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UNIVERSITÀ DEGLI STUDI

neQo

EXPERIMENTAL FUNCTION ESTIMATION FROM QUANTUM PHASE MEASUREMENTS

DR. ILARIA GIANANI

QSPRING SEMINAR - 20/04/2021

OVERVIEW

QUANTUM COMPUTERS

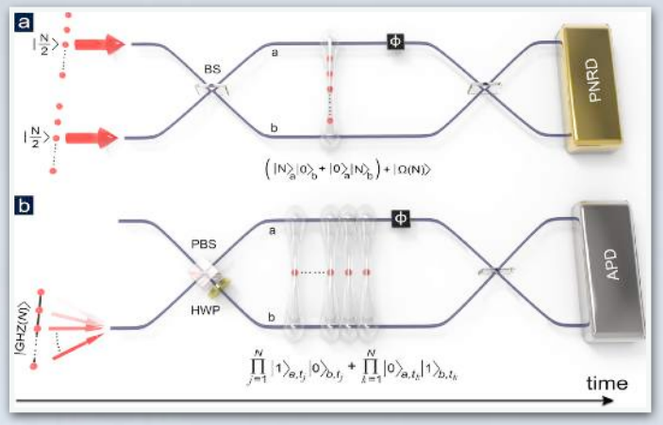
TRAPPED IONS AND SUPERCONDUCTING CIRCUITS

QUANTUM SIMULATORS

QUANTUM COMMUNICATIONS

QUANTUM-SAFE CRYPTOGRAPHY

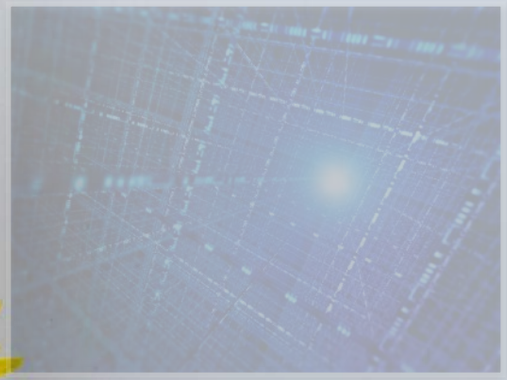
QUANTUM REPEATERS



QUANTUM SENSING

PARAMETER ESTIMATION

OVERVIEW



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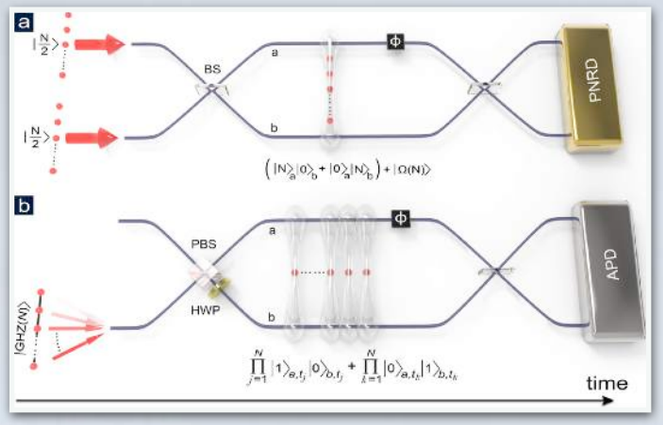
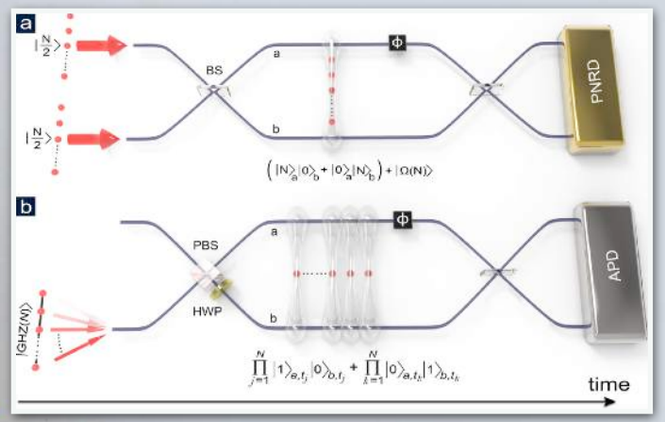


QUANTUM SENSING

PARAMETER ESTIMATION

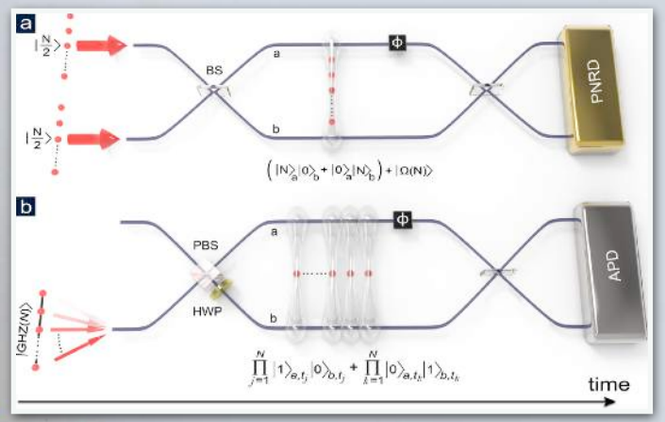
OVERVIEW



QUANTUM SENSING

PARAMETER ESTIMATION

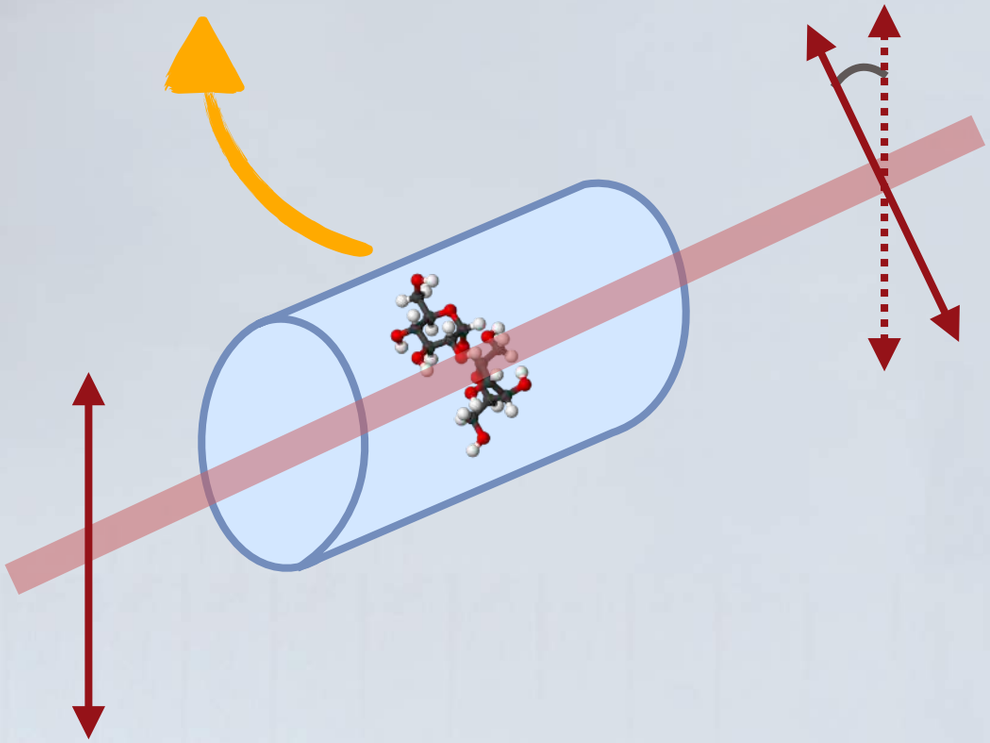
OVERVIEW



QUANTUM SENSING

PARAMETER ESTIMATION

CHIRAL STUFF



OVERVIEW

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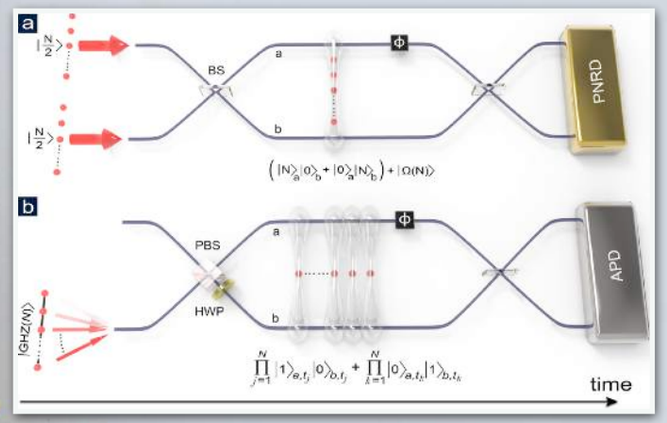
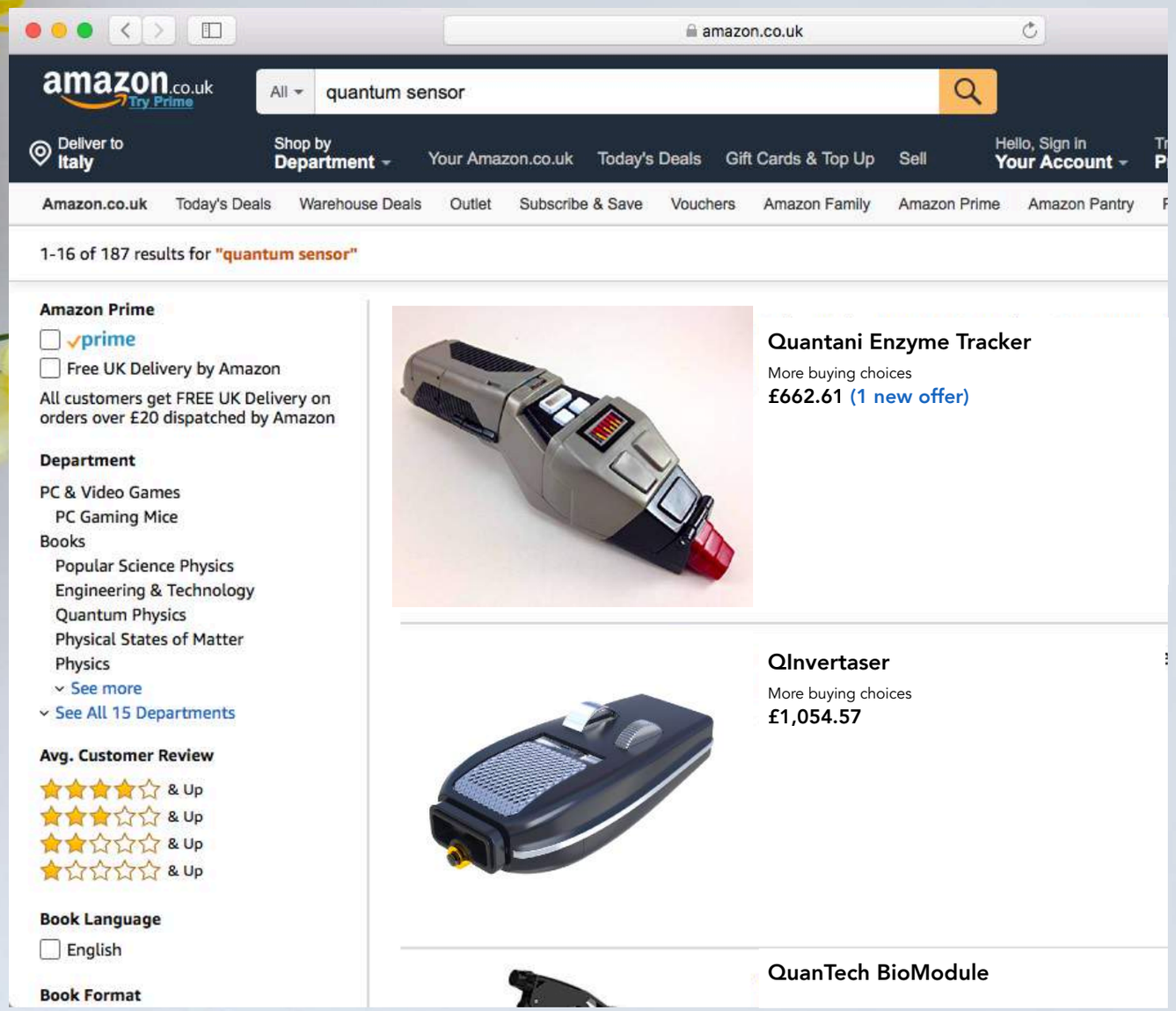
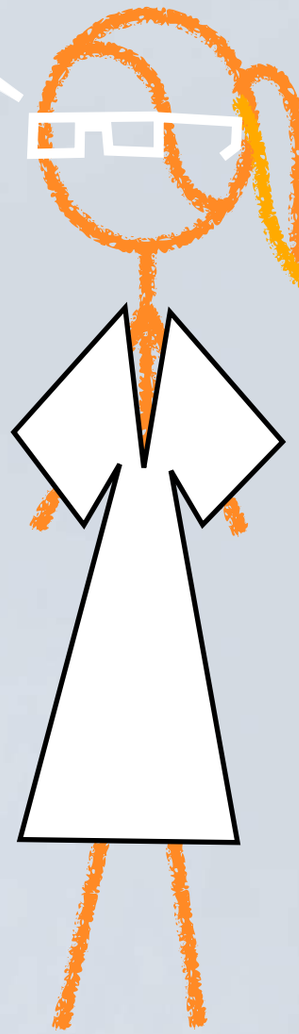


QUANTUM SENSING

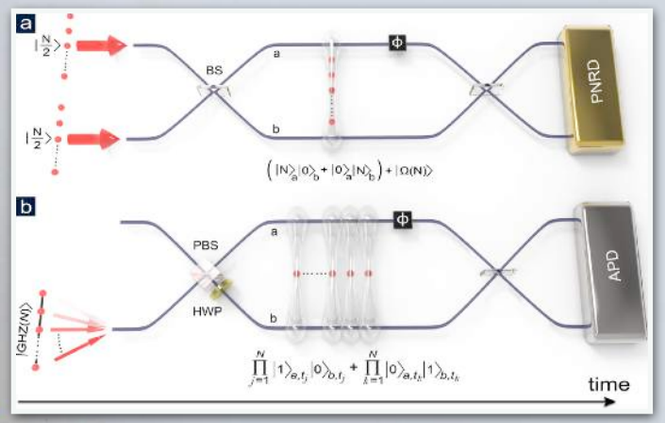
PARAMETER ESTIMATION

COOL CAN I BUY IT ON AMAZON?



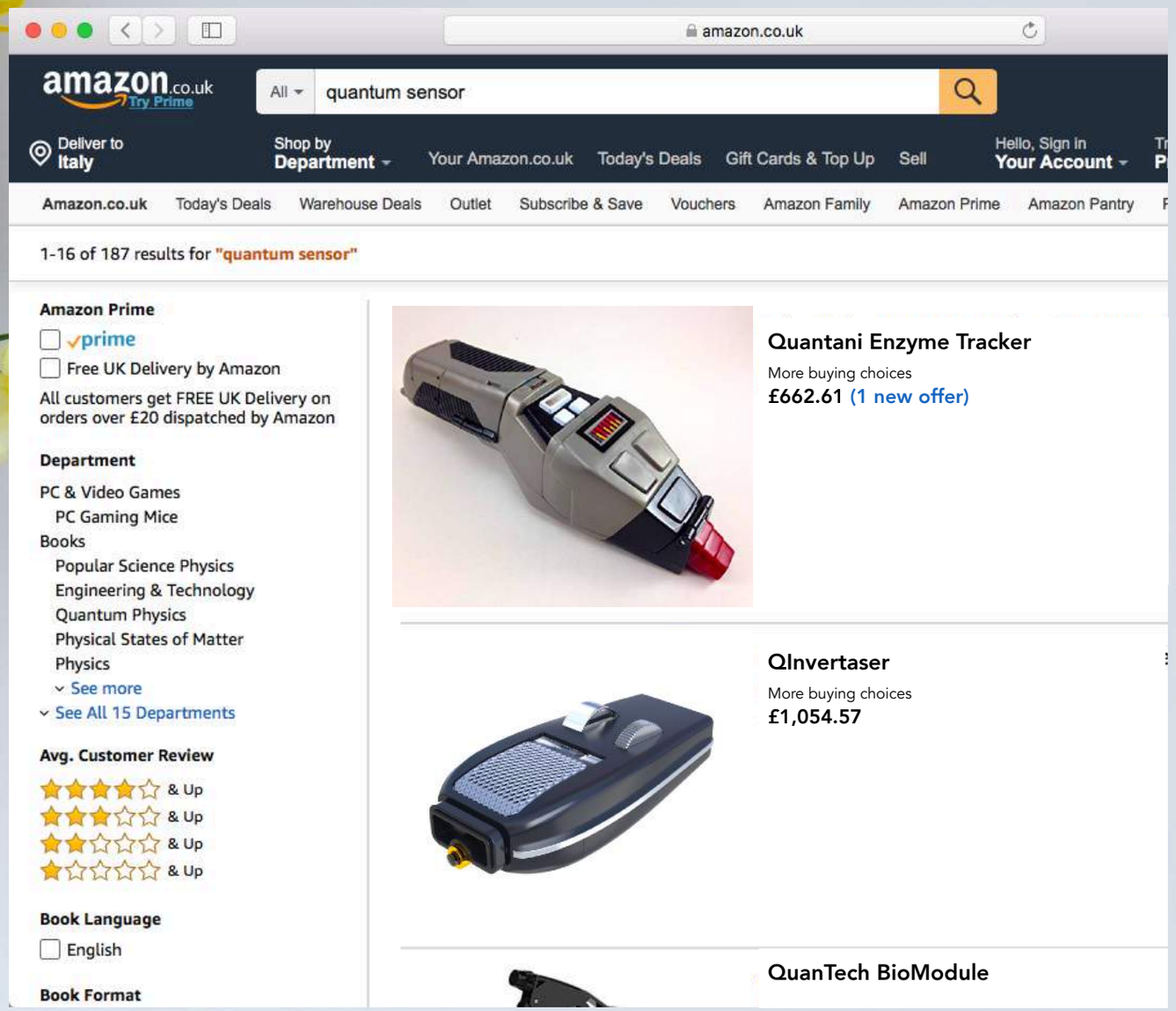


OVERVIEW



QUANTUM SENSING

PARAMETER ESTIMATION



TECHNOLOGY

(SOURCE, EFFICIENCY, DETECTORS, ETC)

FUNDAMENTALS

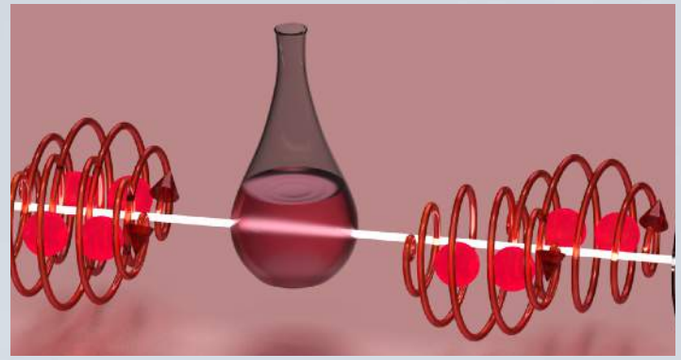
FROM LAB TO REAL OPERATIONS

OUTLINE

MULTIPARAMETER ESTIMATION

PARAMETER ESTIMATION

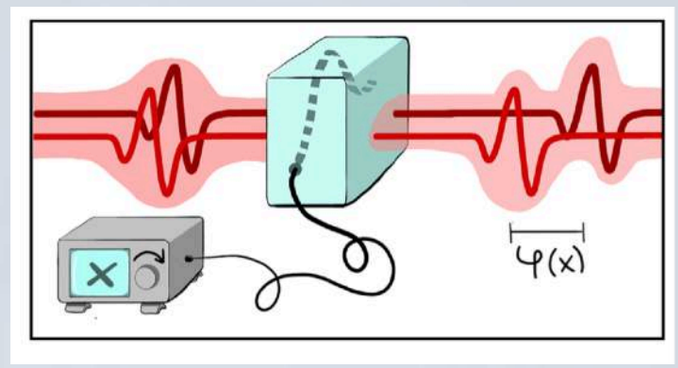
MULTIPARAMETER ESTIMATION OF A CHIRAL SOLUTION



FUNCTION ESTIMATION

ESTIMATION OF THE RESPONSE FUNCTION OF A LIQUID CRYSTAL

SIMULATIONS



PARAMER ESTIMATION

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PROBE PREPARATION

PROBE INTERACTION WITH SYSTEM

PROBE MEASUREMENT

PARAMER ESTIMATION

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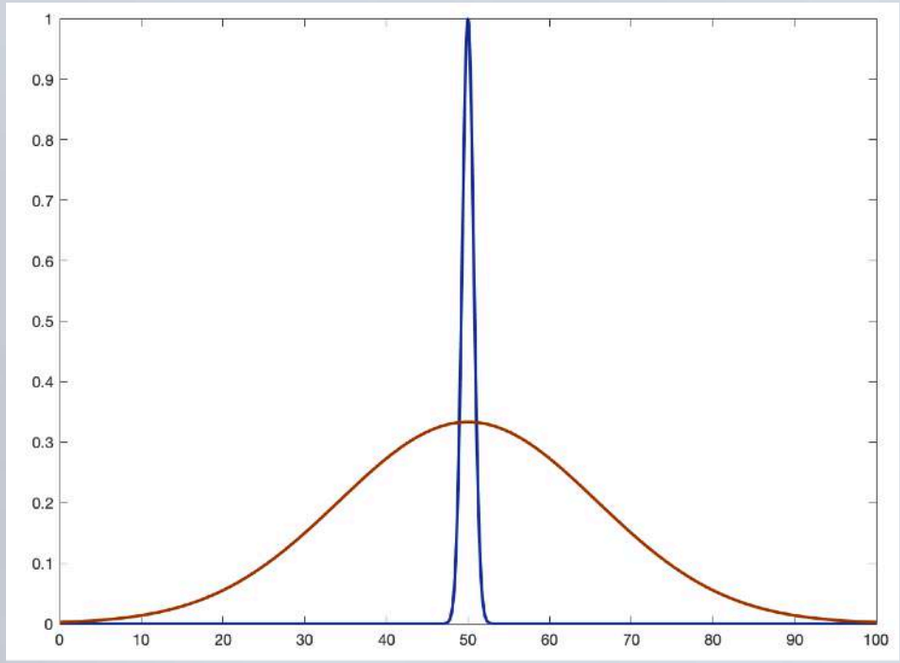
PROBABILITY DISTRIBUTION $P(K|\lambda)$



K

λ PARAMETER

K MEASUREMENT OUTCOME

$\chi = k_1, k_2 ... k_M$

$\hat{\lambda}(\chi)$ ESTIMATOR

PARAMER ESTIMATION

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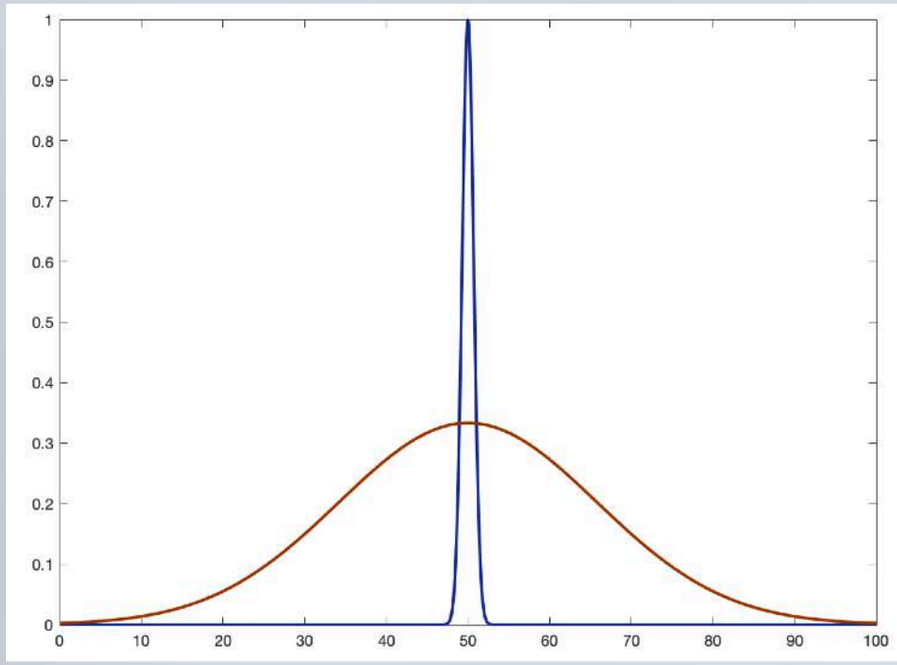
PROBABILITY DISTRIBUTION $P(K|\lambda)$



K

λ PARAMETER

K MEASUREMENT OUTCOME

$\chi = k_1, k_2 ... k_M$

$\hat{\lambda}(\chi)$ ESTIMATOR

FISHER INFORMATION

$$\mathcal{F}[p(k|\lambda)] = \sum_k p(k|\lambda) \left(\partial_\lambda \log p(k|\lambda)\right)^2$$

CRAMER-RAO BOUND (CRB)

$$\mathbb{E}\left[(\hat{\lambda} - \lambda)^2\right] \geq \frac{1}{M\mathcal{F}[p(k|\lambda)]}$$

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MEASUREMENTS

$$\chi = k_1, k_2 ... k_M$$

ESTIMATOR

$$\hat{\lambda}(\chi)$$

???

BAYESIAN ESTIMATOR

$$p(\chi|\lambda) = \prod_{j=1}^{M} p(k_j|\lambda)$$

$$P(\lambda|\chi) = \frac{p(\chi|\lambda)p(\lambda)}{p(\chi)}$$

A-POSTERIORI PROBABILITY

PARAMER ESTIMATION

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MEASUREMENTS

$$\chi = k_1, k_2 ... k_M$$

ESTIMATOR

$$\hat{\lambda}(\chi)$$

???

BAYESIAN ESTIMATOR

$$\hat{\lambda}_B(\chi) = \int \lambda P(\lambda|\chi) d\lambda$$

$$Var(\lambda) = \int (\lambda - \hat{\lambda}_B(\chi))^2 P(\lambda|\chi) d\lambda$$

MULTIPARAMER ESTIMATION

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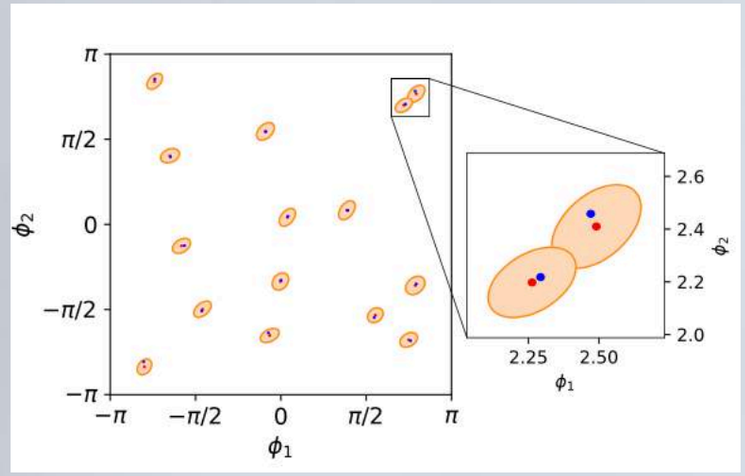
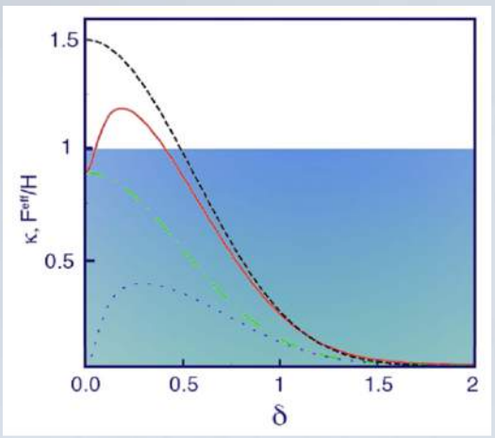
INTERESTING PARAMETERS

MULTIPLE PHASES



M VALERI ET AL, NPJQI 6,96 (2020)

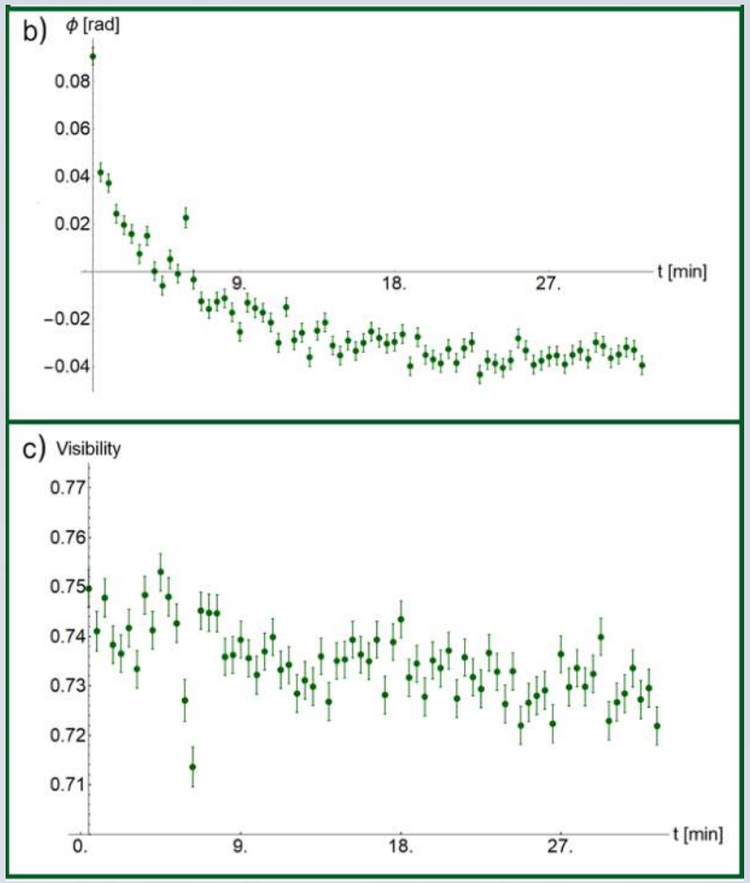
PHASE - DEPHASING



E. ROCCIA ET AL, QST 3 (2018)
M. VIDRIGHIN ET AL, NAT COM 5, 3532 (2014)

NUISANCE PARAMETERS

THINGS WE DO NOT CARE ABOUT BUT IF WE DON'T THEY AFFECT OUR ESTIMATION



E. ROCCIA ET AL, OPTICA 5,10 (2018)
V. CIMINI ET AL, OP. EX., 27, 24 (2019)

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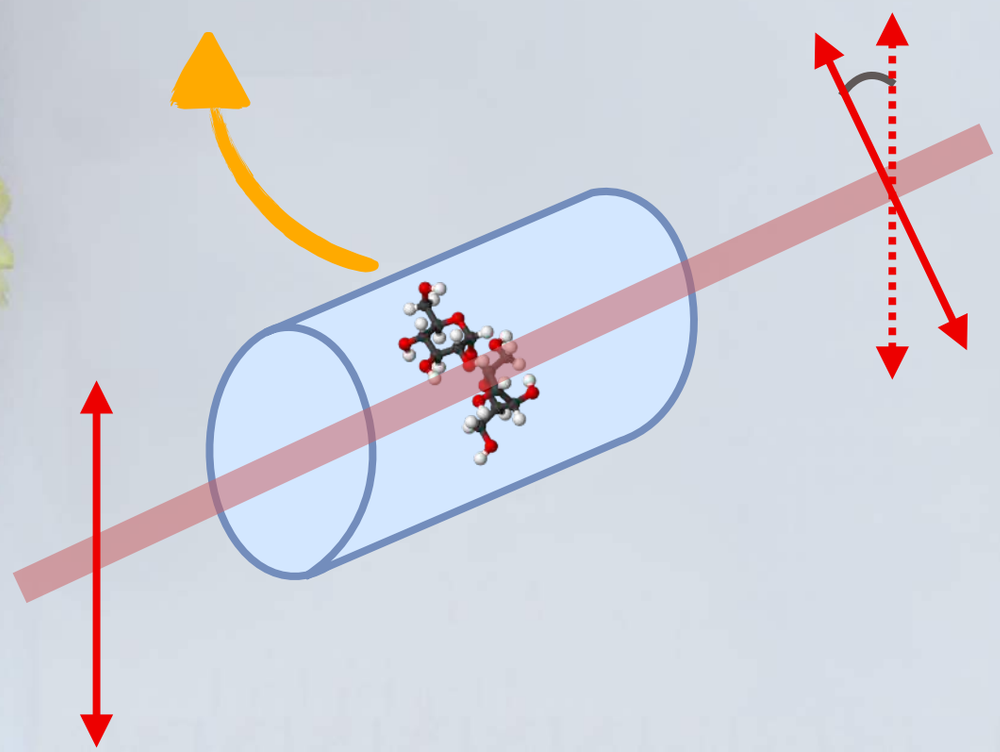
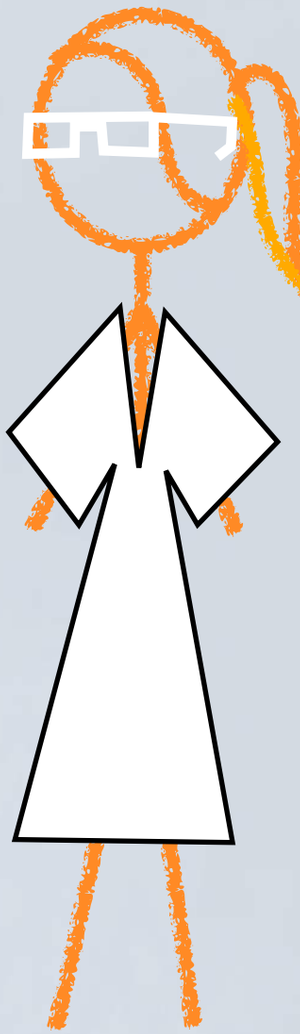
NUISANCE PARAMETERS

CHIRAL STUFF

PHASE BETWEEN
CIRCULAR POLARIZATION





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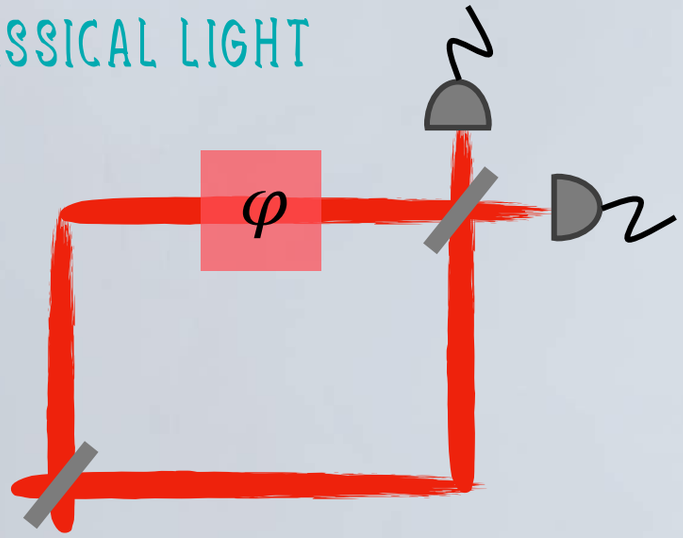
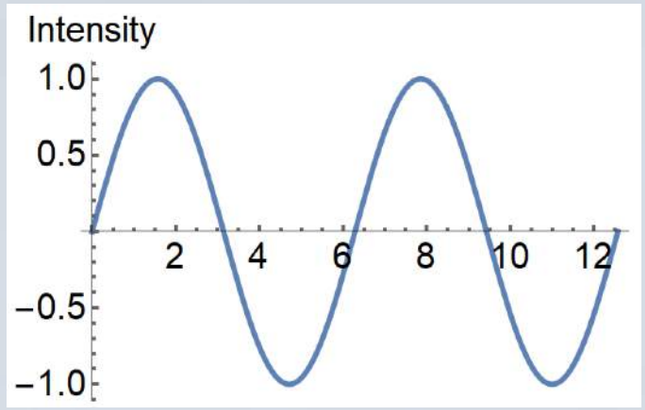
NUISANCE PARAMETERS

CLASSICAL LIGHT

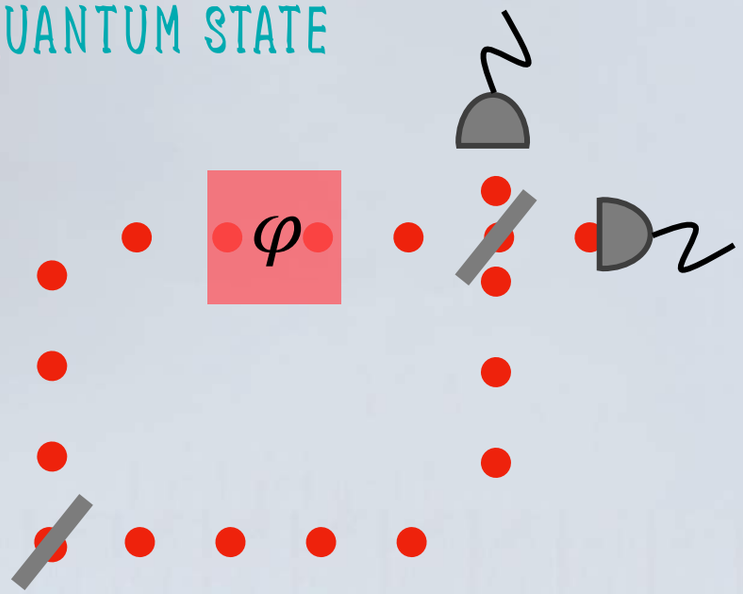




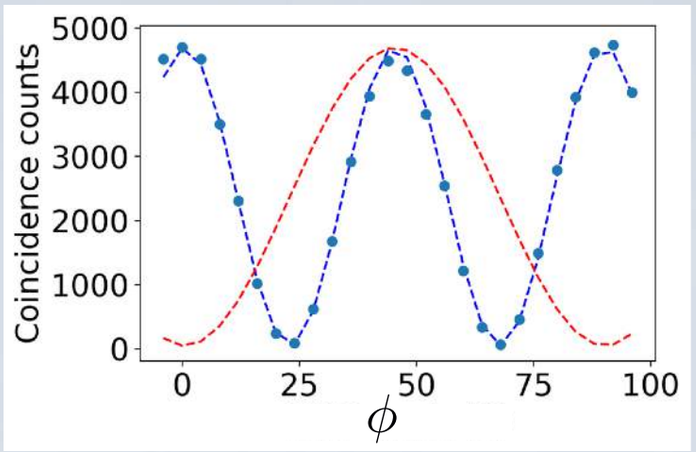
SHOT NOISE LIMIT

$$\Delta\varphi \geq \frac{1}{\sqrt{N}}$$

QUANTUM STATE



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|N0\rangle + |0N\rangle\right)$$



HEISENBERG LIMIT

$$\Delta\varphi \geq \frac{1}{N}$$

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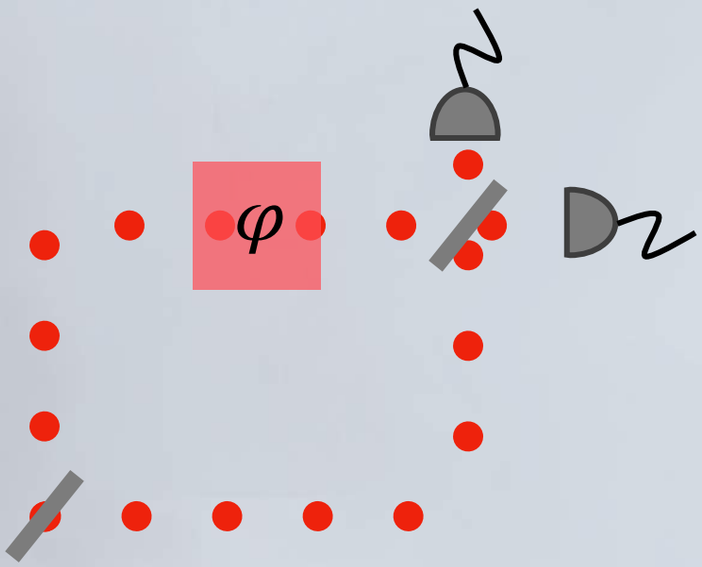
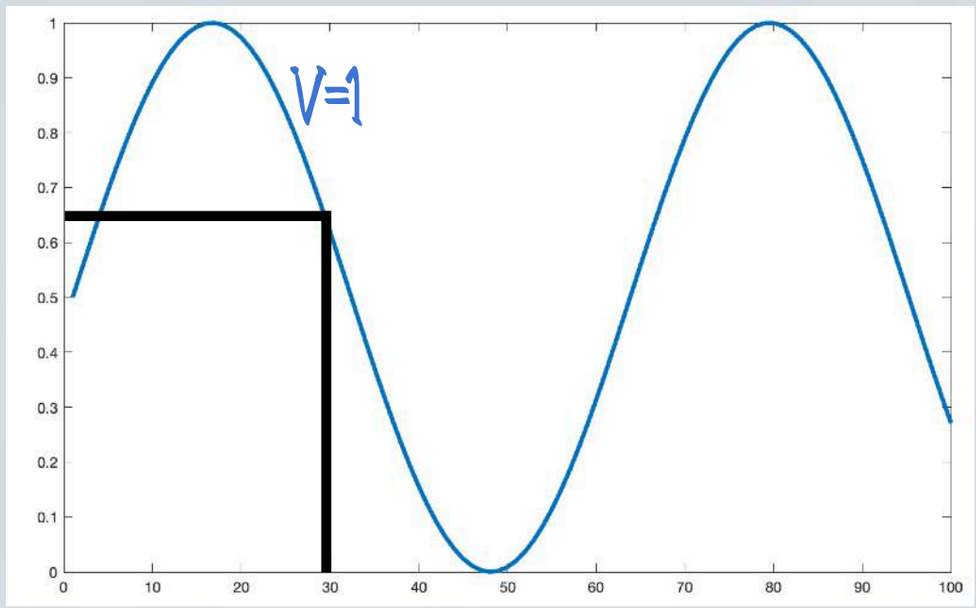
NUISANCE PARAMETERS





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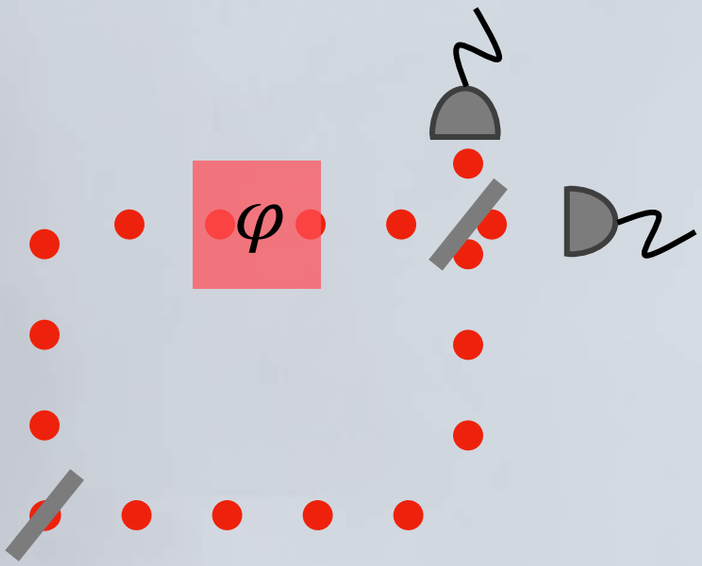
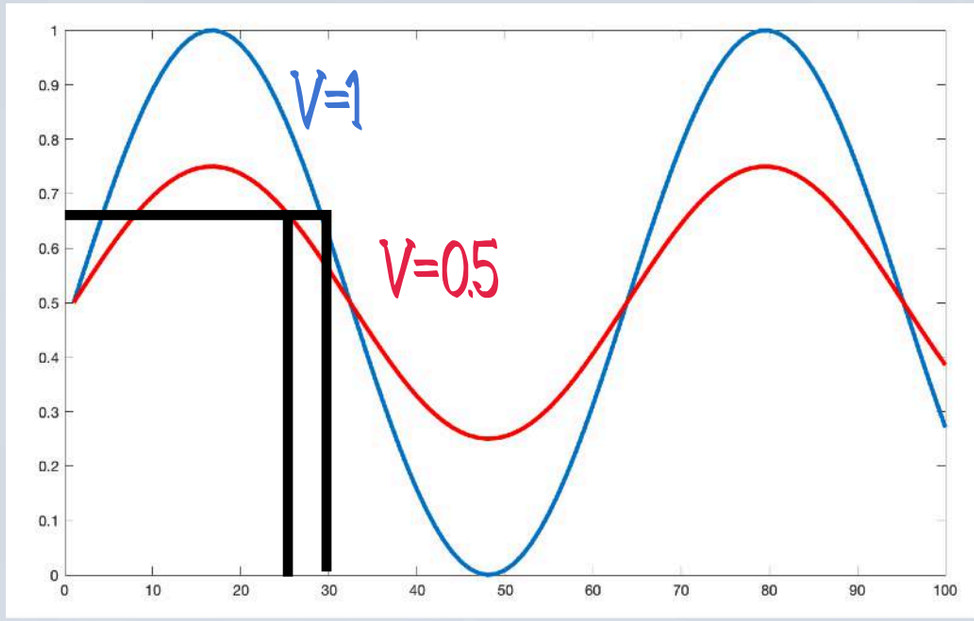
NUISANCE PARAMETERS



BIAS!



MEASURE PHASE AND VISIBILITY AT THE SAME TIME

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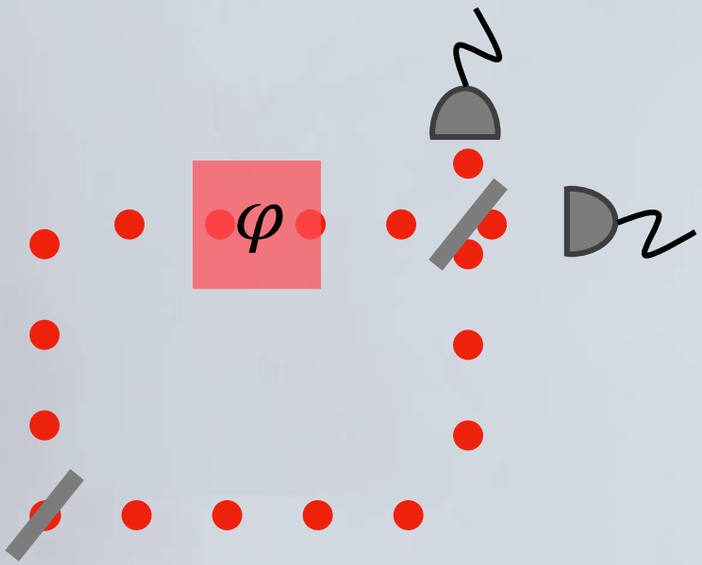
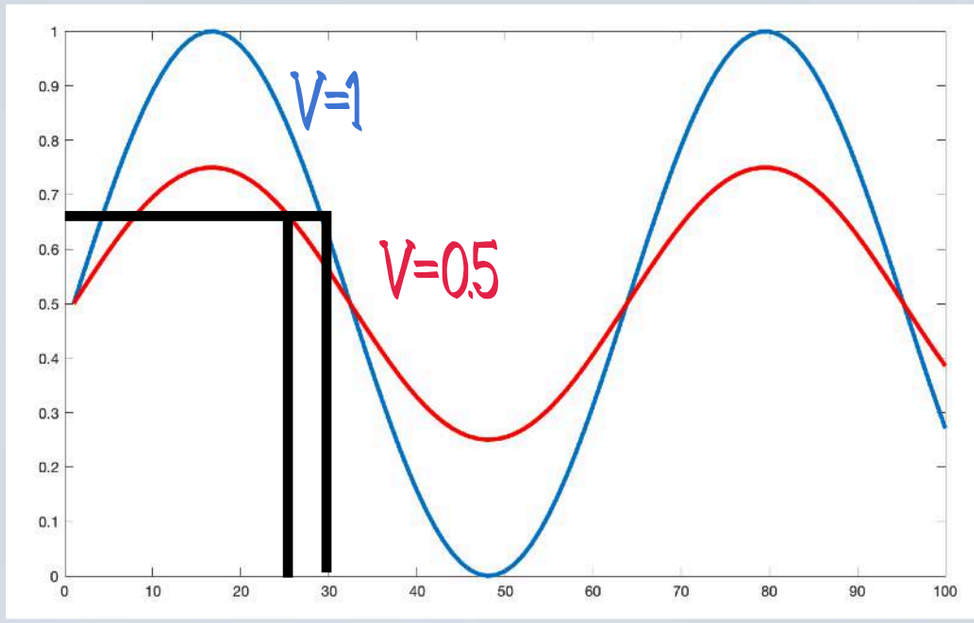
NUISANCE PARAMETERS



BIAS!



MEASURE PHASE AND VISIBILITY AT THE SAME TIME

FISHER INFORMATION

$$\mathcal{F}_{\mu\nu} = \sum_k p(k|\varphi, v)(\partial_\mu \log p(k|\varphi, v)(\partial_\nu \log p(k|\varphi, v)$$

CRAMER-RAO BOUND (CRB)

$$Var[\varphi] \geq \frac{\mathcal{F}^{-1}_{\varphi,\varphi}}{M} \qquad Var[v] \geq \frac{\mathcal{F}^{-1}_{v,v}}{M}$$

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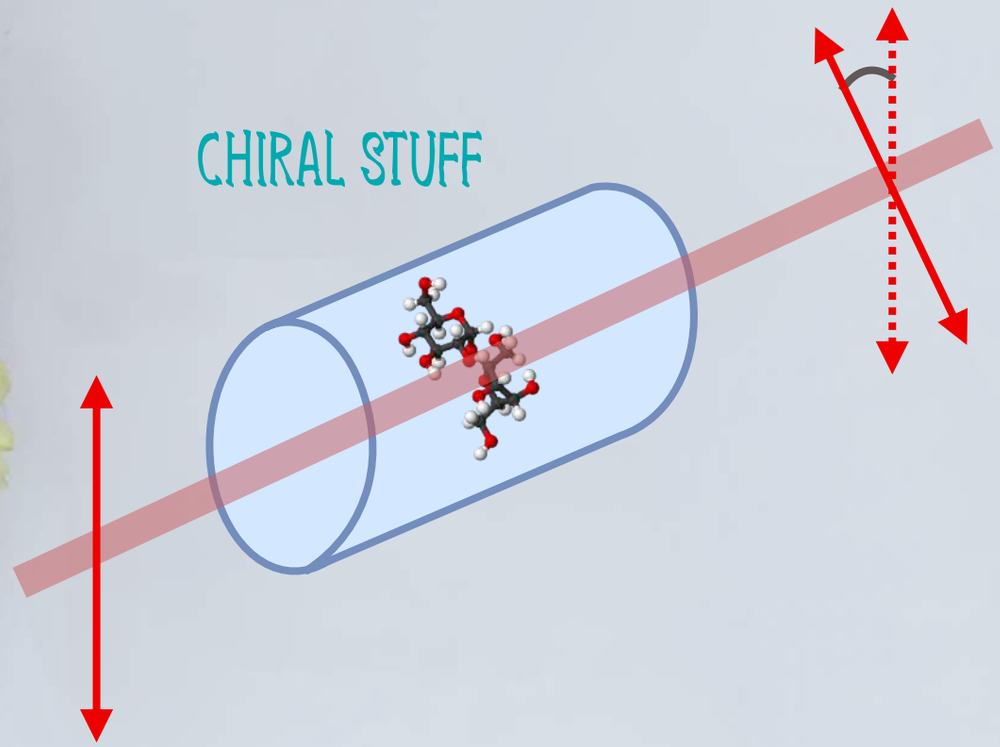
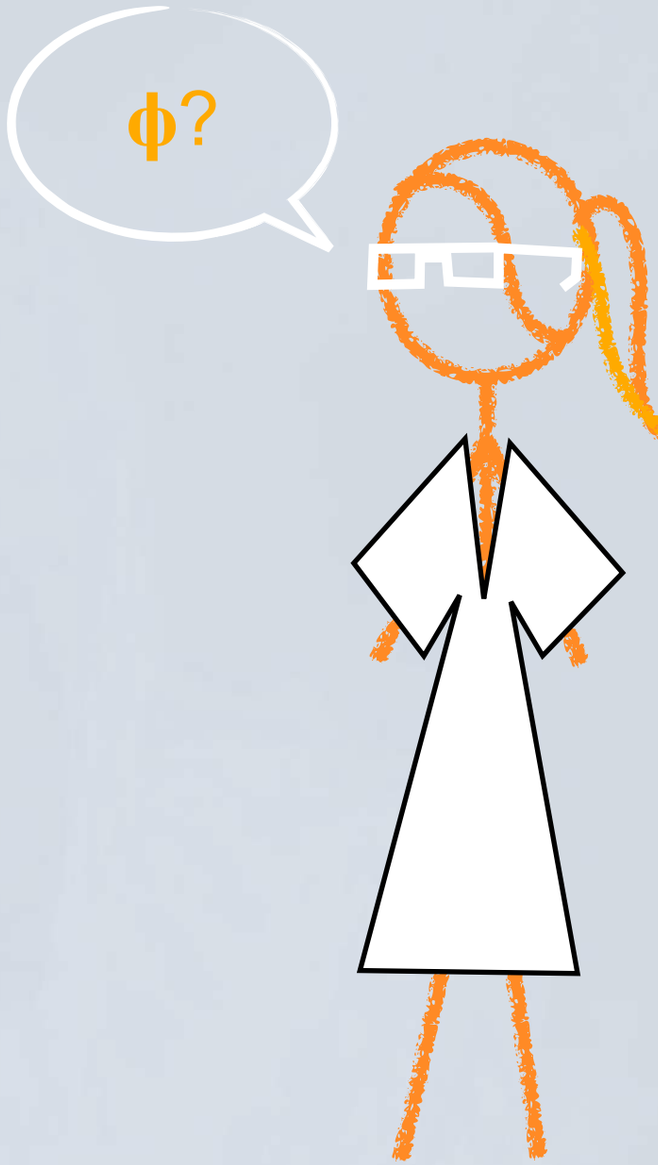
NUISANCE PARAMETERS

CHIRAL STUFF





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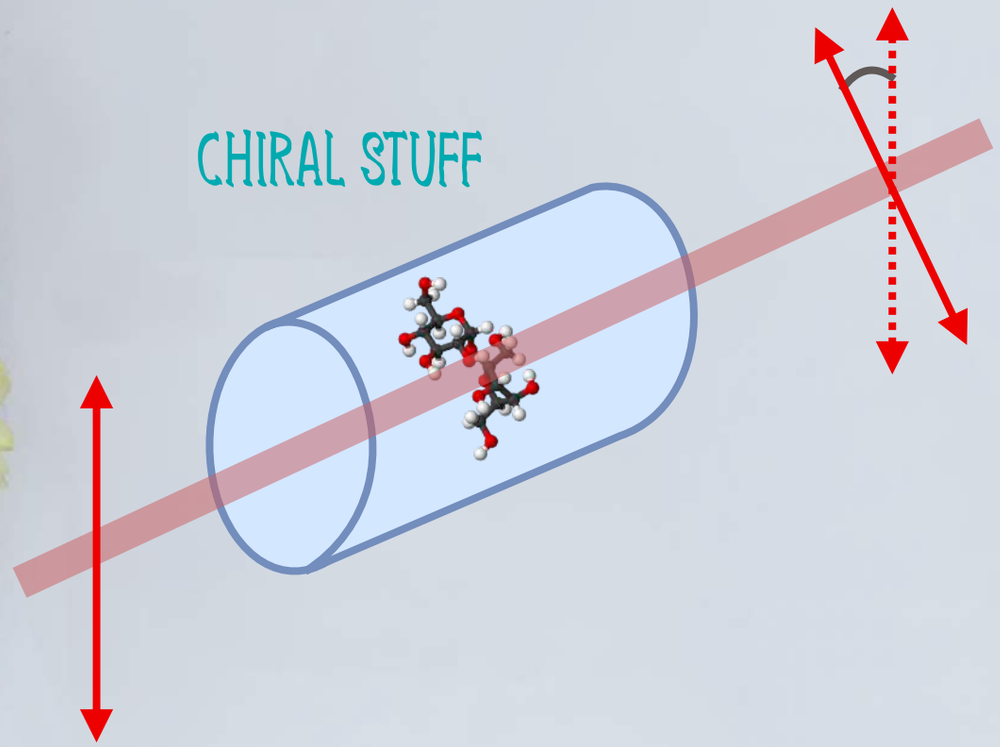
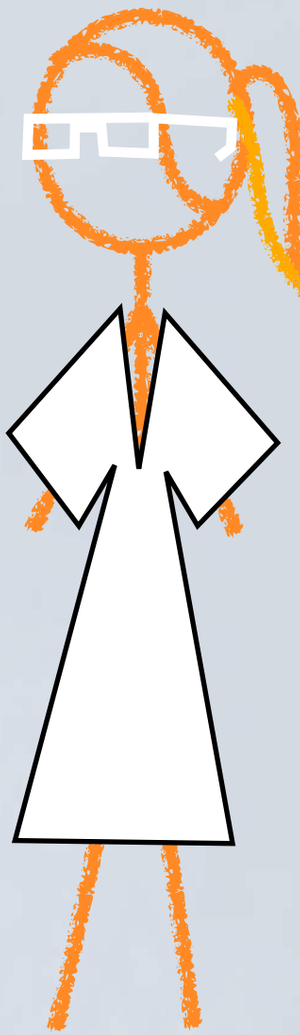
NUISANCE PARAMETERS

POLARIZATION INTERFEROMETER

CHIRAL STUFF





PREPARE POL R+L

ADD A PHASE BETWEEN R AND L

MEASURE POL

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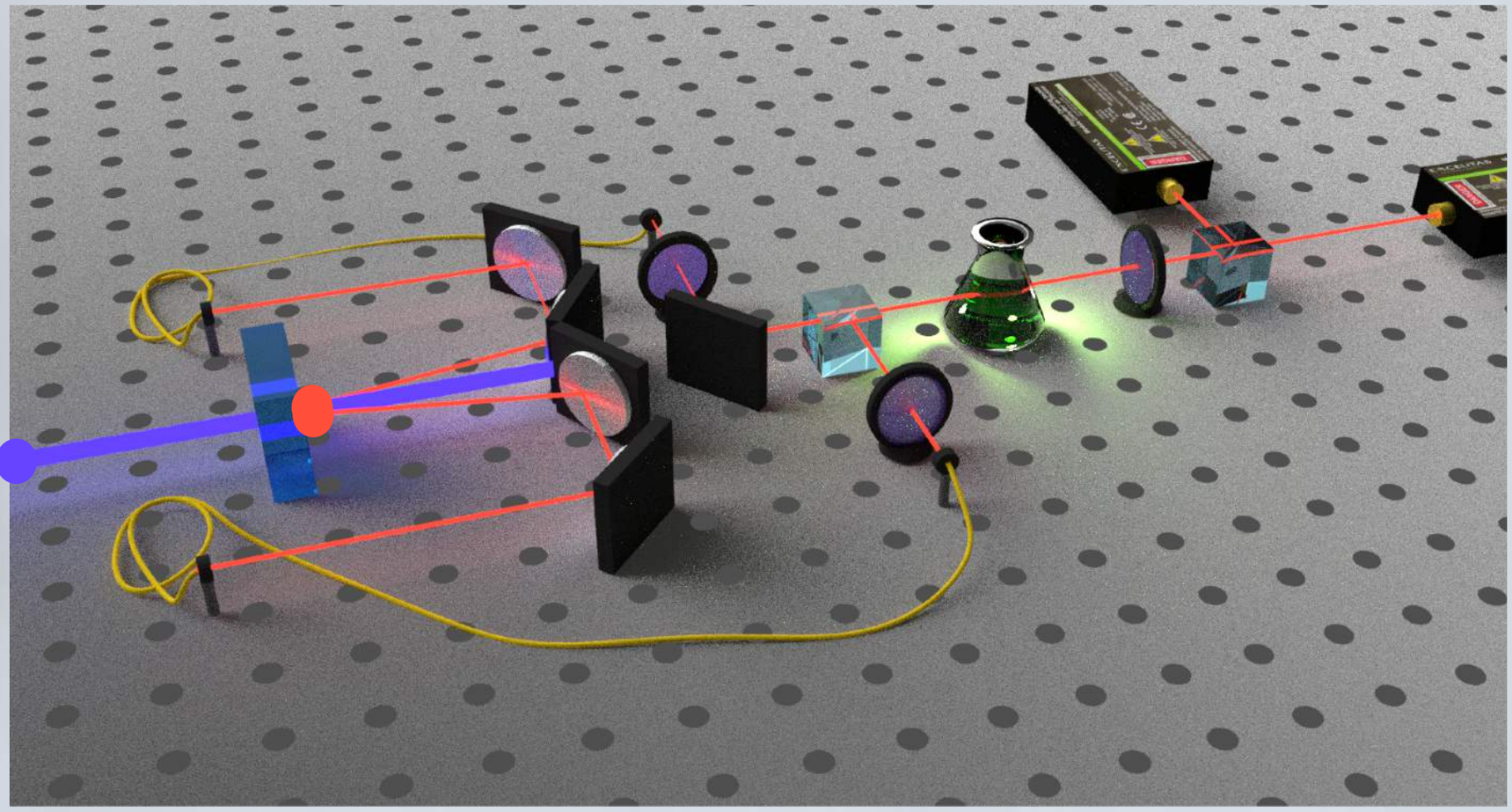

PROBE PREPARATION

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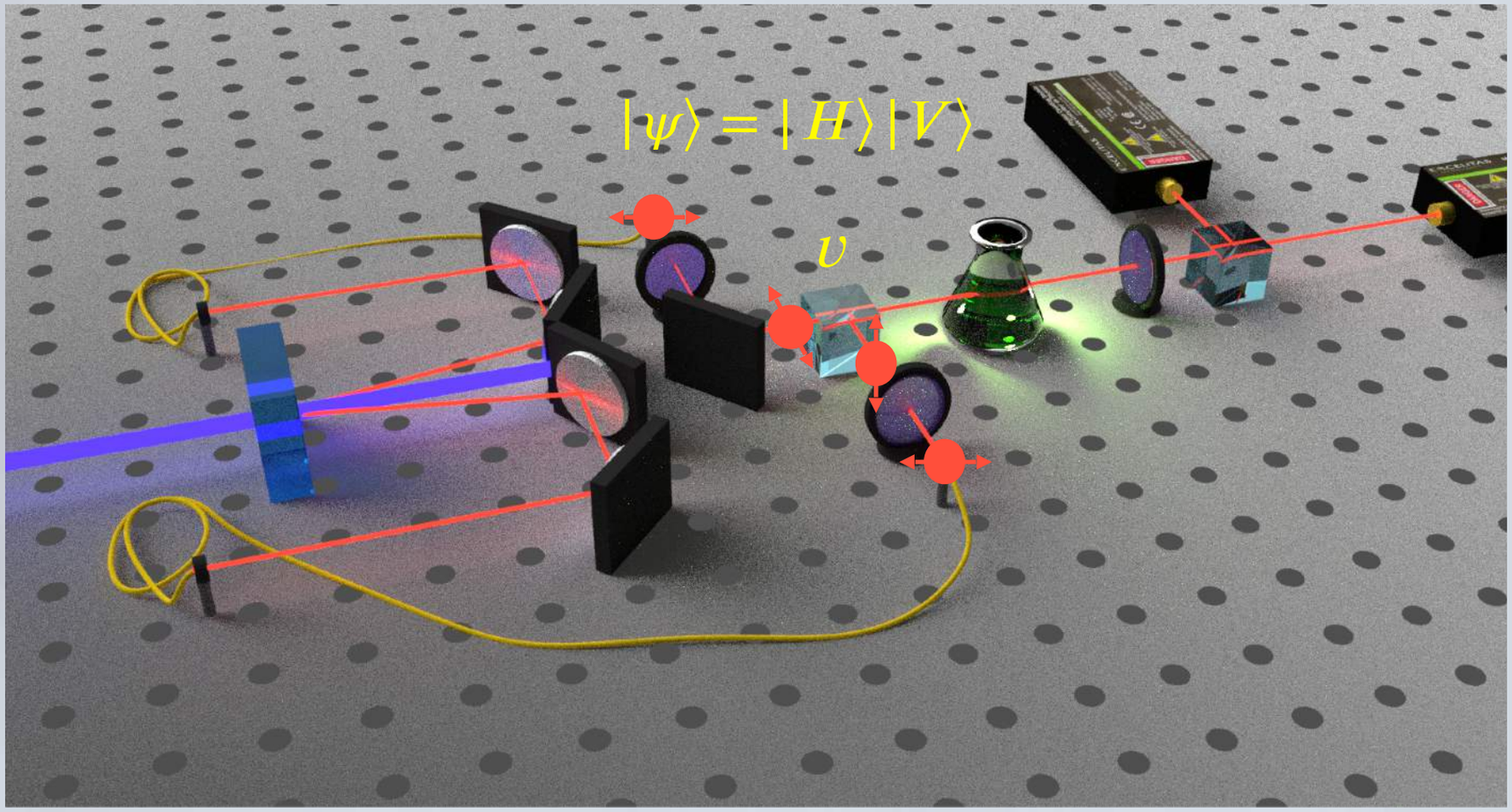
NUISANCE PARAMETERS



PROBE PREPARATION

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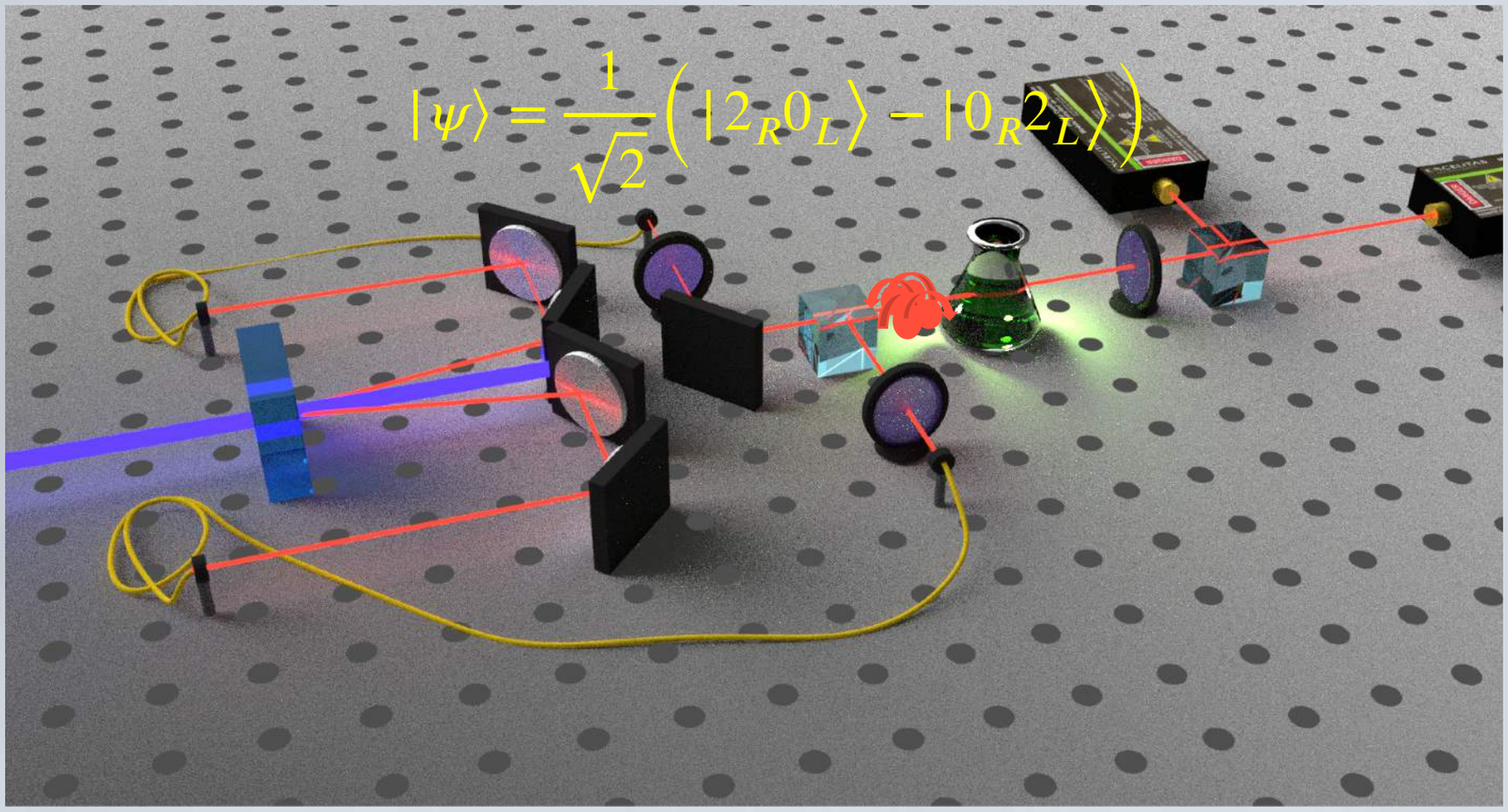
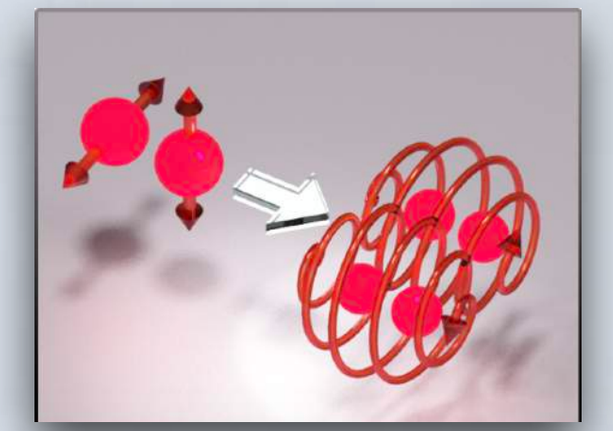


PROBE PREPARATION

NOON STATE IN THE CIRCULAR POLARISATION



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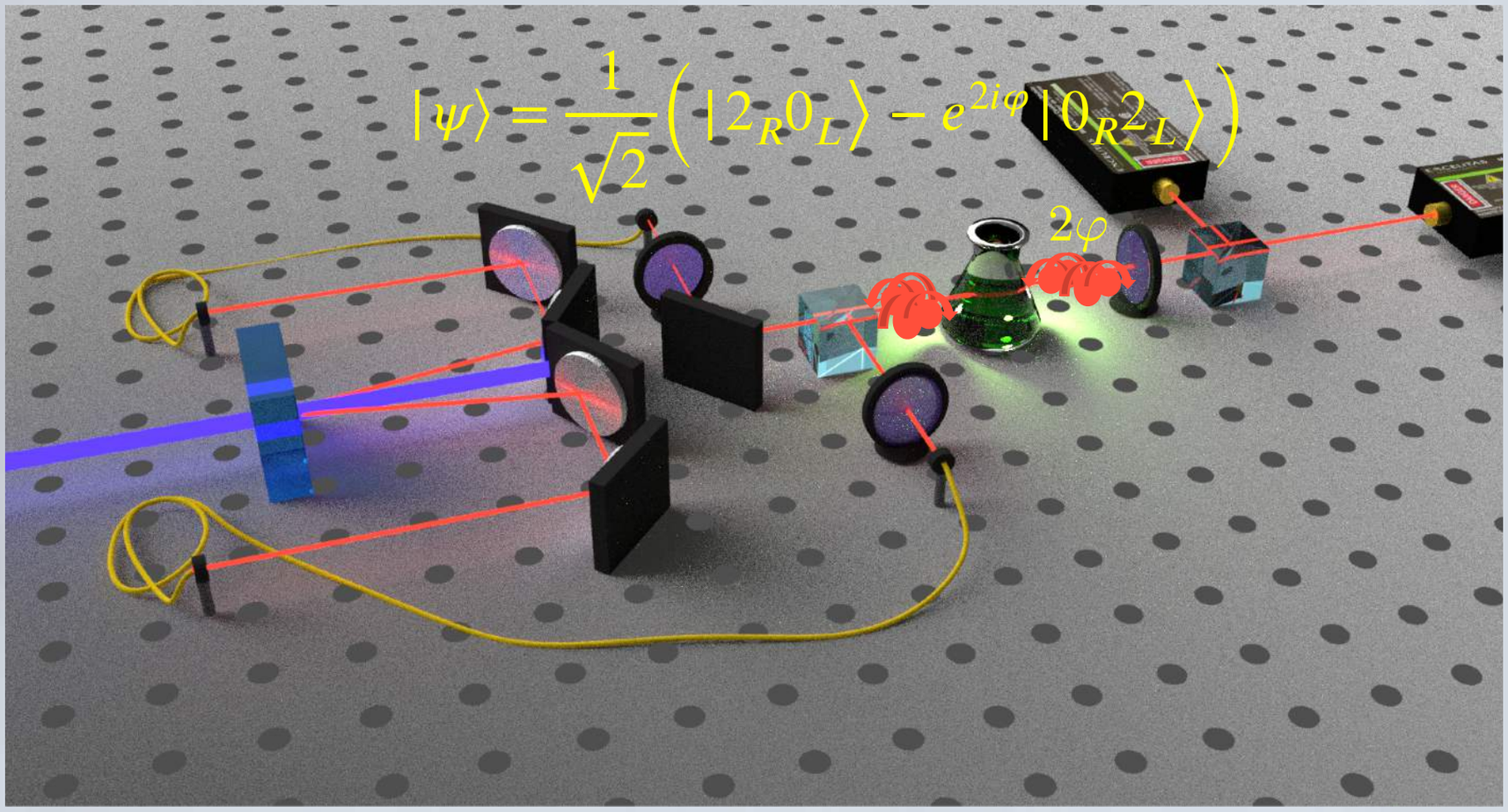
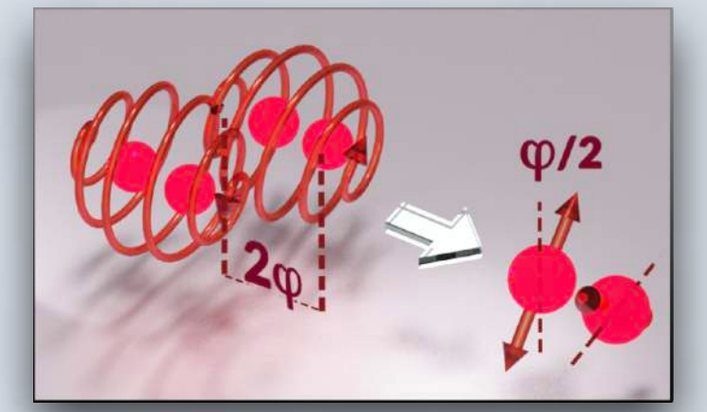


PROBE INTERACTION WITH SYSTEM

SAMPLE IMPARTS PHASE BETWEEN R AND L POLARISATION



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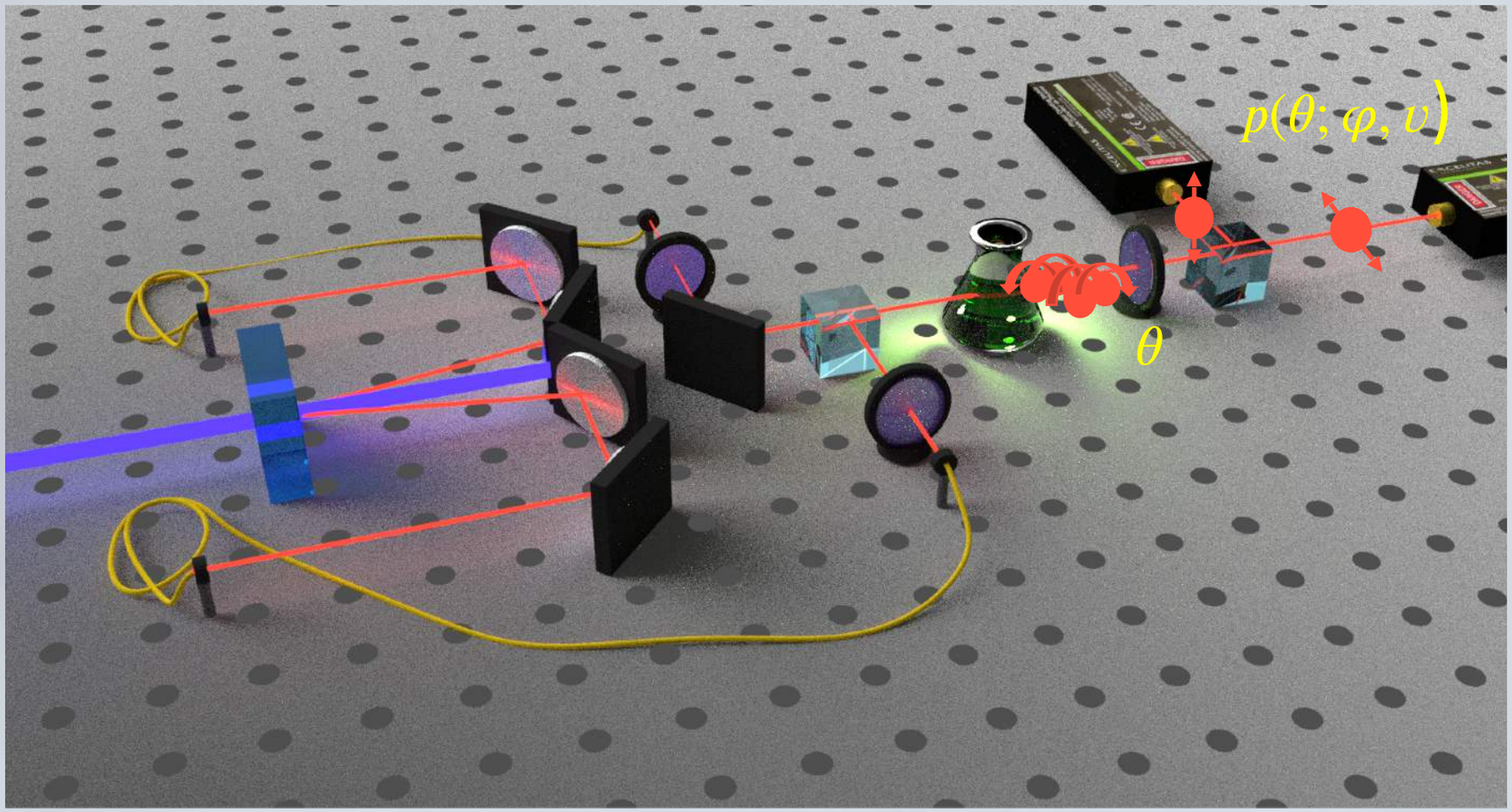
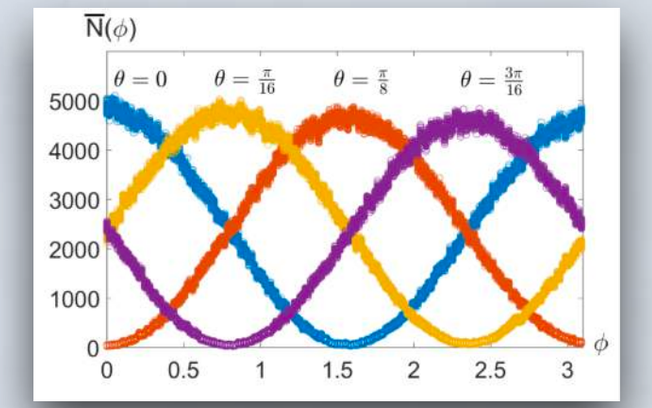


PROBE MEASUREMENT

PERFORM MEASUREMENTS FOR DIFFERENT VALUES OF θ



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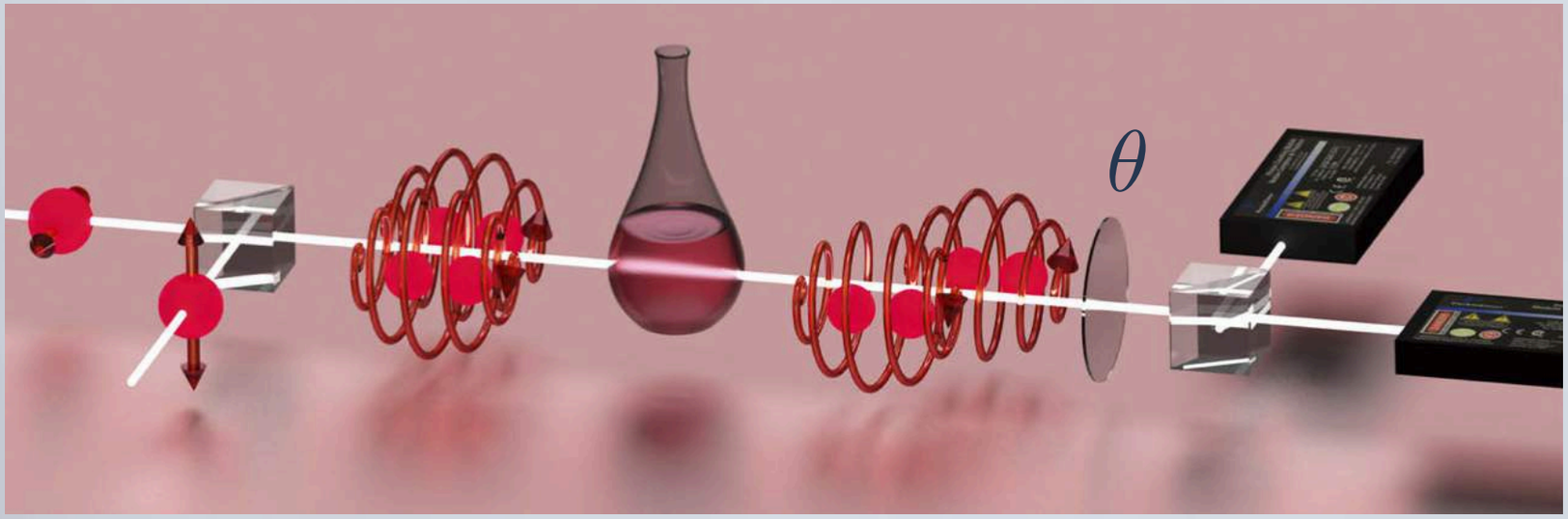

$$p(\theta|\varphi, v) = \frac{1}{4}[1 + v\cos(8\theta + 2\varphi)]$$

FROM EACH: n_θ

$$P_B(\varphi, v|n) = \mathcal{N}\prod_\theta p(\theta|\varphi, v)^{n_\theta} p(\varphi, v)$$

BAYESIAN PROBABILITY

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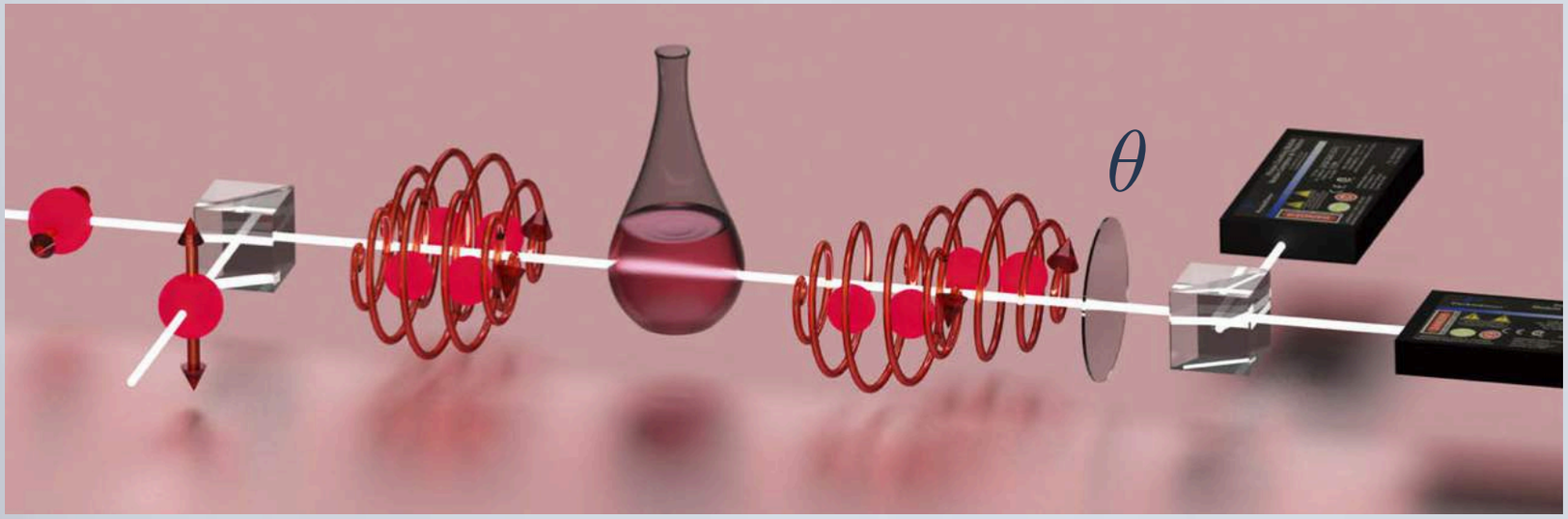

$$p(\theta|\varphi,v)=\frac{1}{4}[1+v\cos(8\theta+2\varphi)]$$

FROM EACH: n_θ

$$P_B(\varphi,v|n)=\mathcal{N}\prod_\theta p(\theta|\varphi,v)^{n_\theta}p(\varphi,v)$$

BAYESIAN PROBABILITY

$$\varphi_B=\int\varphi P_B(\varphi,v|n)d\varphi dv \quad \Delta^2\varphi_B=\int(\varphi-\varphi_B)^2P_B(\varphi,v|n)d\varphi dv$$

$$v_B=\int vP_B(\varphi,v|n)d\varphi dv \quad \Delta^2 v_B=\int(v-v_B)^2P_B(\varphi,v|n)d\varphi dv$$

MULTIPARAMER ESTIMATION

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UNIVERSITÀ DEGLI STUDI

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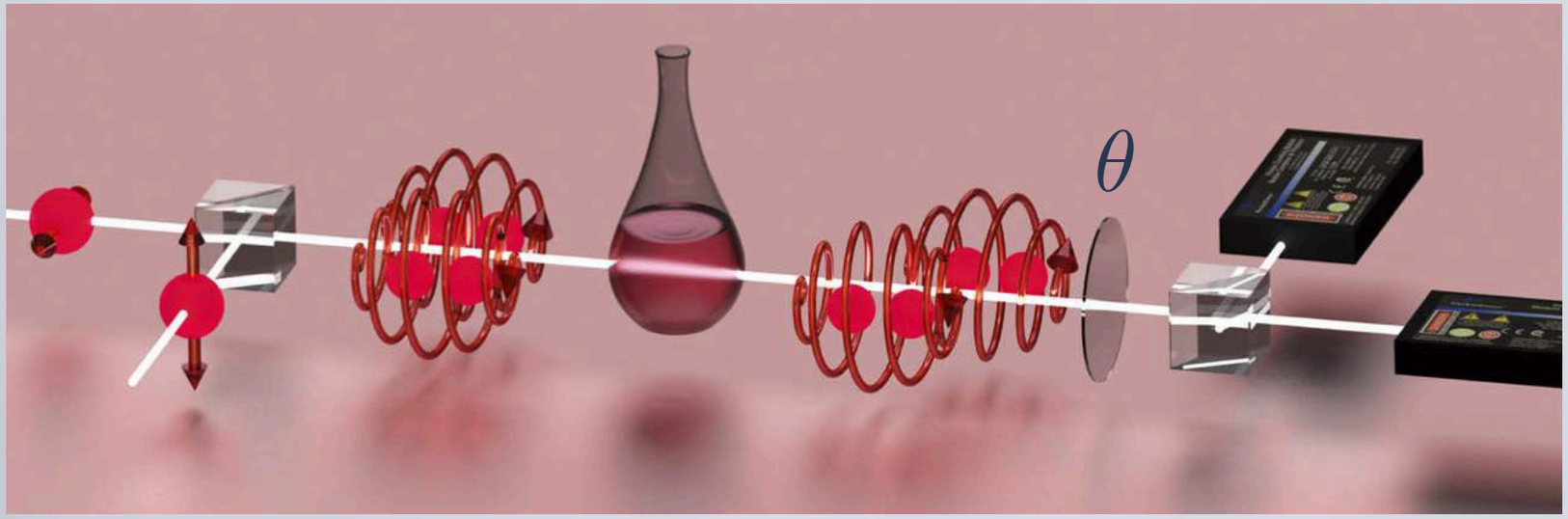
NUISANCE PARAMETERS



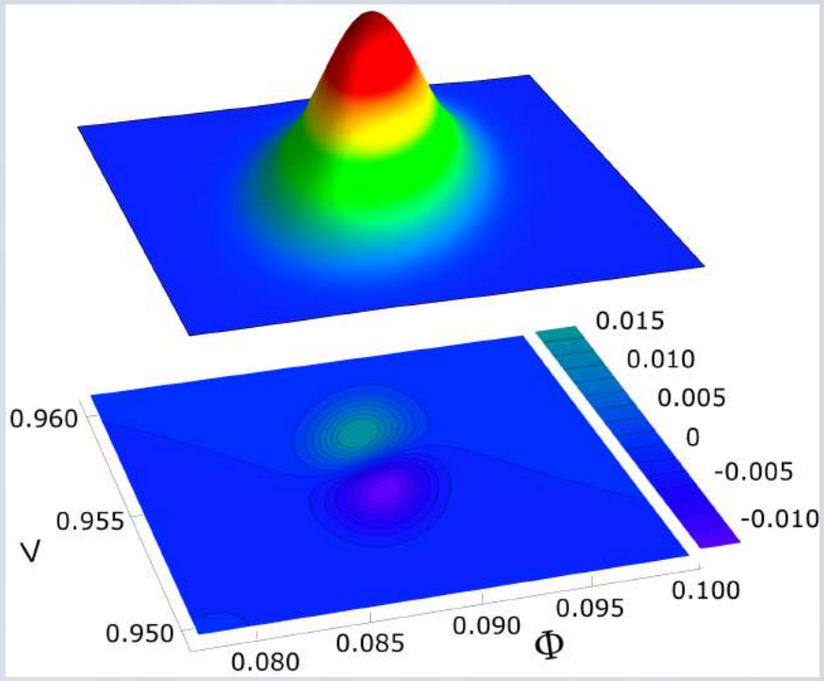
$$p(\theta|\varphi,v)=\frac{1}{4}[1+v\cos(8\theta+2\varphi)]$$

$$P_B(\varphi,v|n)=\mathcal{N}\prod_{\theta}p(\theta|\varphi,v)^{n_\theta}p(\varphi,v)$$

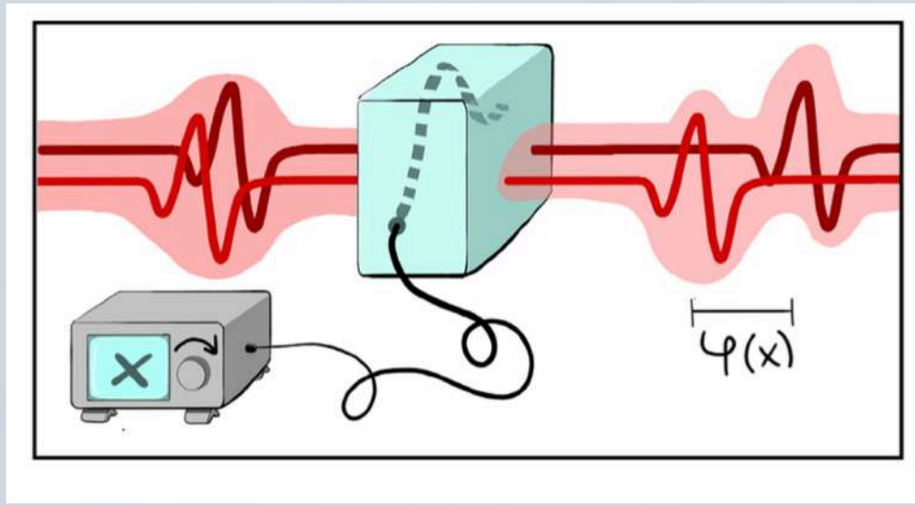
$$\varphi_B=\int\varphi P_B(\varphi,v|n)d\varphi dv$$

$$v_B=\int vP_B(\varphi,v|n)d\varphi dv$$

GLUCOSE SOLUTION



FUNCTION ESTIMATION



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FUNCTION ESTIMATION

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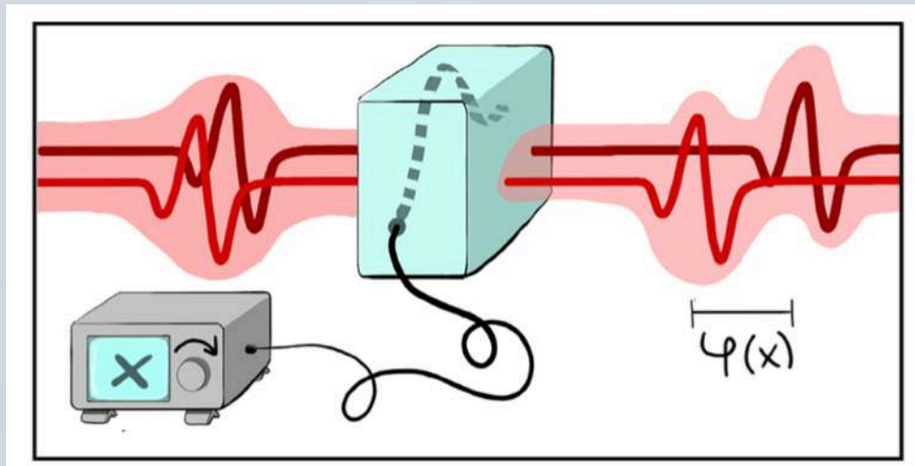

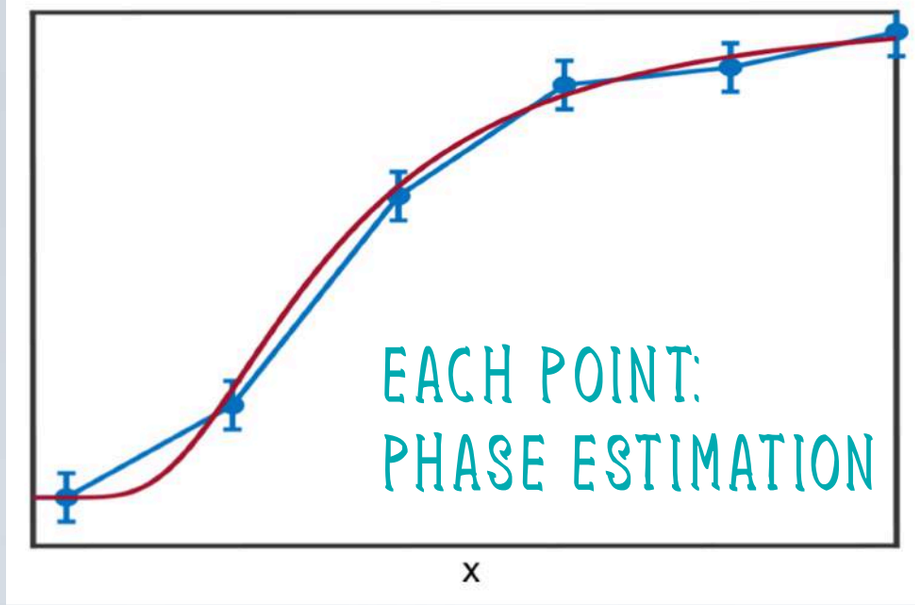

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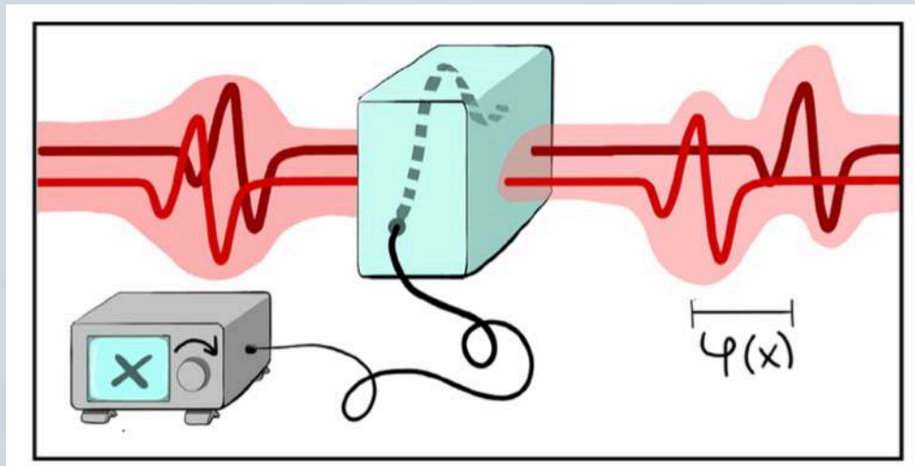

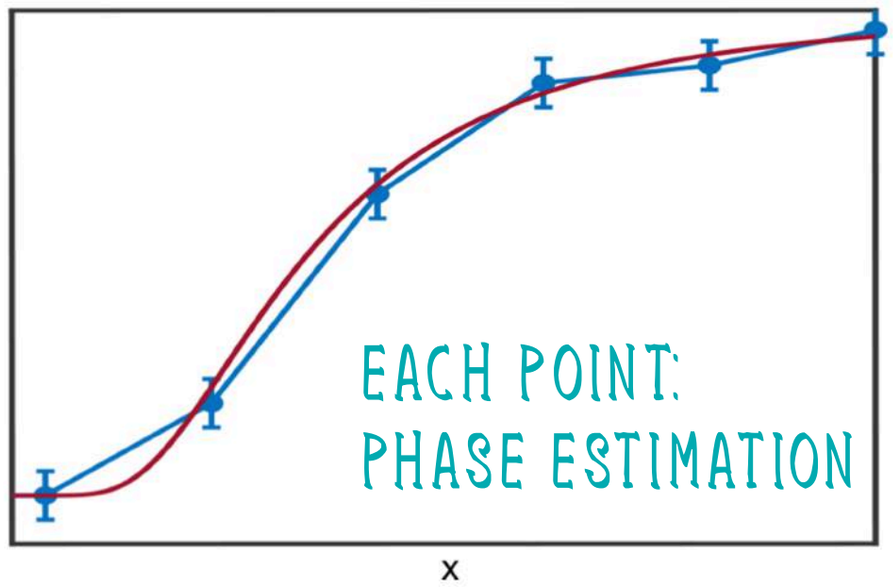

$$\delta_0^2 = \frac{1}{L}\mathbb{E}\left[\int |\varphi(x) - \tilde{\varphi}(x)|^2 dx\right]$$

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FUNCTION ESTIMATION

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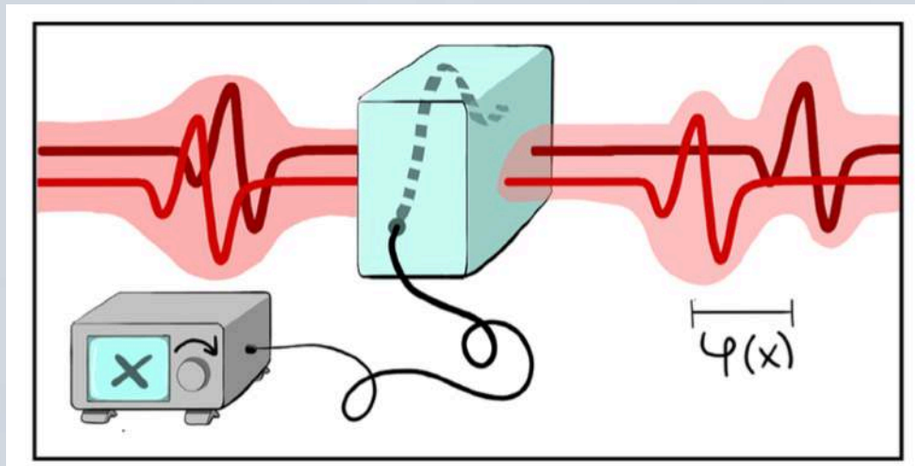

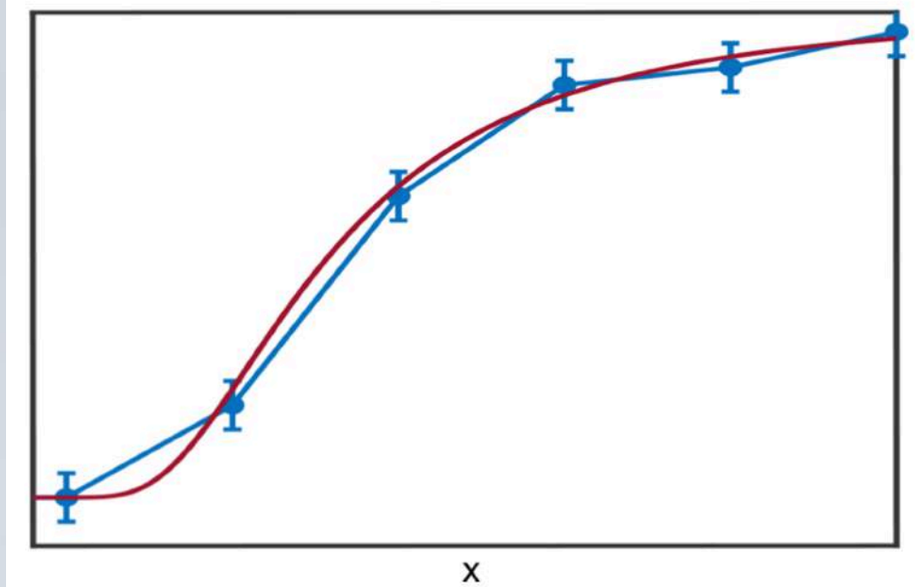

N_R = RES PER POINT

N_S = # OF POINTS

-> δ_0^2

CLASSICAL VS QUANTUM

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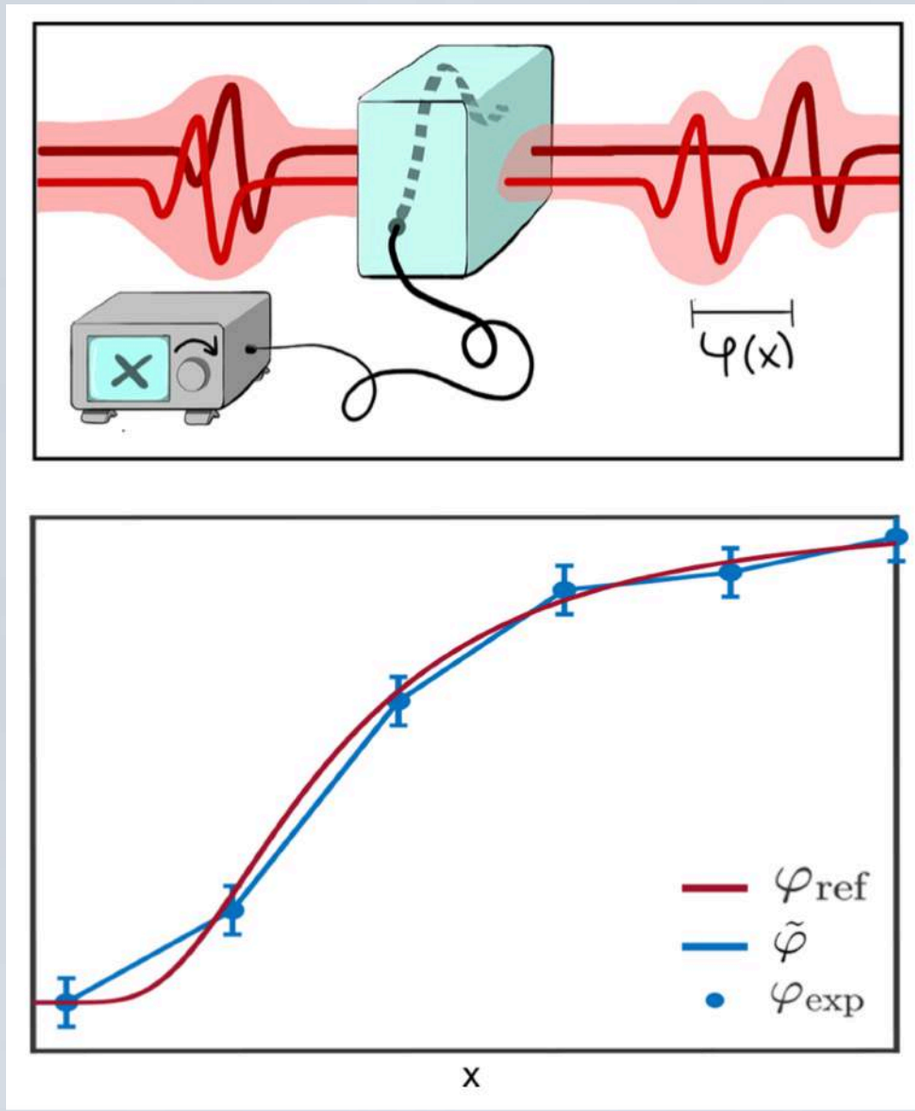
FUNCTION ESTIMATION



$$\delta_0^2 = \frac{1}{L}\sum_{x=0}^{L} \mathbb{E}\left[|\varphi_{ref}(x) - \tilde{\varphi}(x)|^2\right] \Delta x_{ref}$$

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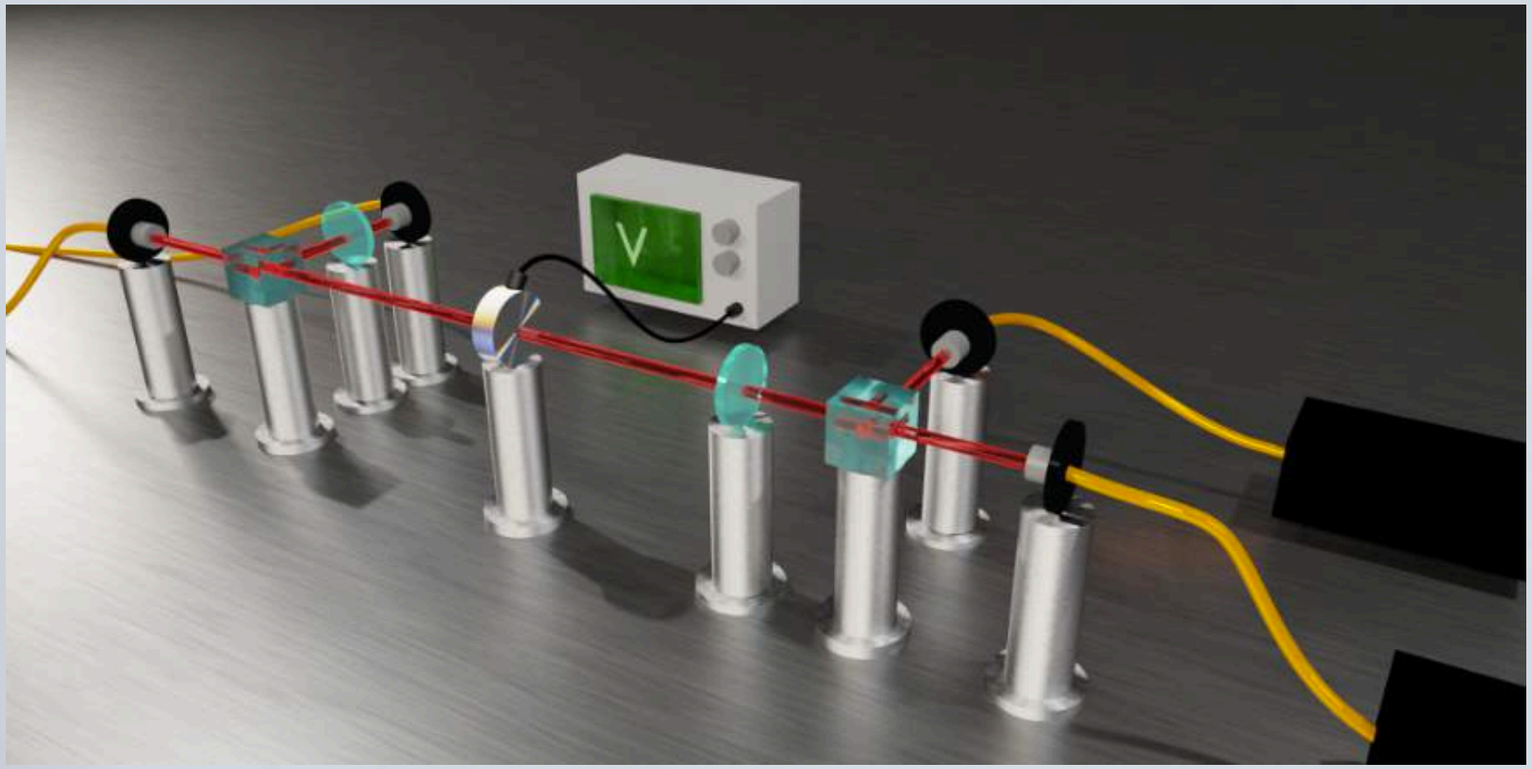
FUNCTION ESTIMATION



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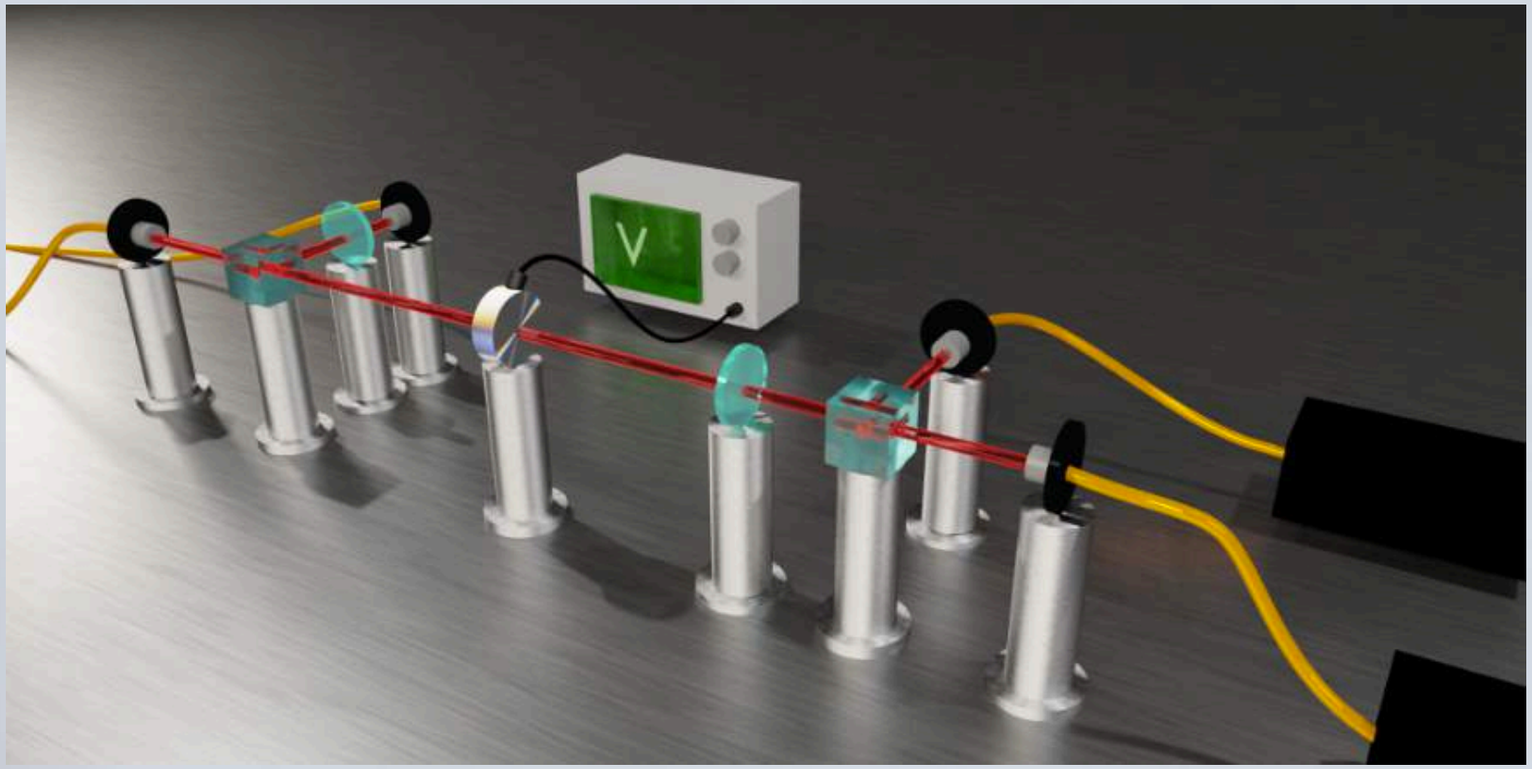
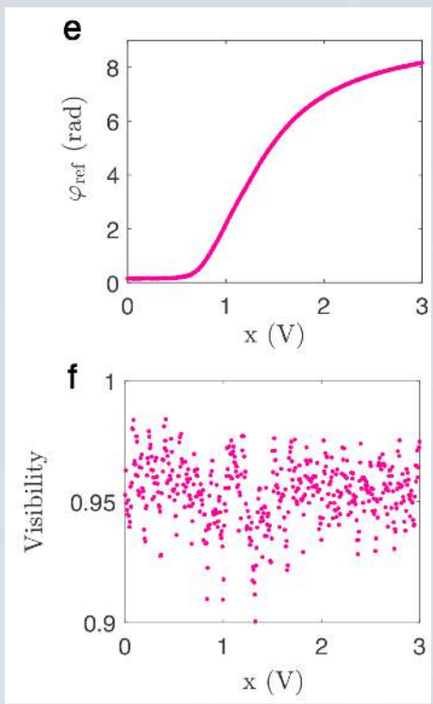
FUNCTION ESTIMATION





REFERENCE
N_R =60K
N_S =500

EACH POINT:
MULTIPARAMETER
PHASE ESTIMATION

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FUNCTION ESTIMATION

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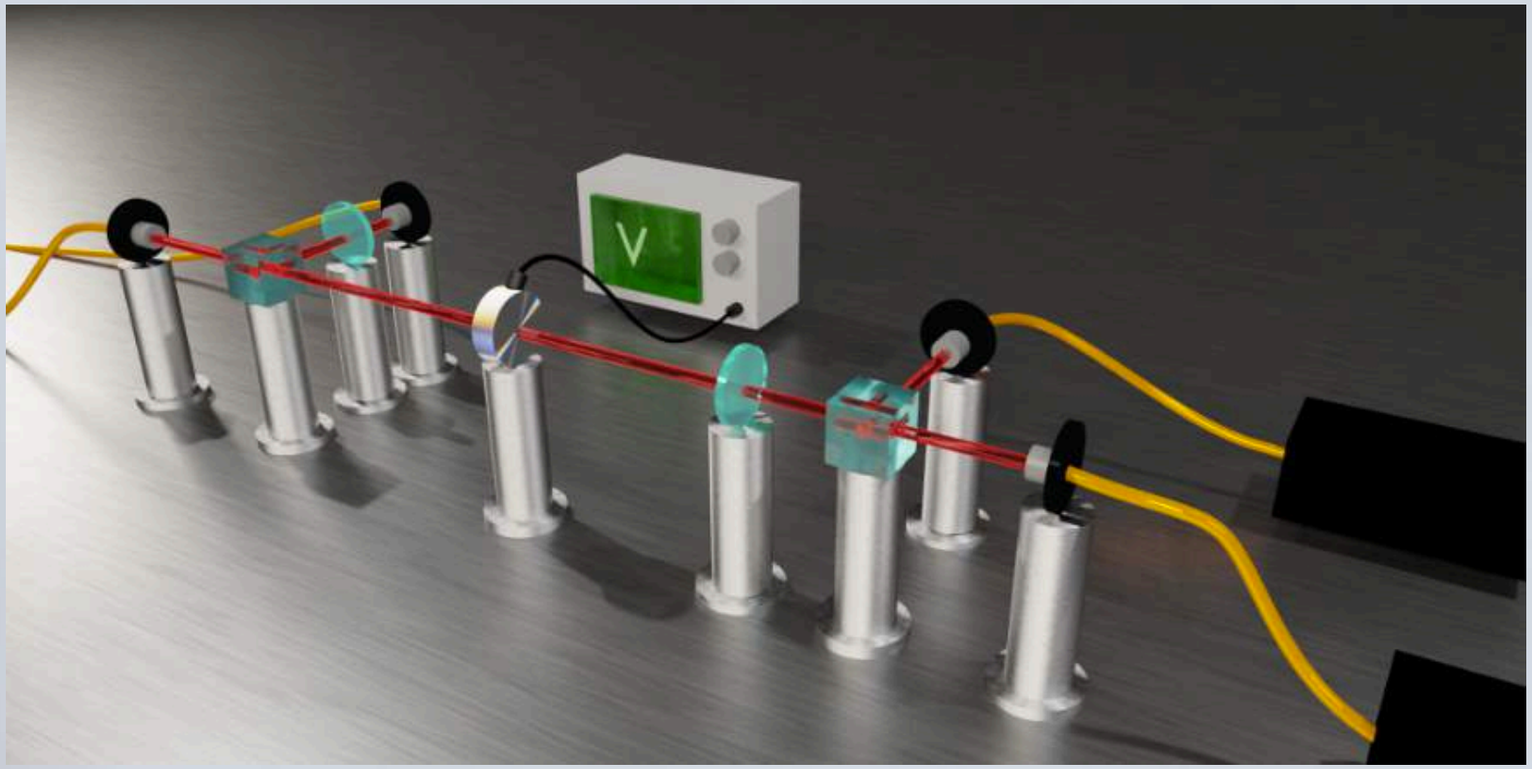
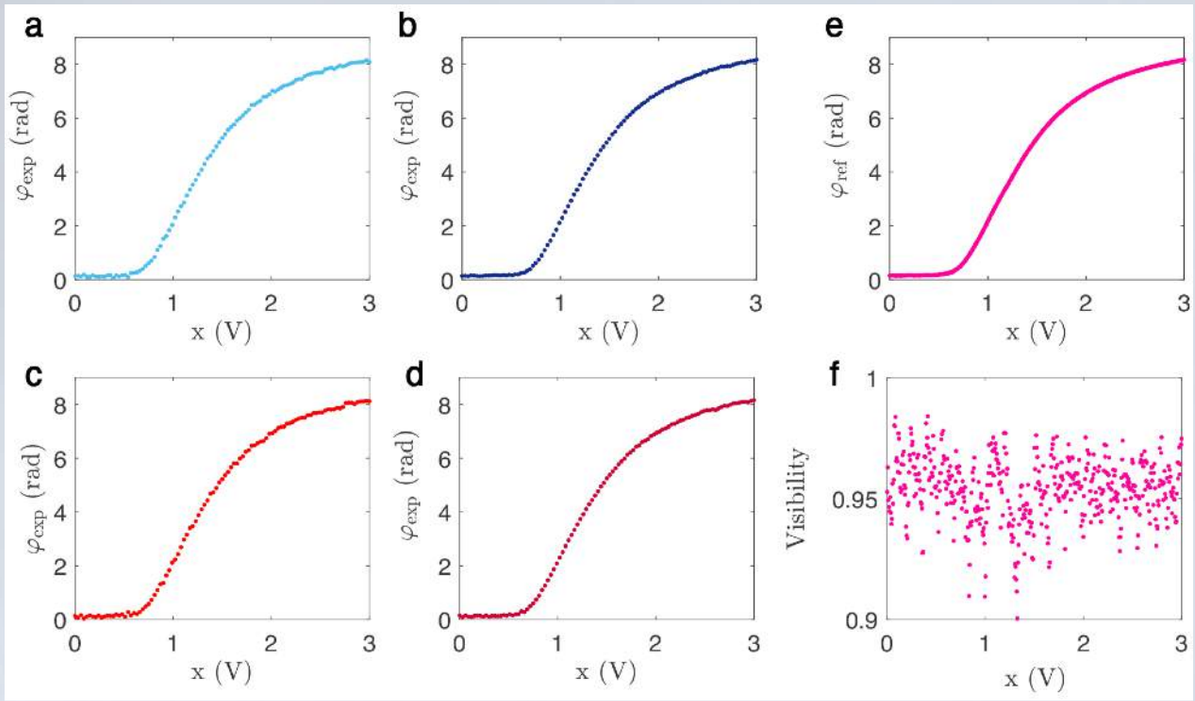


$N_R = 800$ $N_R = 3800$

NOON
100 POINTS

SP
100 POINTS

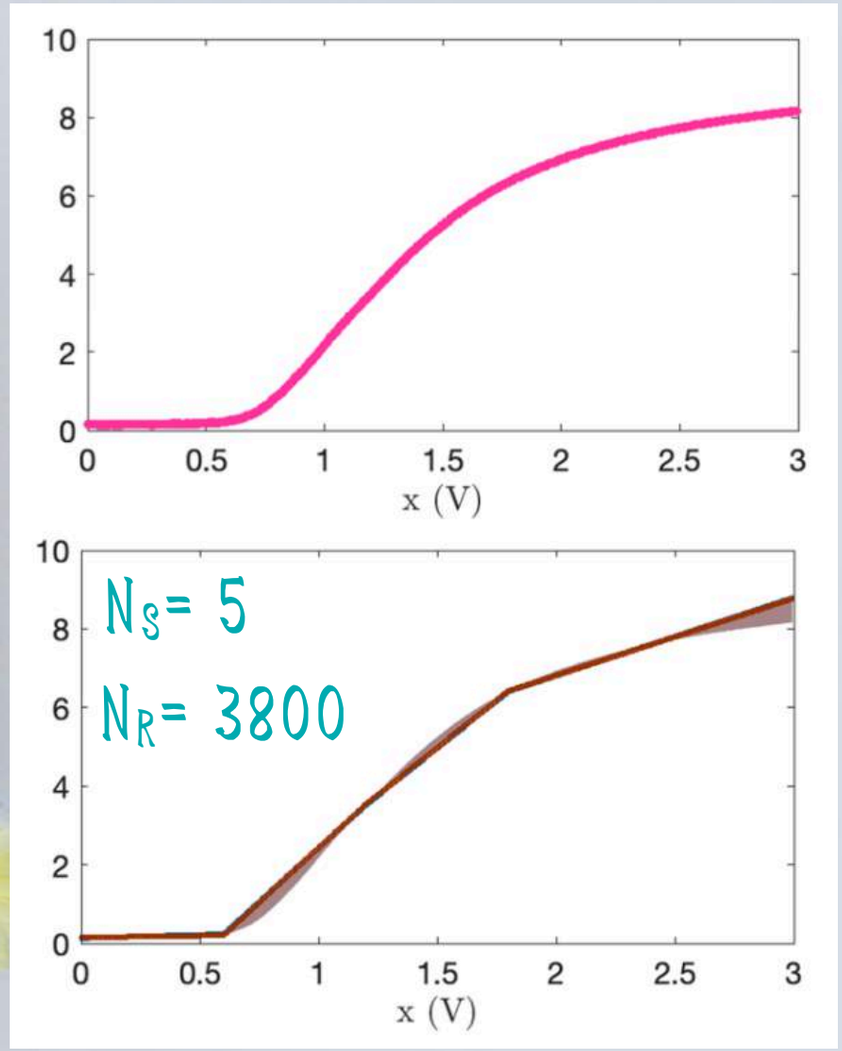


REFERENCE
$N_R = 60K$
$N_S = 500$

EACH POINT:
MULTIPARAMETER
PHASE ESTIMATION

I. GIANANI ET AL, PRA 103 (2021)

FUNCTION ESTIMATION



VARYING N_S=5-100

INTERPOLATE TO N_S=500 (REF)

(LINEAR INTERPOLATION)

$$\Delta\tilde{\varphi}(x) = \varphi_{ref}(x) - \tilde{\varphi}(x)$$

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FUNCTION ESTIMATION

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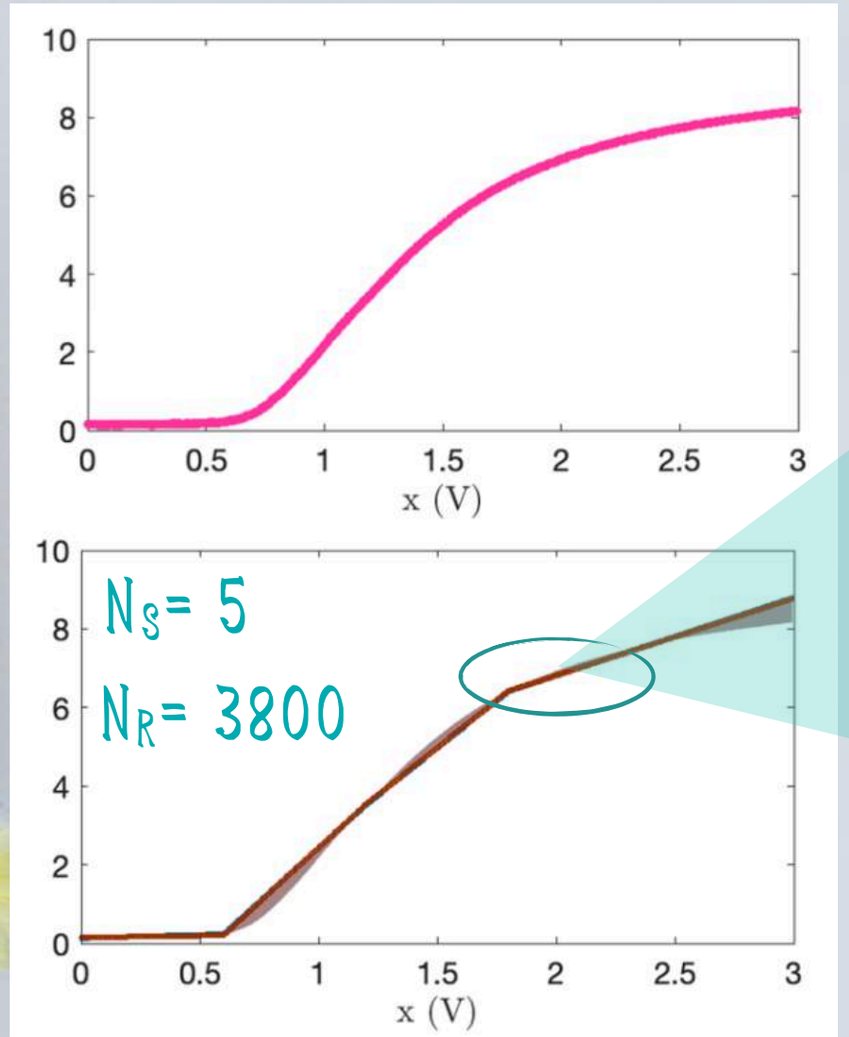
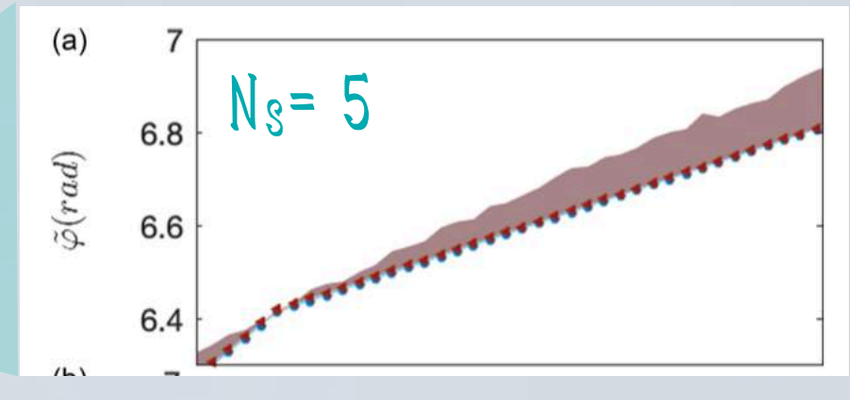


VARYING N_S=5-100

INTERPOLATE TO N_S=500 (REF)

(LINEAR INTERPOLATION)



$$\Delta\tilde{\varphi}(x) = \varphi_{ref}(x) - \tilde{\varphi}(x)$$

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FUNCTION ESTIMATION

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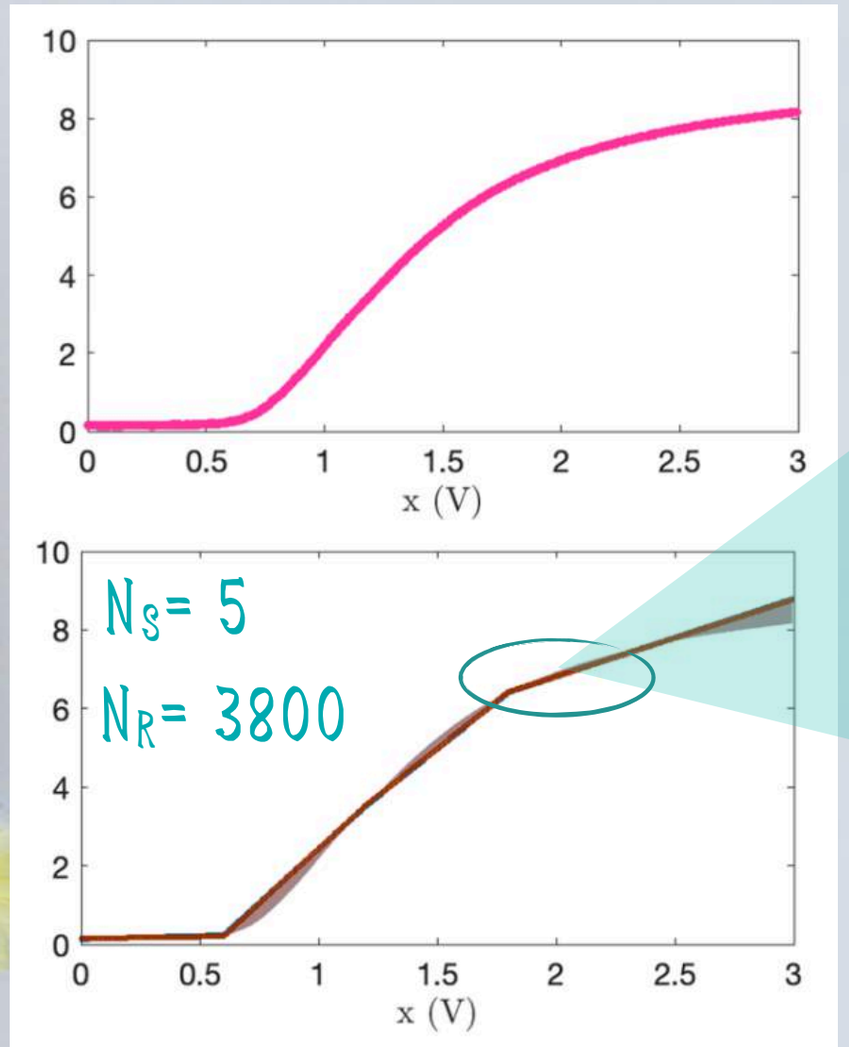
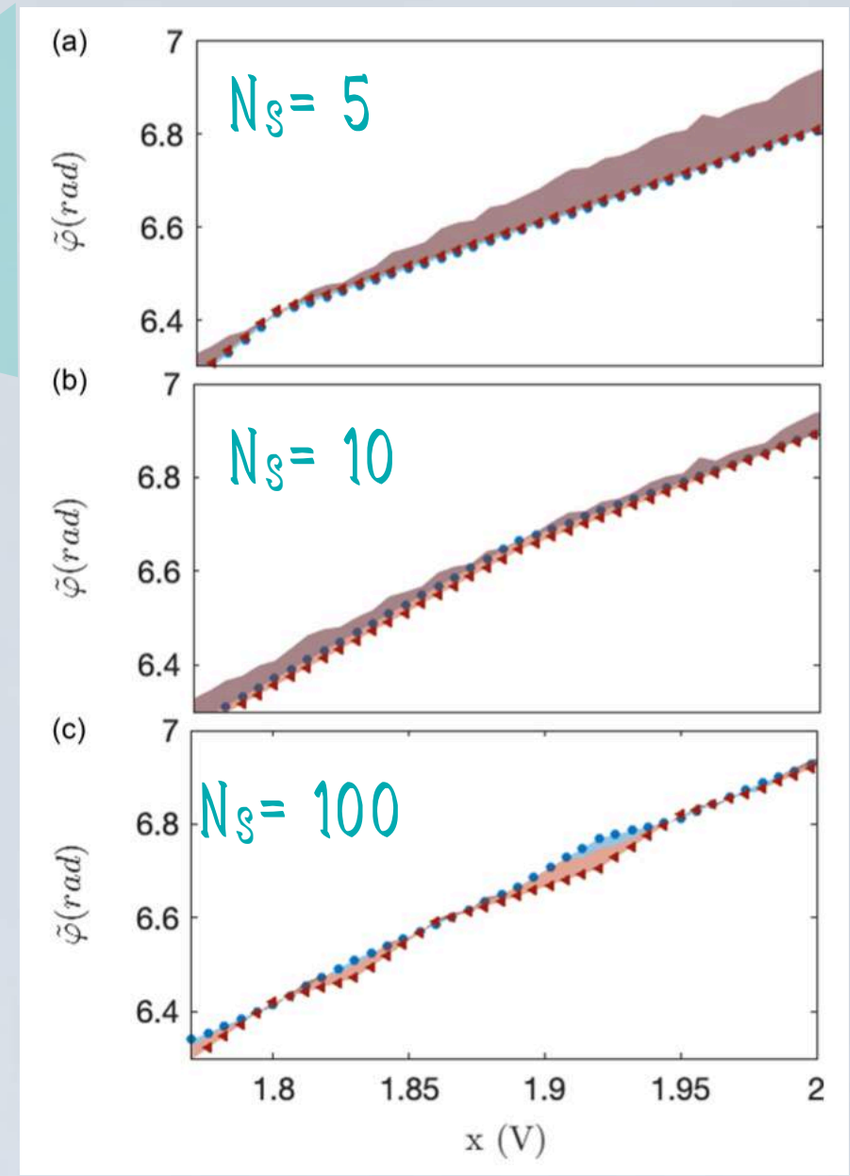


$$\Delta\tilde{\varphi}(x) = \varphi_{ref}(x) - \tilde{\varphi}(x)$$

VARYING N_S=5-100

INTERPOLATE TO N_S=500 (REF)

(LINEAR INTERPOLATION)



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FUNCTION ESTIMATION

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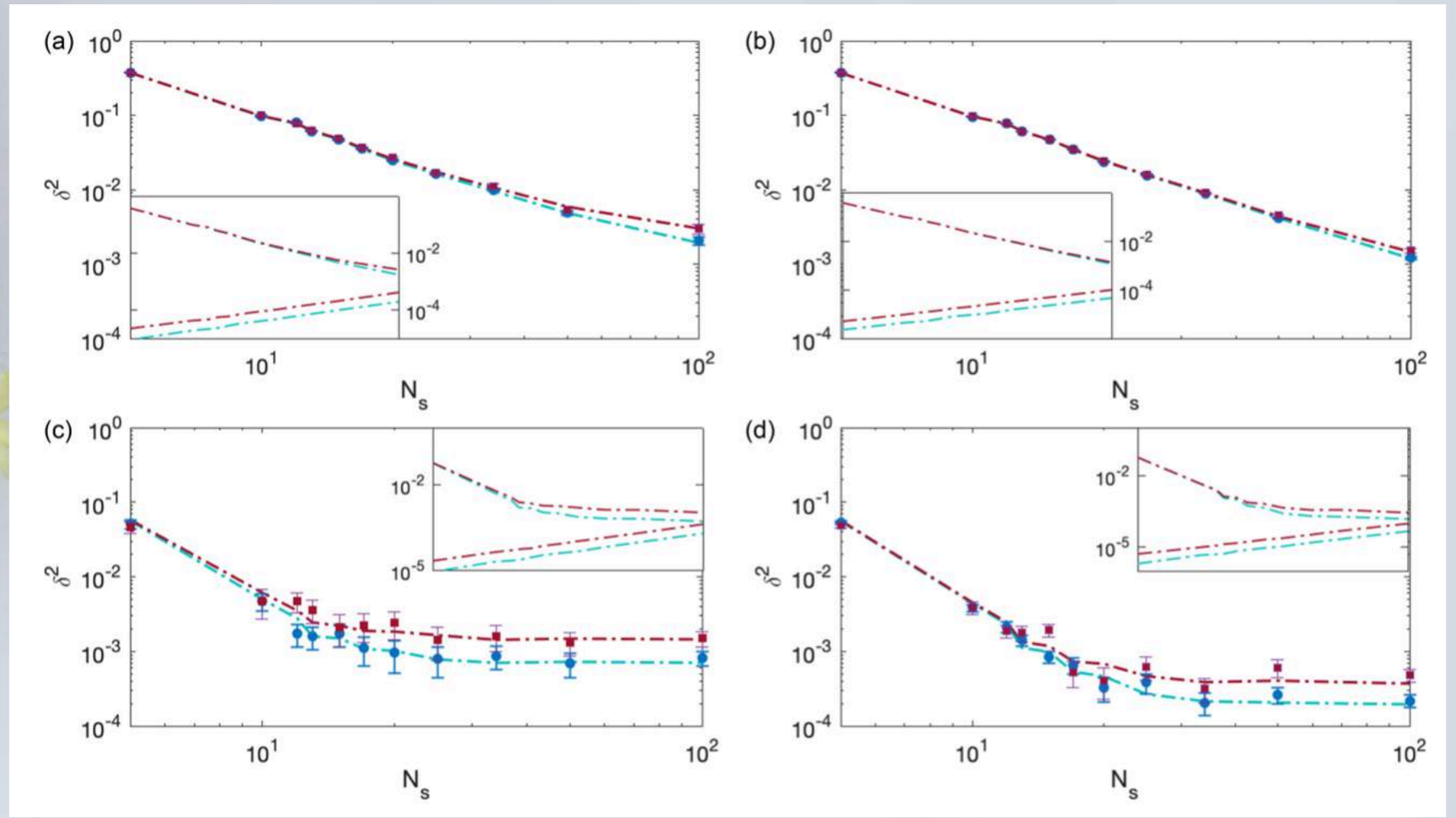
$$\delta_0^2 = \frac{1}{L}\sum_{x=0}^{L} \mathbb{E}\left[|\varphi_{ref}(x) - \tilde{\varphi}(x)|^2\right] \Delta x_{ref}$$



NEAREST NEIGHBOUR

LINEAR

FIXED N_R= 800

FIXED N_R=3800

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FUNCTION ESTIMATION

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TWO SOURCES OF ERROR

STATISTICAL

INCREASE RESOURCES PER POINT

N_R

INTERPOLATION

INCREASE NUMBER OF POINTS

N_S

TOTAL NUMBER OF RESOURCES

$$N = N_R N_S$$

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FUNCTION ESTIMATION

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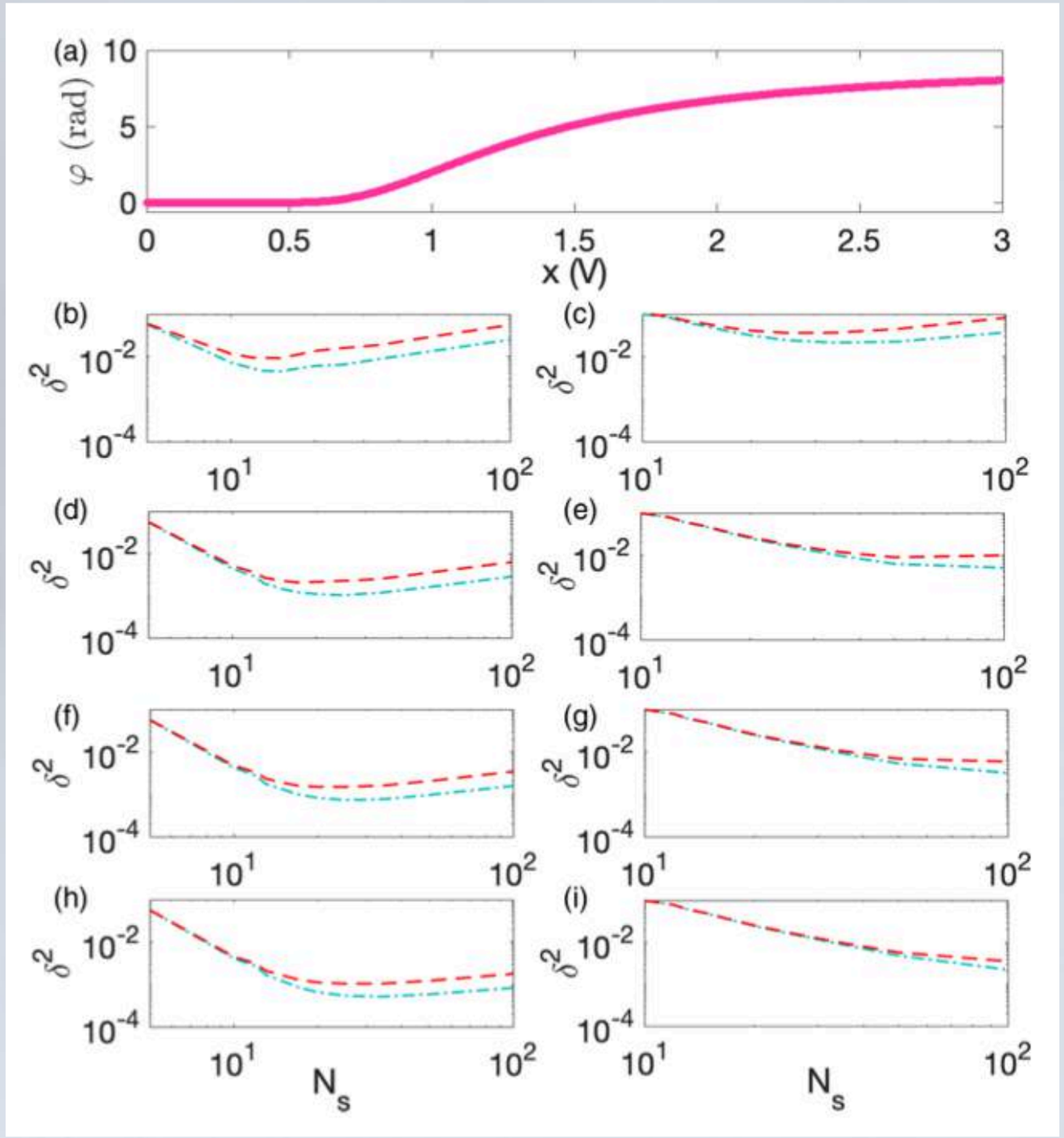
FIXED TOTAL NUMBER OF RESOURCES $N=N_R N_S$



N= 1K

N= 10K

N= 20K

N= 50K

LINEAR

NEAREST NEIGHBOUR

FUNCTION ESTIMATION

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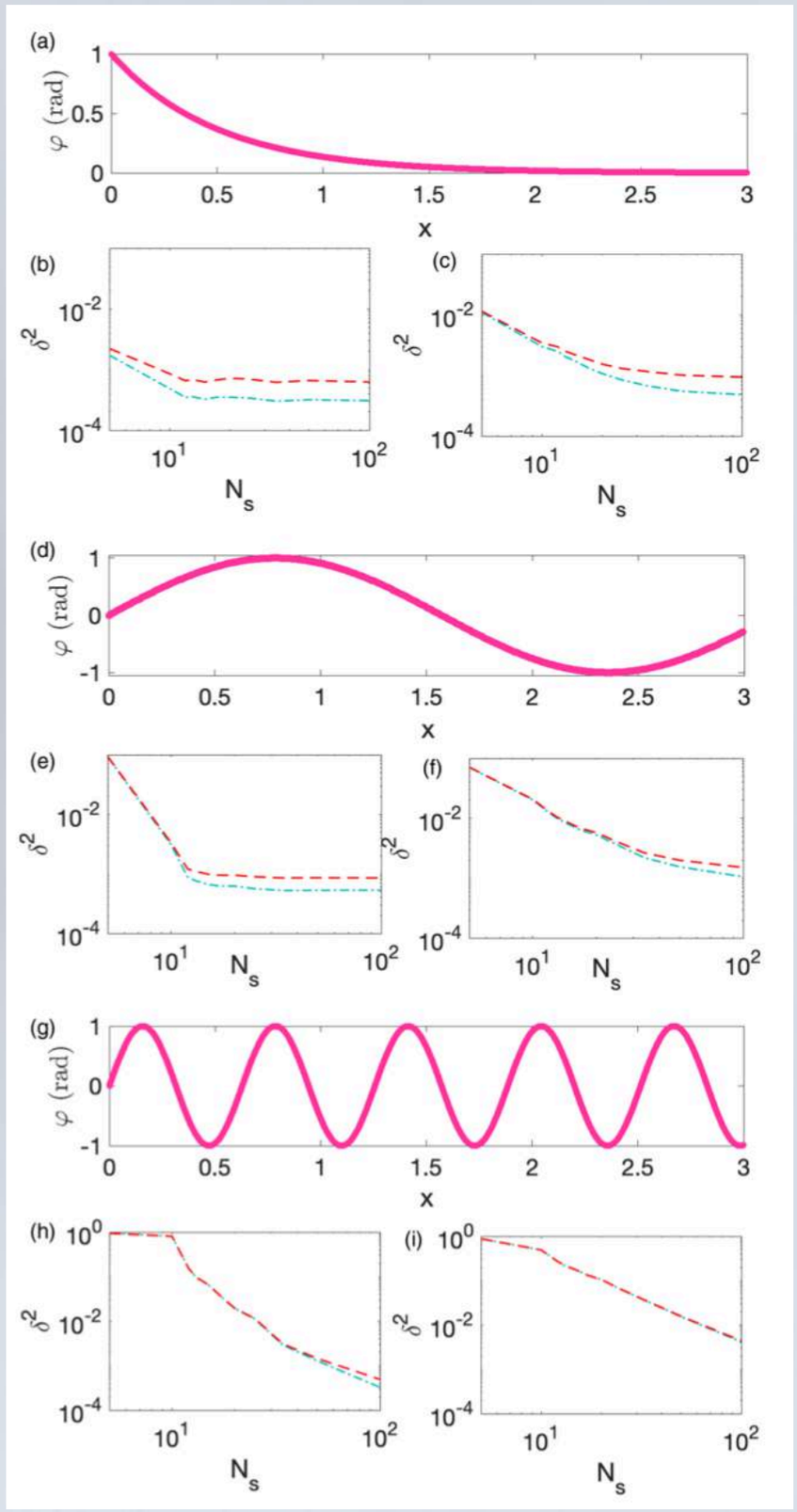

FIXED N_R=3000 N_S=5-100

YAY ADVANTAGE

SOME ADVANTAGE

NO ADVANTAGE

LINEAR

NEAREST NEIGHBOUR

CONCLUSIONS

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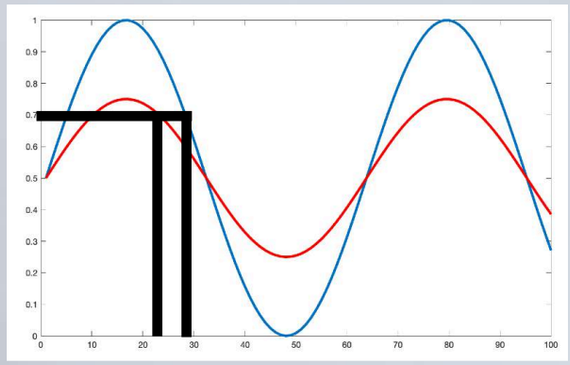
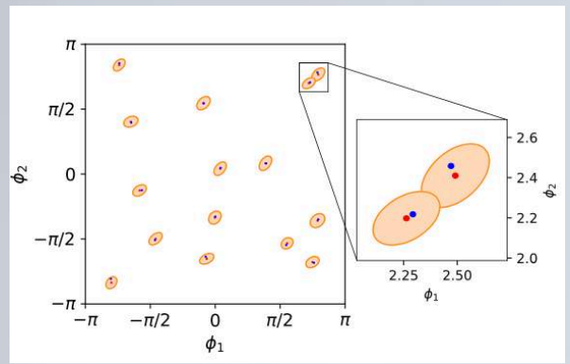
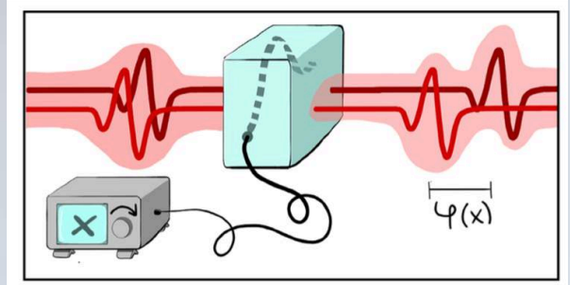
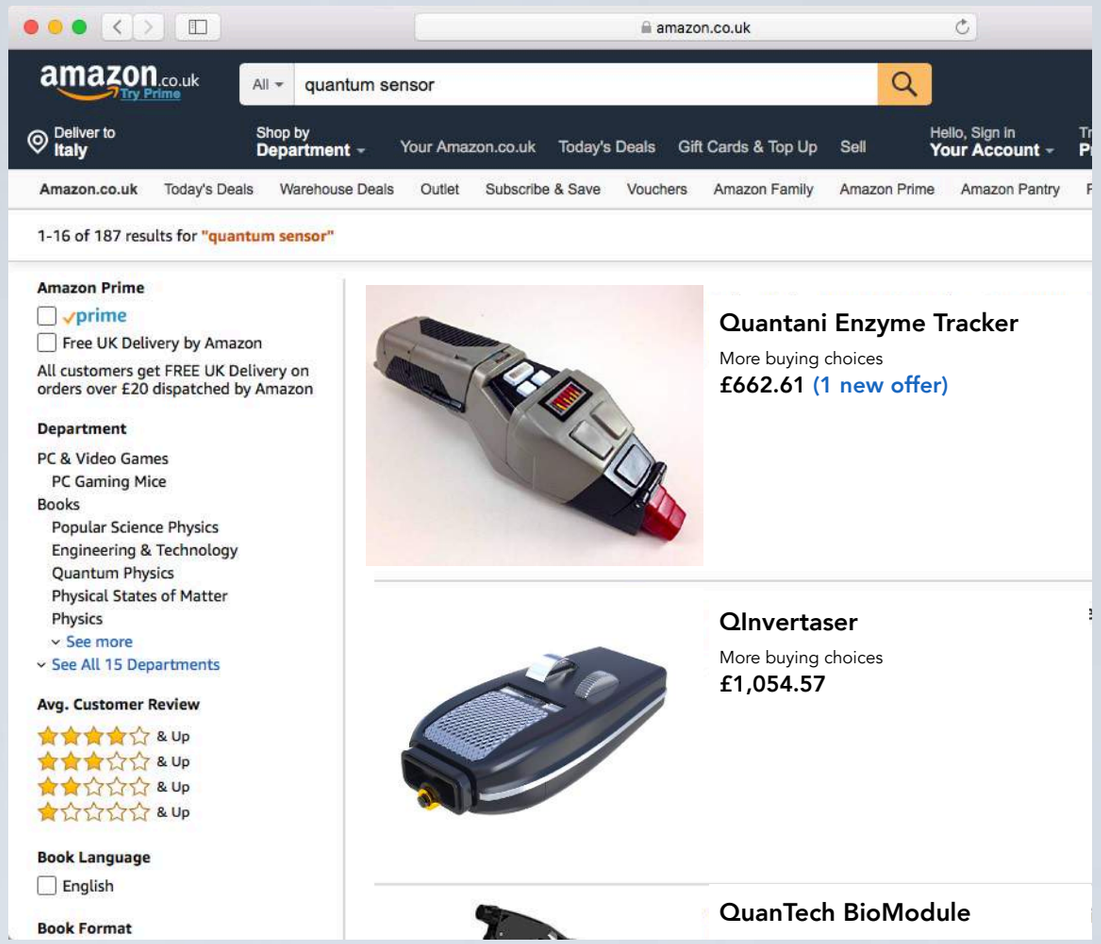
ESTIMATION OF MULTIPLE PARAMETERS







NECESSARY STEP



CONCLUSIONS

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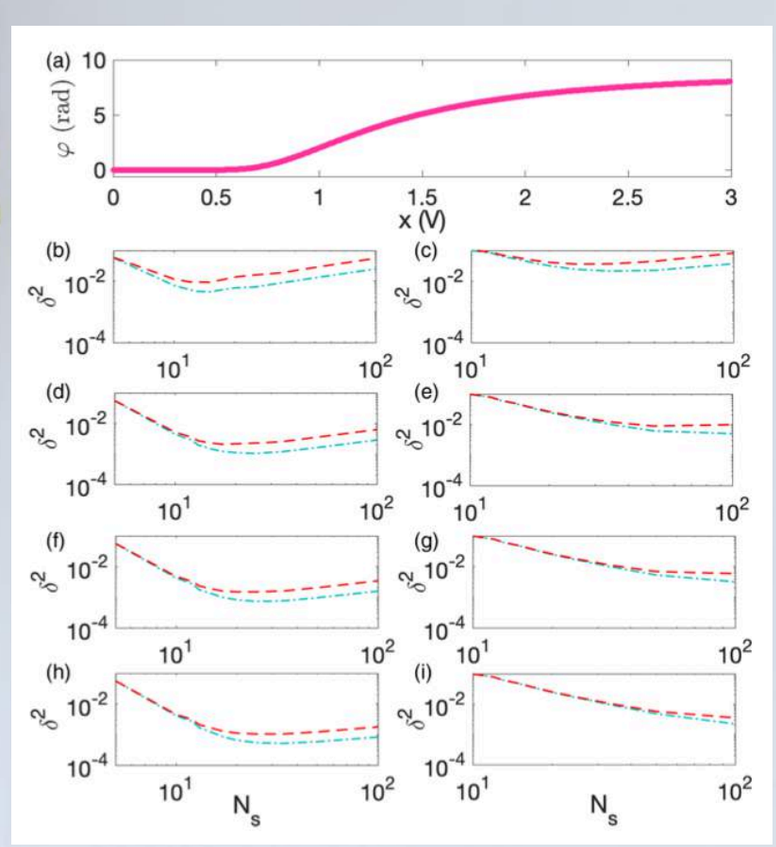
RESOLUTION IS AN ISSUE

RESOURCES SHOULD BE EMPLOYED CAREFULLY

$N = N_R N_S$



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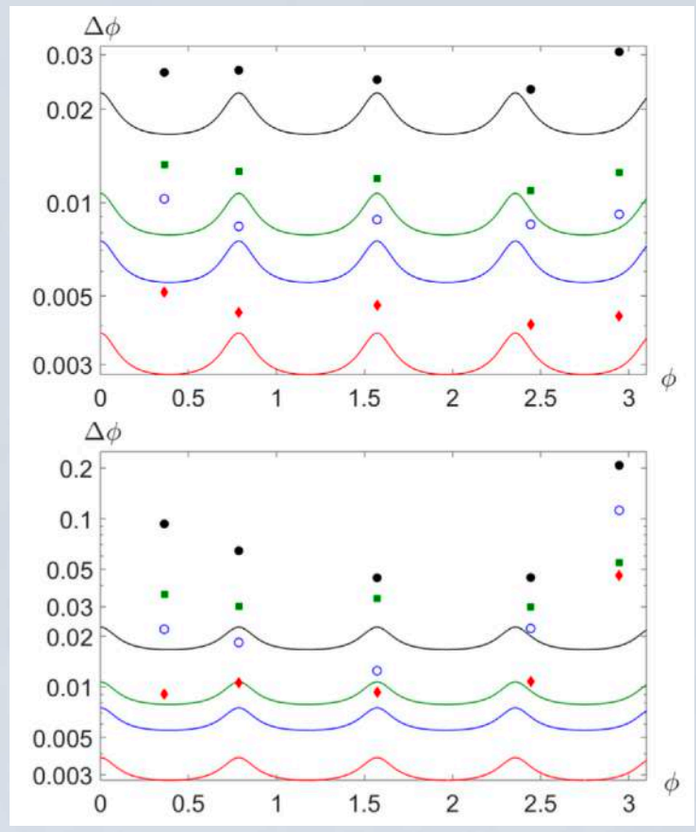

V. CIMINI ET AL, PRL 123 (2019)

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GIORDANO RAMPIONI
MARTA MELLINI

MILAN:

MARCO GENONI

WARSAW:

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THANK YOU FOR YOUR ATTENTION!