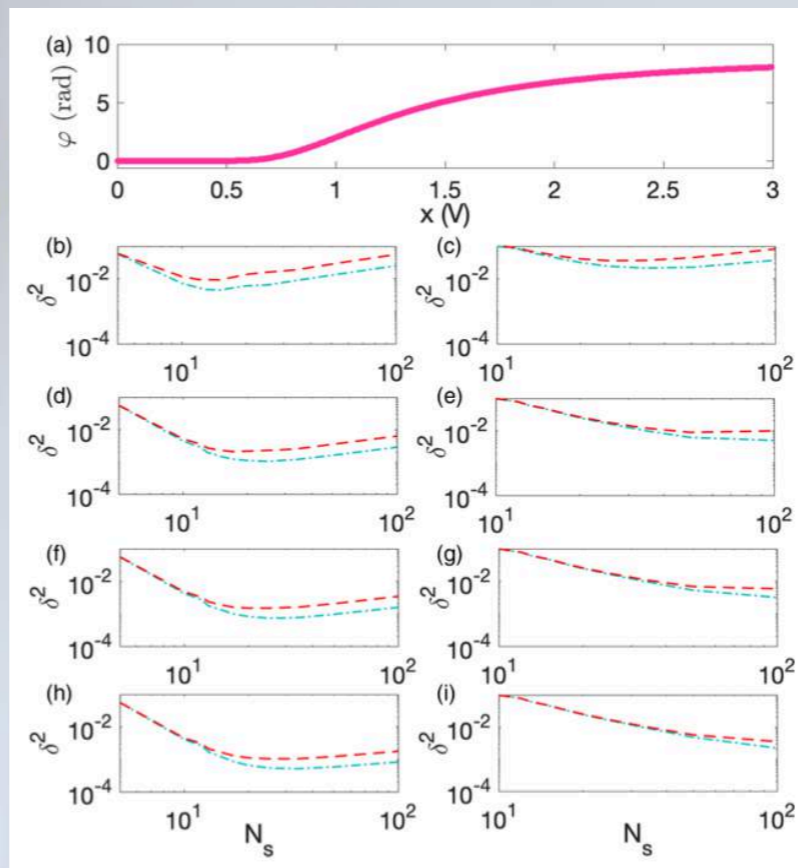
The screenshot shows the Amazon.co.uk website with a search for "quantum sensor". The search results show 1-16 of 187 results. The first result is the "Quantani Enzyme Tracker" priced at £662.61. The second result is the "QInvertaser" priced at £1,054.57. The third result is the "QuanTech BioModule". The page also displays filters for Amazon Prime, Department, Avg. Customer Review, Book Language, and Book Format.

CONCLUSIONS

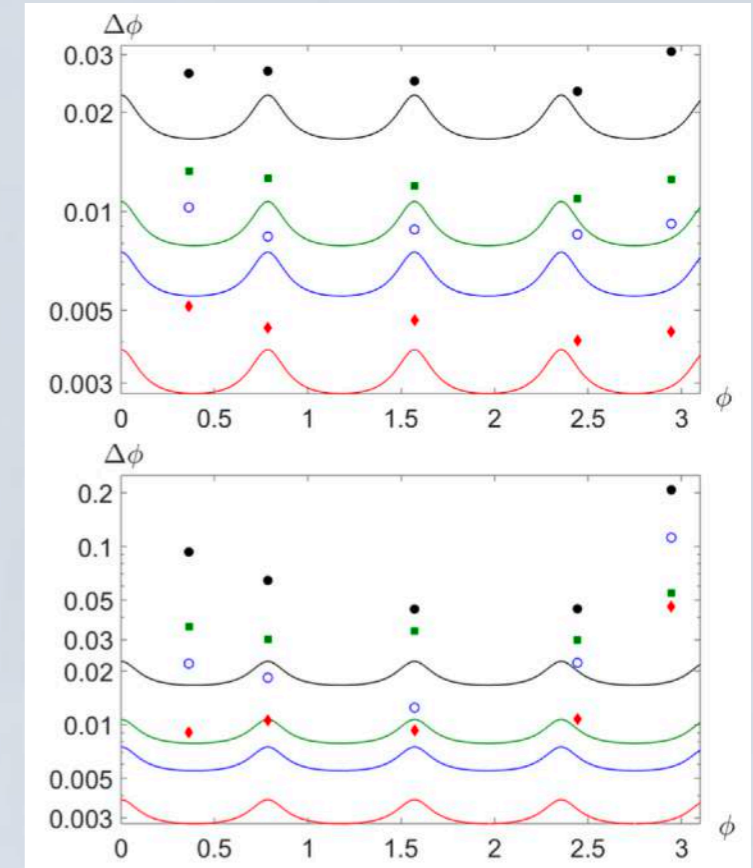
RESOLUTION IS AN ISSUE

RESOURCES SHOULD BE EMPLOYED CAREFULLY

$$N = N_R N_s$$



I. GIANANI ET AL, PRA 103 (2021)



V. CIMINI ET AL, PRL 123 (2019)



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- FRANCESCO ALBARELLI

THANK YOU FOR YOUR ATTENTION!



Flowers? for QSPRING? Groundbreaking