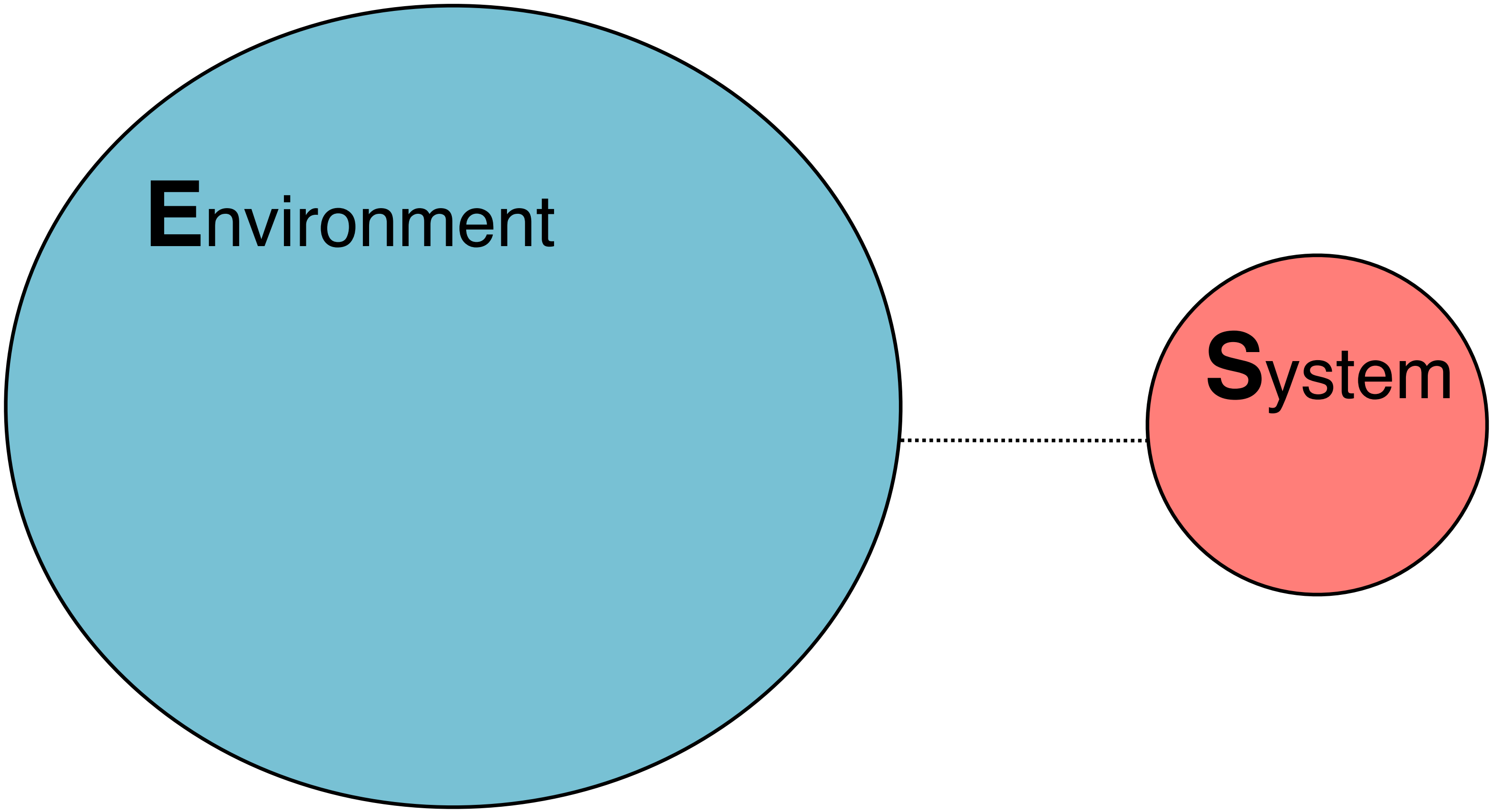
27.04.2021

Information flow in open quantum system dynamics

Nina Megier

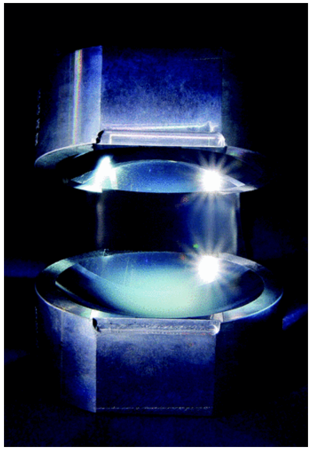
Andrea Smirne, Bassano Vacchini

Open quantum systems



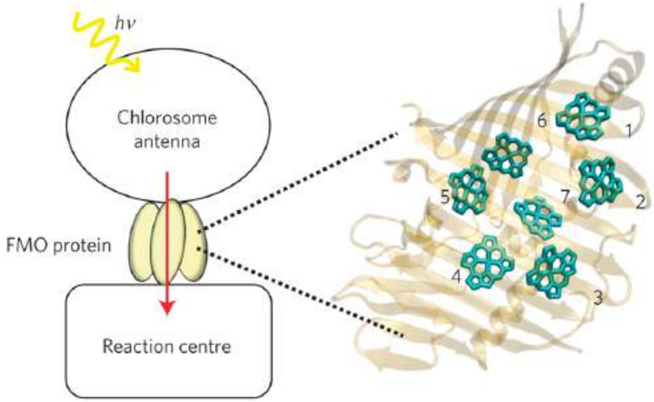
Open quantum systems

quantum optics



Cavity Quantum Electrodynamics[1]

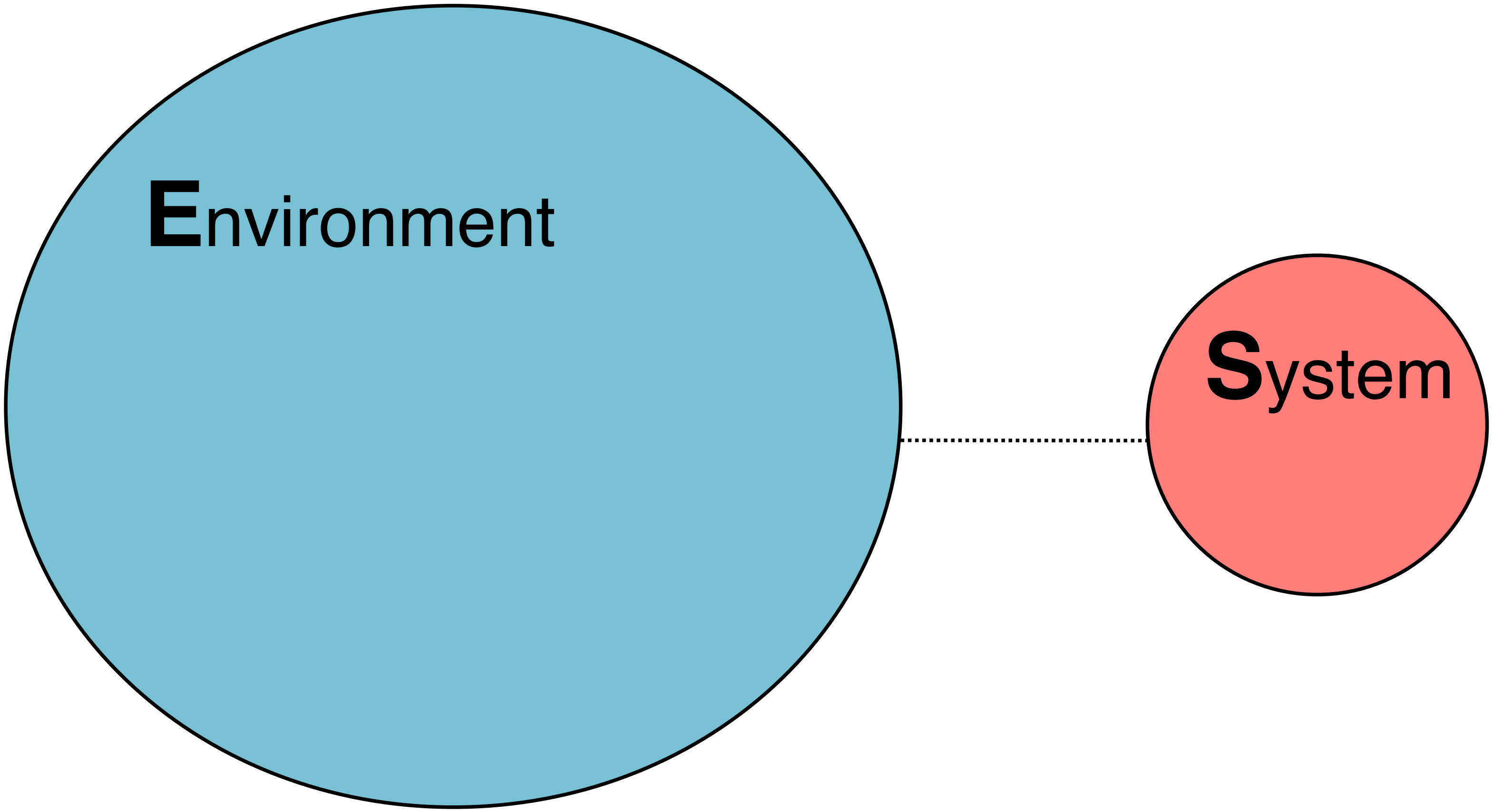
biophysics



Photosynthesis[2]

1. S. Haroche, Rev. Mod. Phys. 85, 1083 (2013)
2. M. Sarovar, A. Ishizaki, G. R. Fleming, K. B. Whaley, Nature Physics 6, 462 (2010)

Open quantum systems



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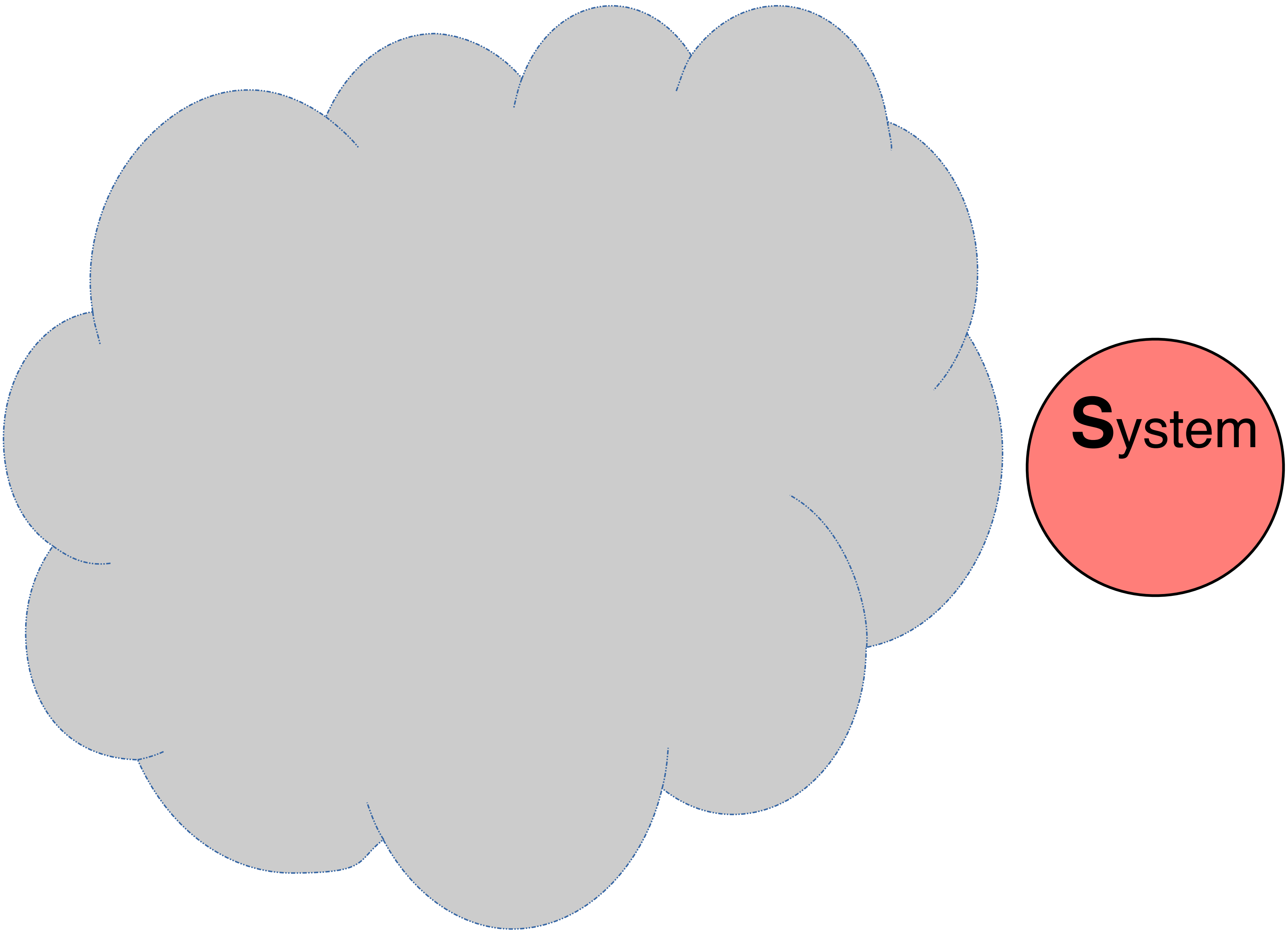

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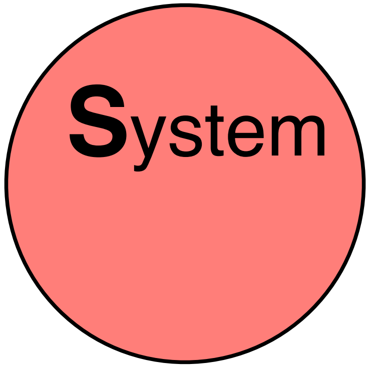

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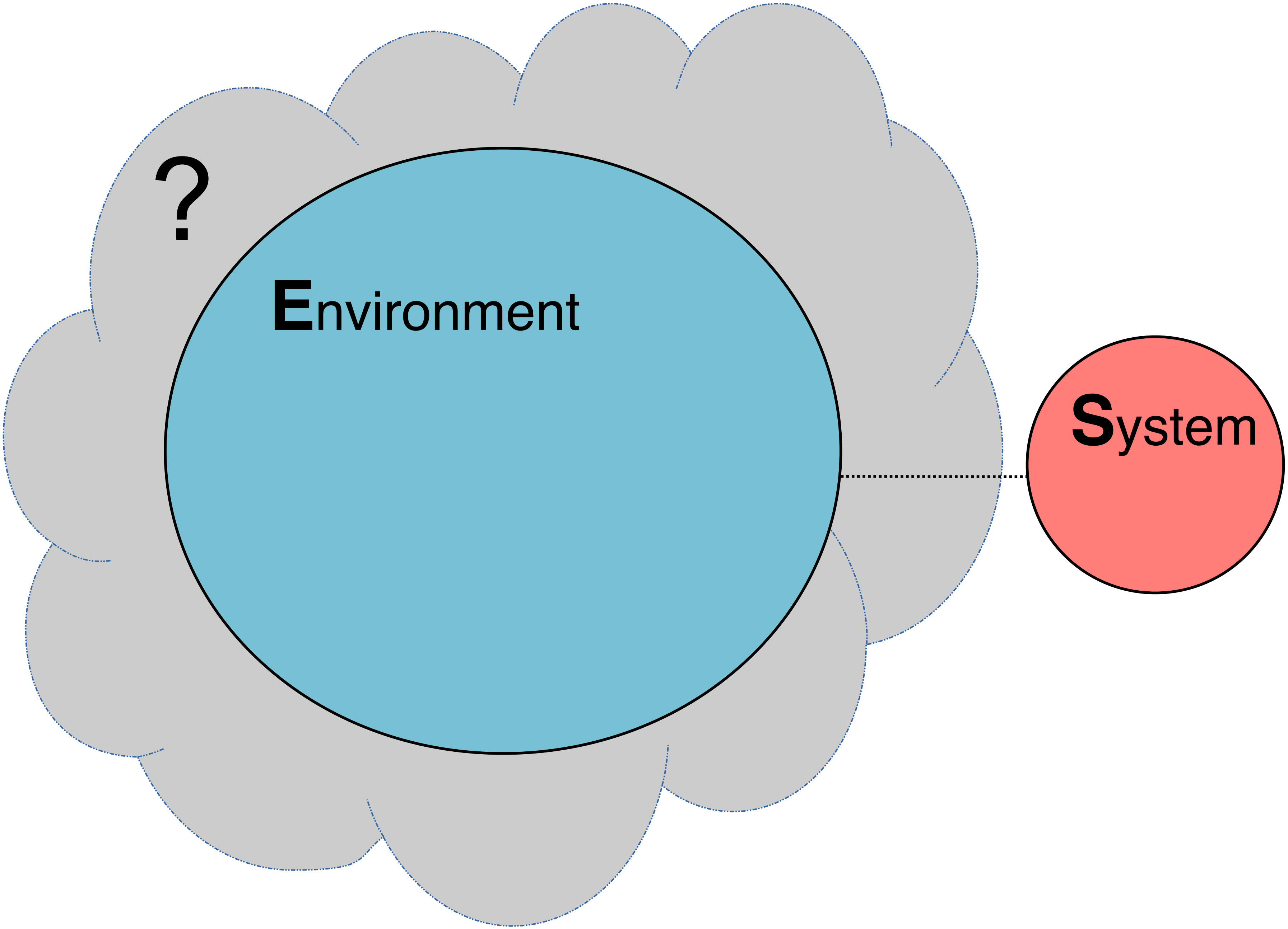

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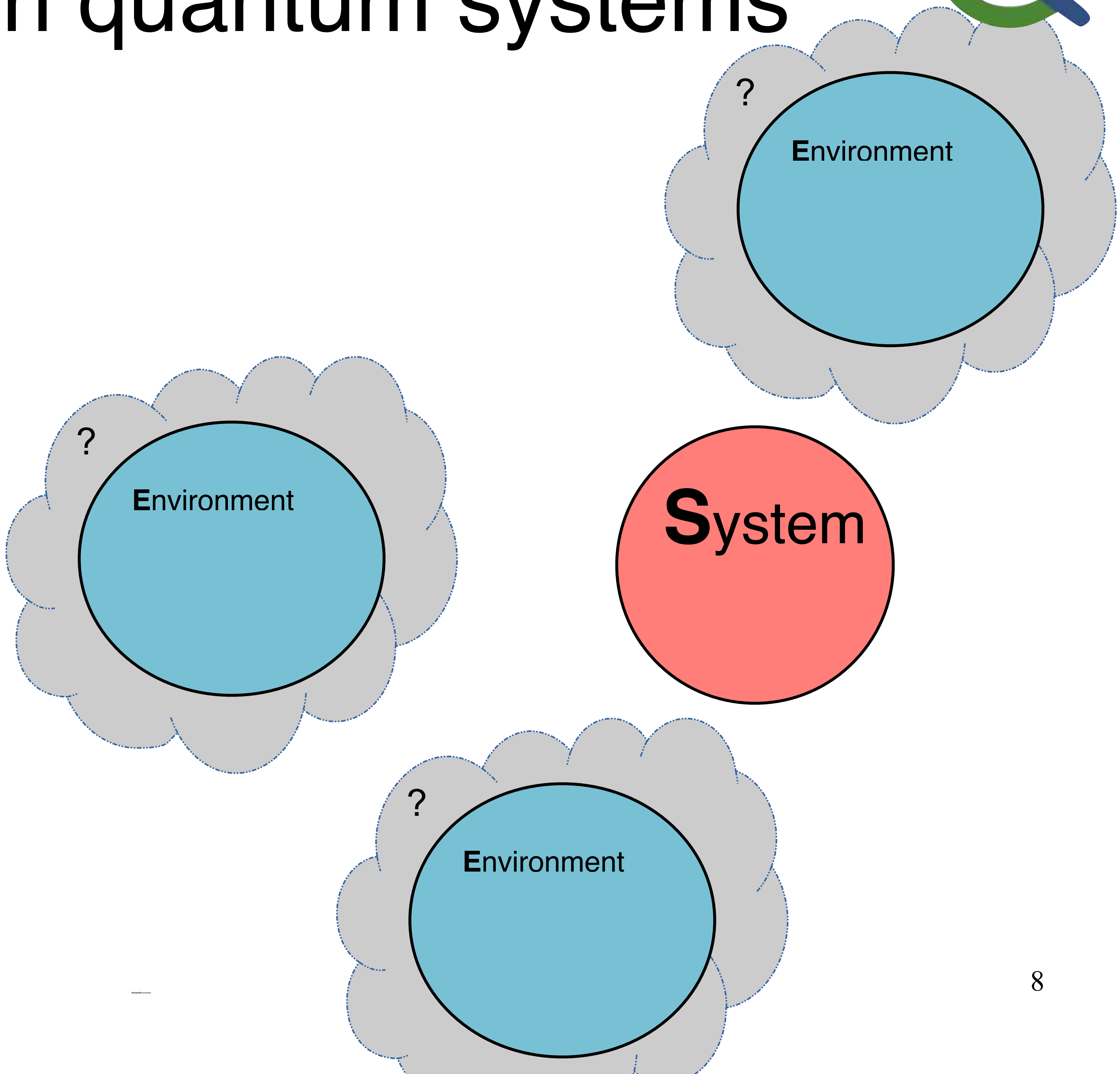
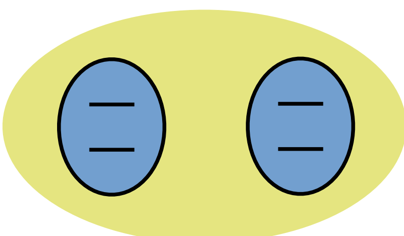
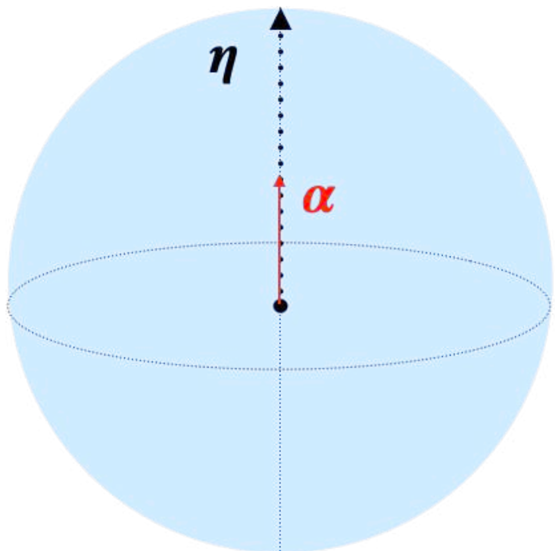
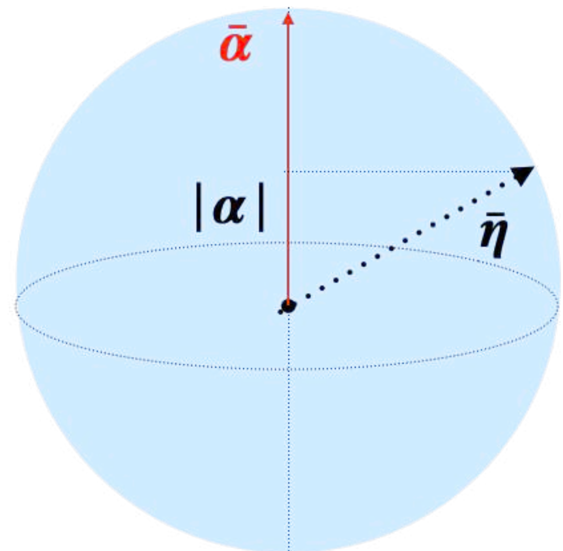


Qubit pure dephasing







$$H_I = g\sigma_z \otimes B$$

$$B = \eta \cdot \sigma$$

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$$\rho_E(0) = \frac{1}{2}(1 + \alpha \cdot \sigma)$$

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A. Smirne, N.Megier, B. Vacchini, Quantum 5, 439 (2021)

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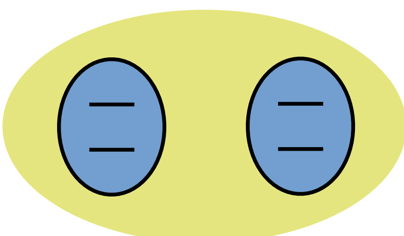

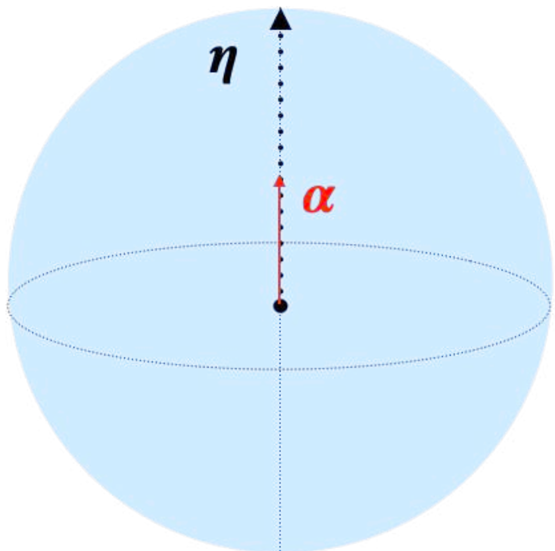

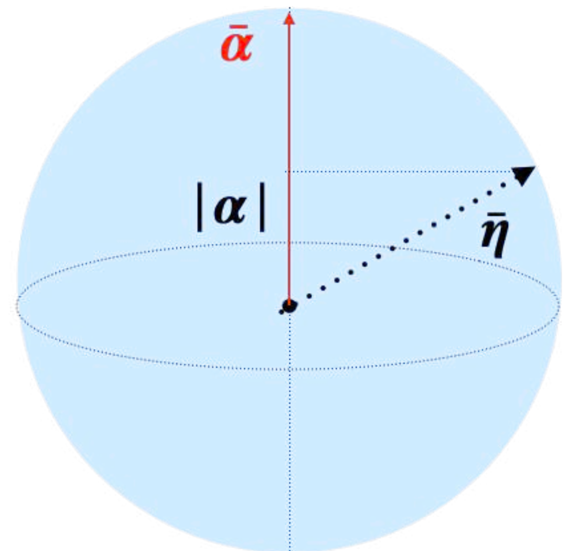

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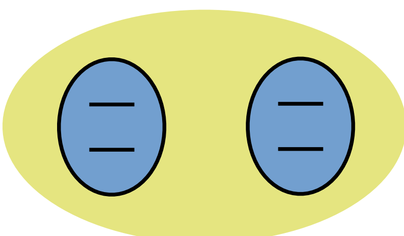

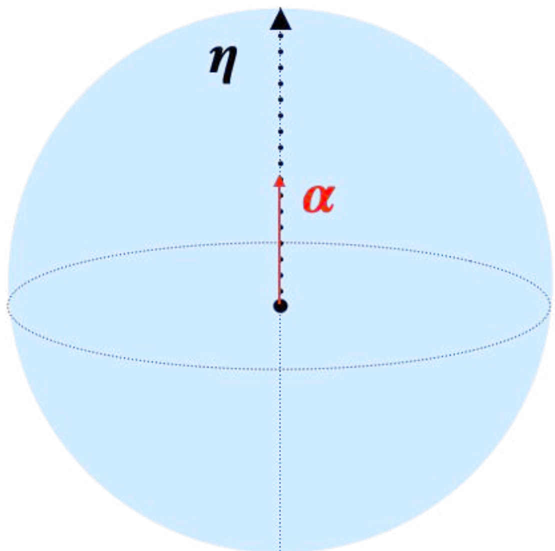

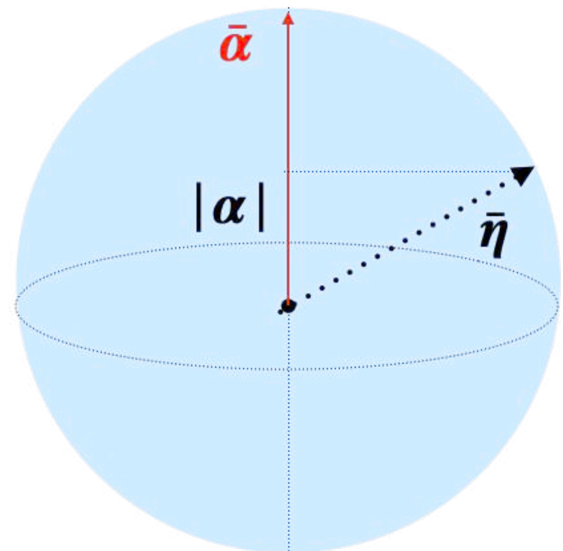

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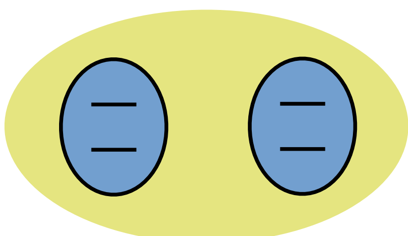

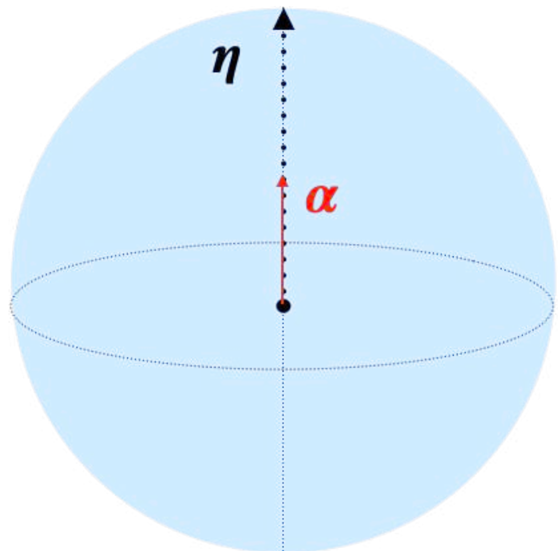

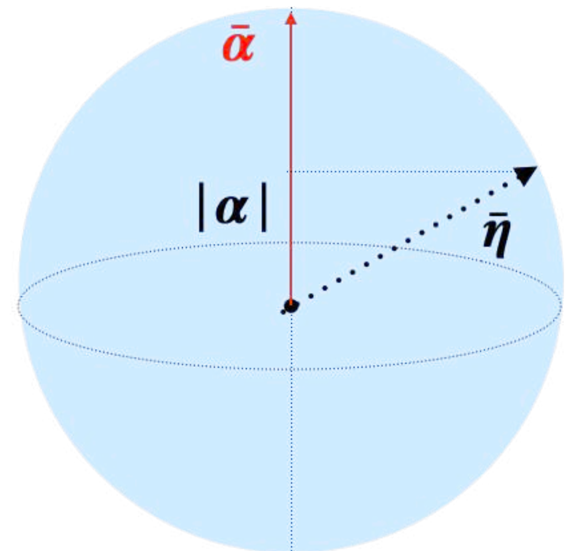

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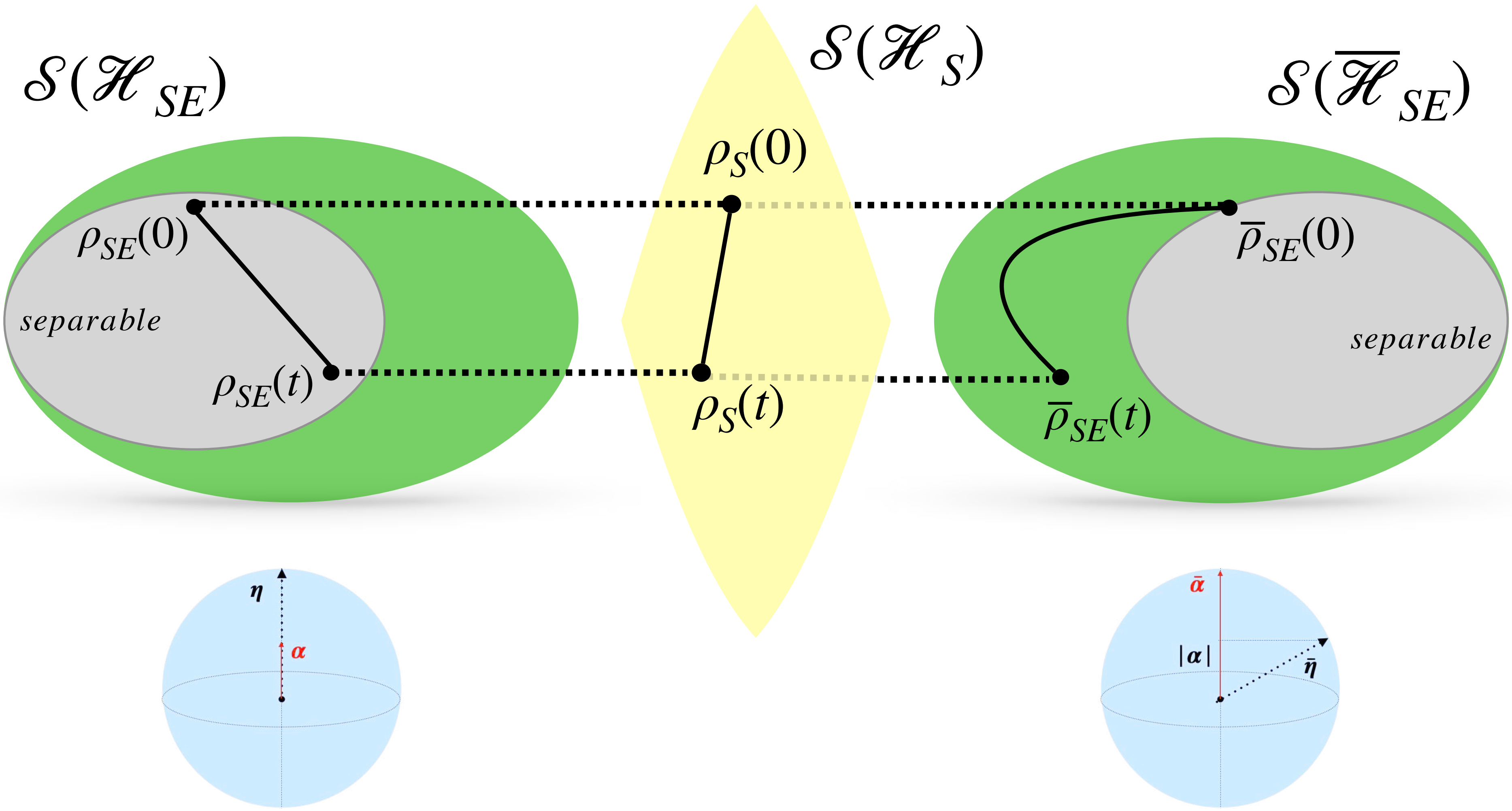
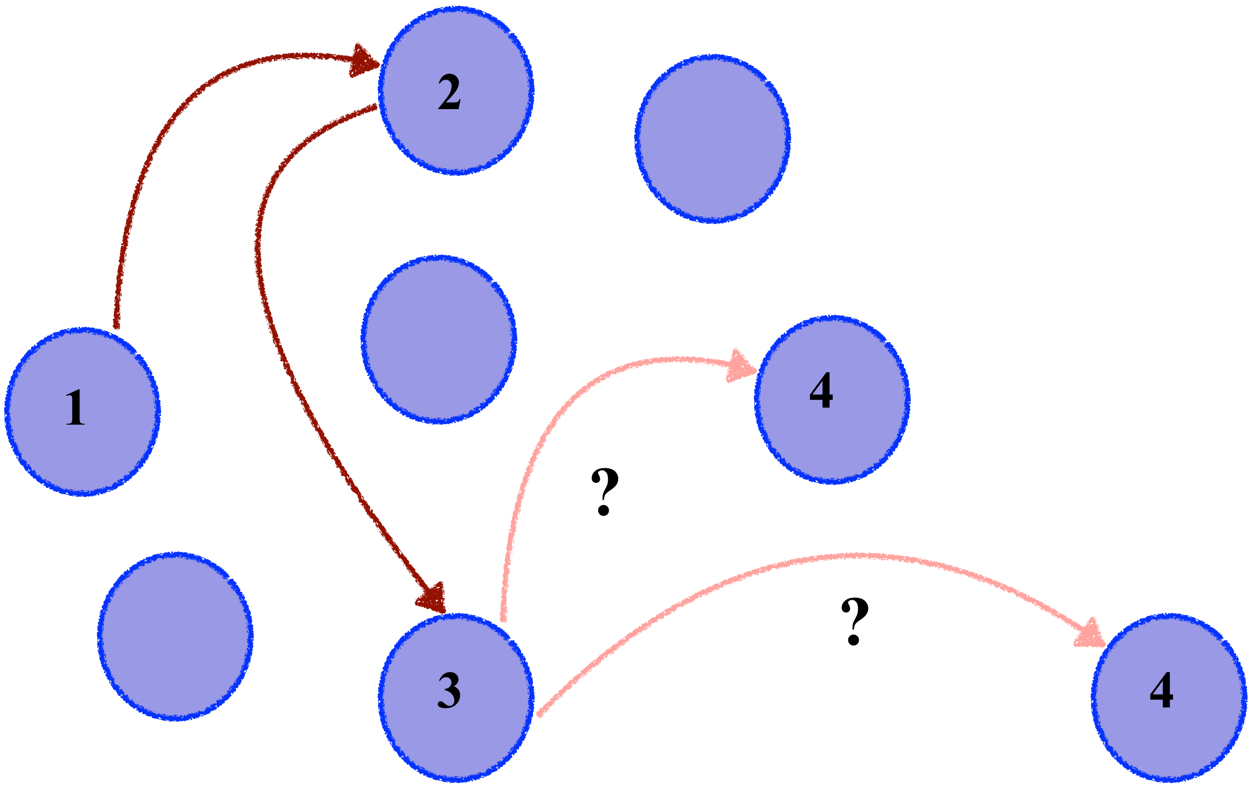


A. Smirne, N.Megier, B. Vacchini, Quantum 5, 439 (2021)

Non-Markovianity



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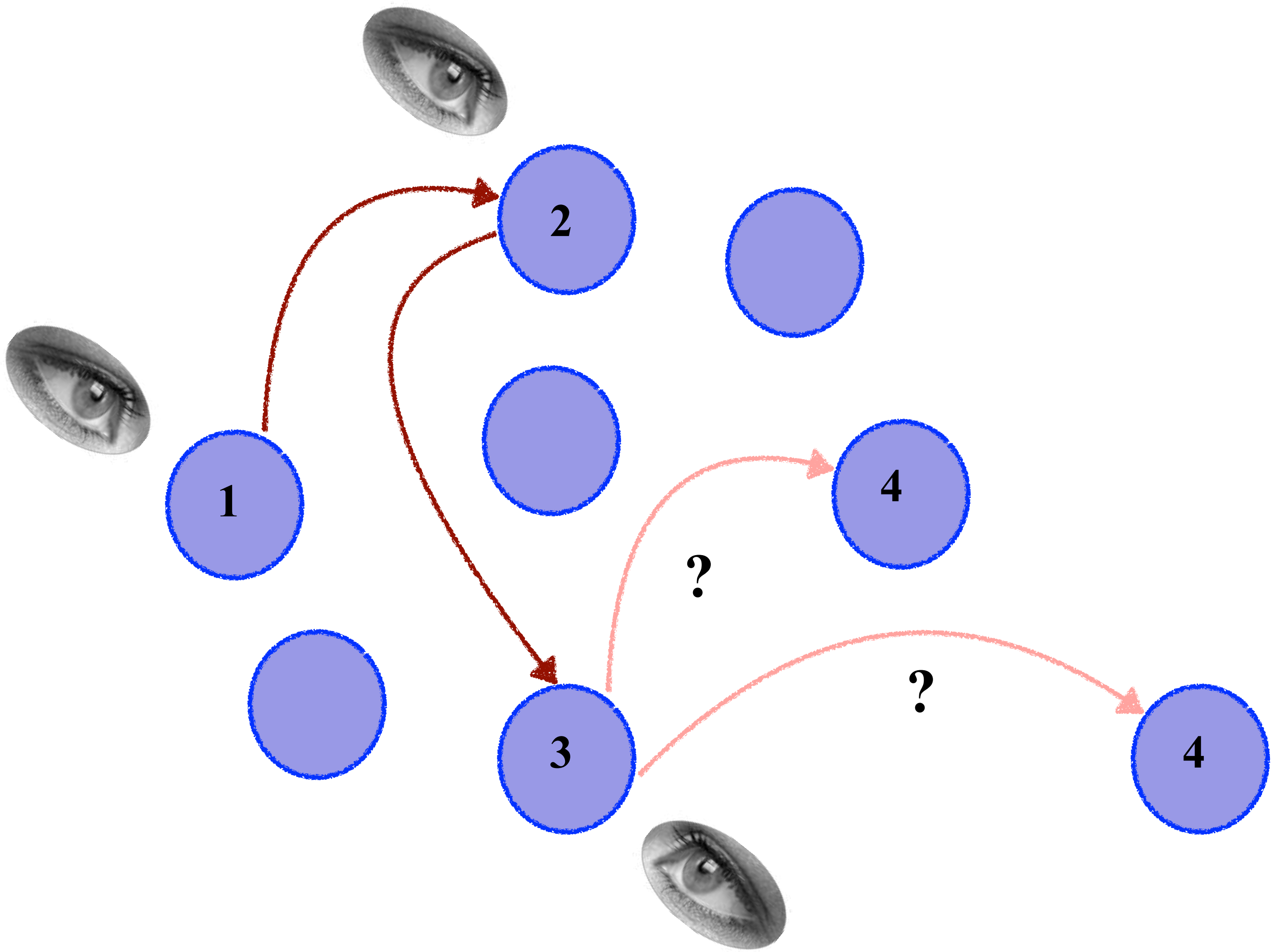
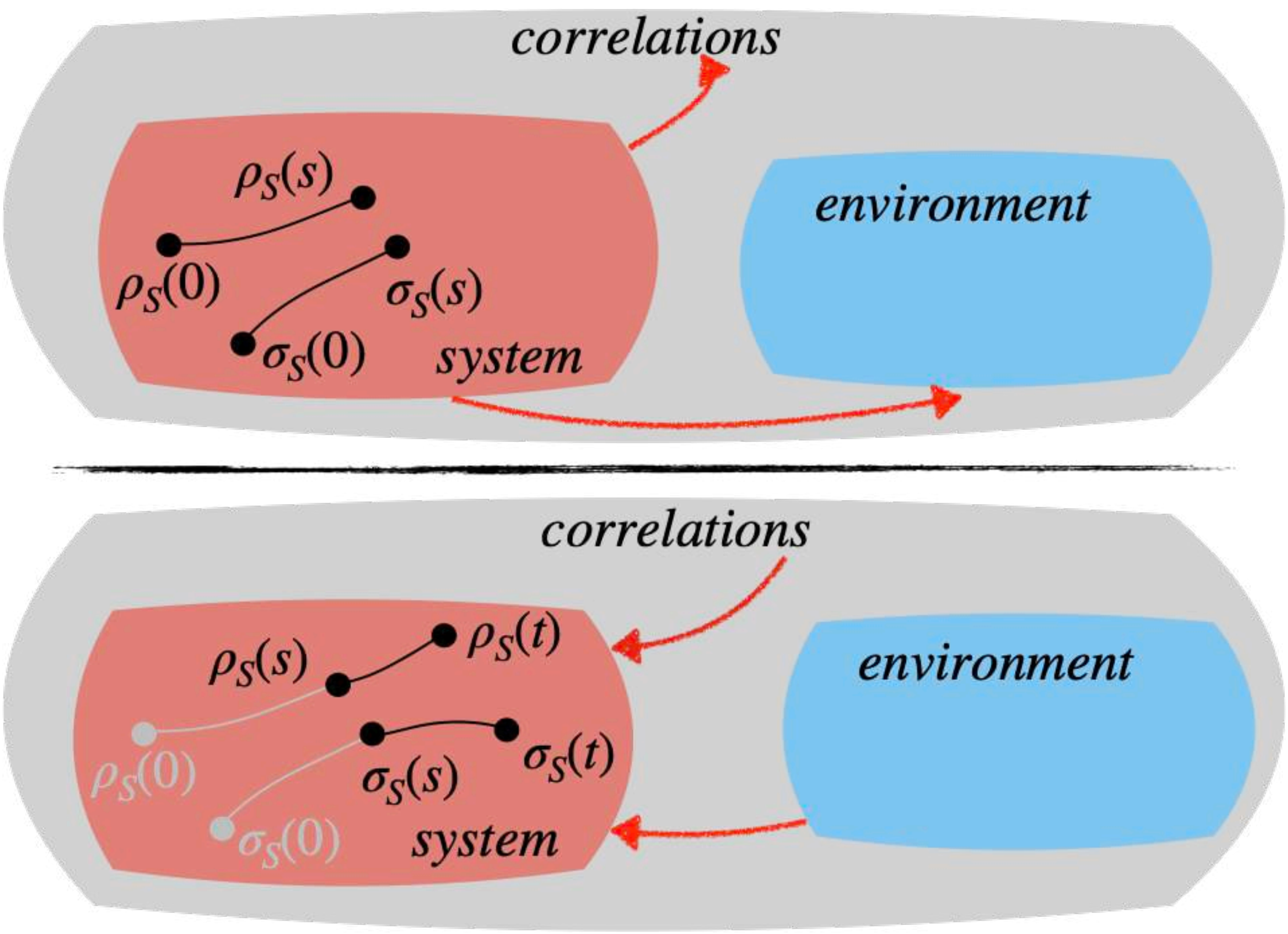


Heinz-Peter Breuer, Elsi-Mari Laine, Jyrki Piilo, and Bassano Vacchini Rev. Mod. Phys. **88**, 021002 (2016)

Quantum non-Markovianity



Heinz-Peter Breuer, Elsi-Mari Laine, Jyrki Piilo Phys. Rev. Lett. 103, 210401 (2010)

Trace distance

$$\mathrm{D}(\varrho, \sigma) = \frac{1}{2}\mathrm{Tr}|\varrho - \sigma|$$

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Elsi-Mari Laine, Jyrki Piilo, Heinz-Peter Breuer *EPL* **92** 60010 (2011)

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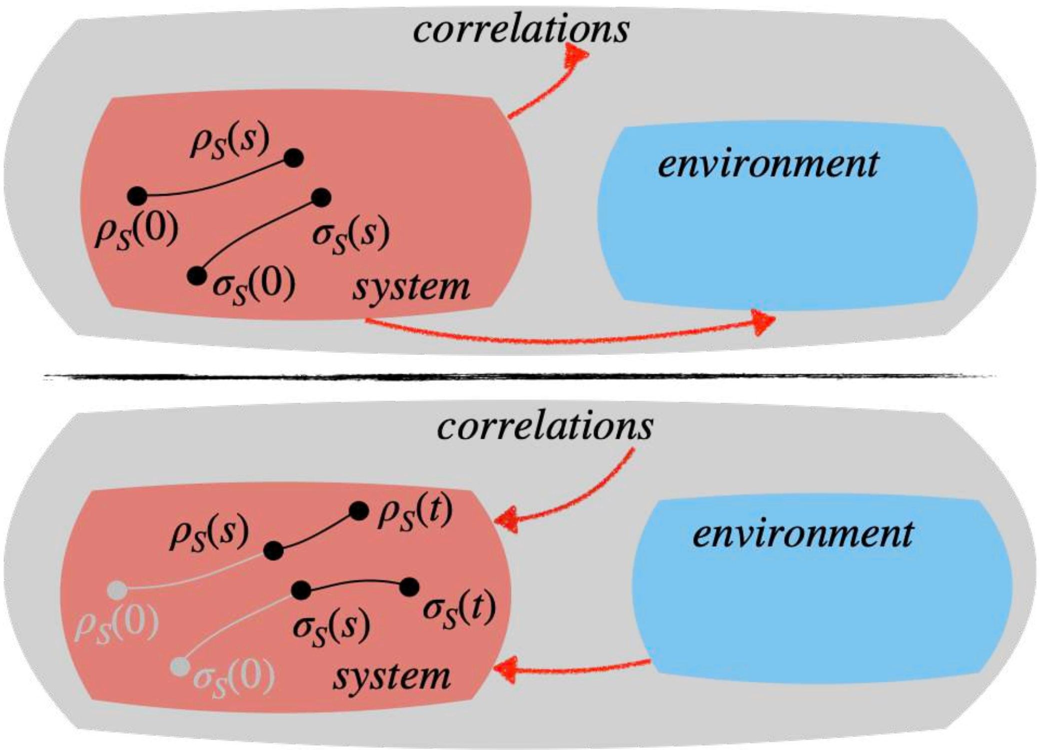

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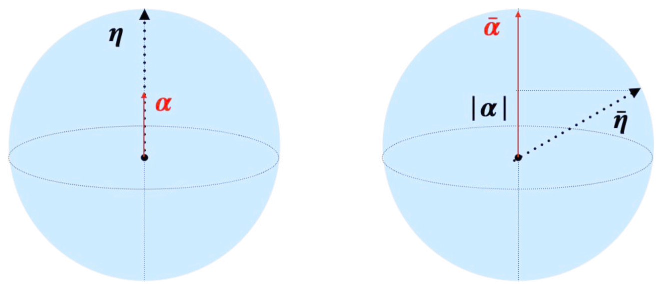

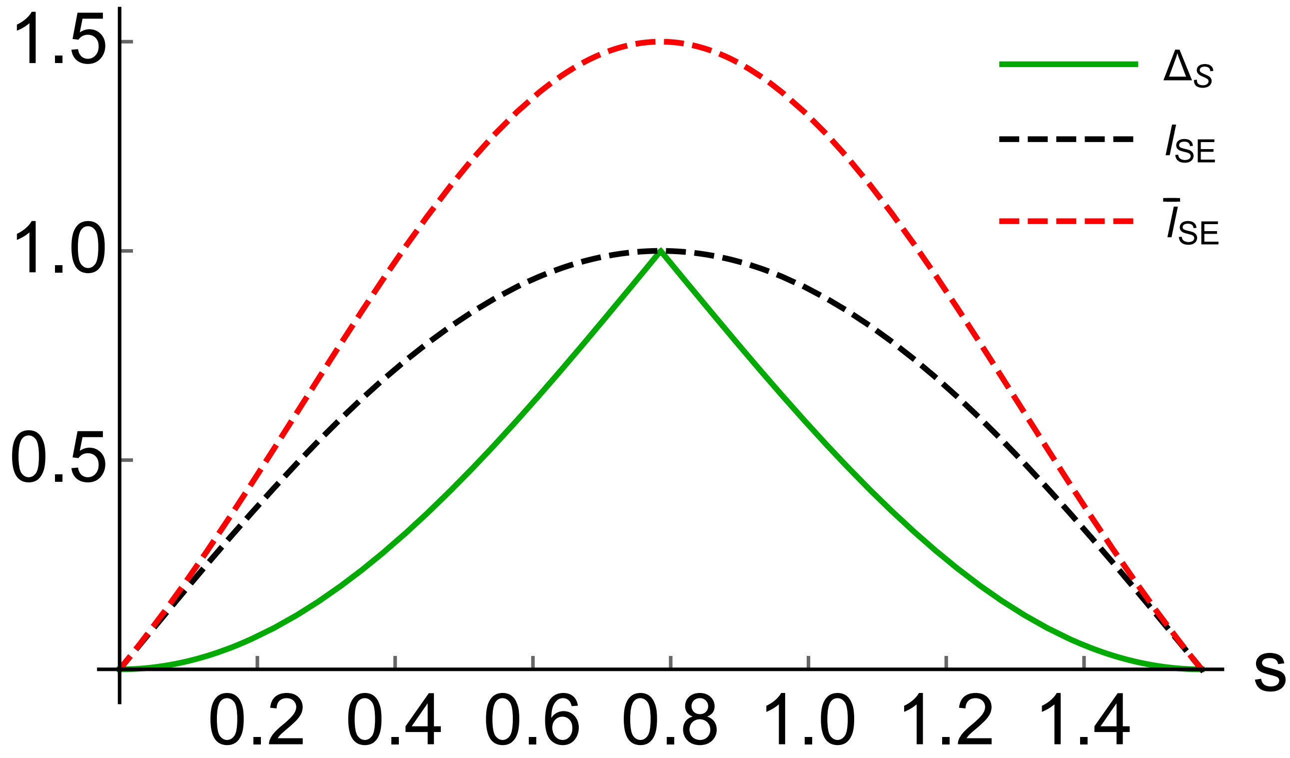

A. Smirne, N.Megier, B. Vacchini, Quantum 5, 439 (2021)

Quantum non-Markovianity

For the proof essential:

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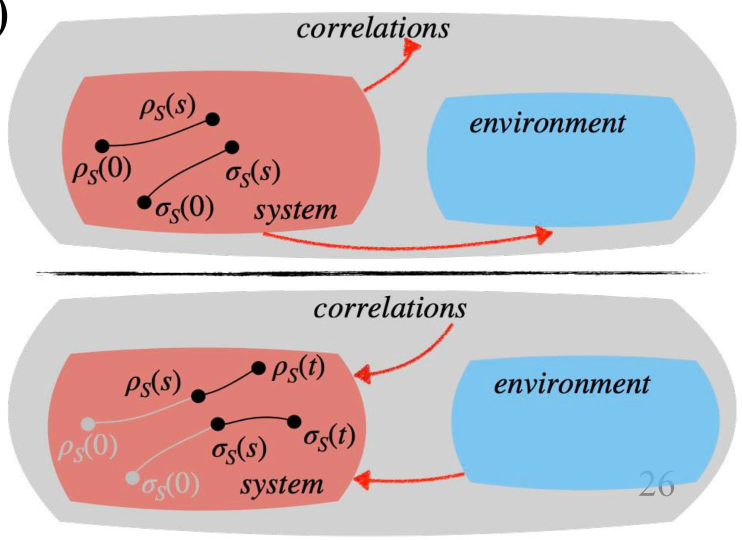

Nina Megier, Andrea Smirne, Bassano Vacchini, arXiv:2101.02720

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- triangle inequality

$$\mathsf{D}\left(\varrho, \sigma\right) \leq \mathsf{D}(\varrho, \tau) + \mathsf{D}(\tau, \sigma)$$

Quantum relative entropy

$$S\left(\varrho,\sigma\right)=\varrho(\log(\varrho)-\log(\sigma))$$

- ~~bounded~~

$$0\leq S\left(\varrho,\sigma\right)\leq 1$$

- contractivity under complete positive maps

$$S\left(\phi\left(\varrho\right),\phi\left(\sigma\right)\right)\leq S(\varrho,\sigma)$$

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Telescopic relative entropy

$$\mathsf{S}_{\mu}\left(\varrho, \sigma\right) = \frac{1}{\log(1/\mu)}\mathsf{S}\left(\varrho, \mu\varrho + (1-\mu)\sigma\right), \qquad 0 < \mu < 1$$

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- quasi-triangle inequalities

$$\mathsf{S}_\mu\left(\sigma,\varrho\right) - \mathsf{S}_\mu(\tau,\varrho) \leq 1 - \mathsf{S}_\mu(1,\mathsf{D}(\sigma,\tau)),$$

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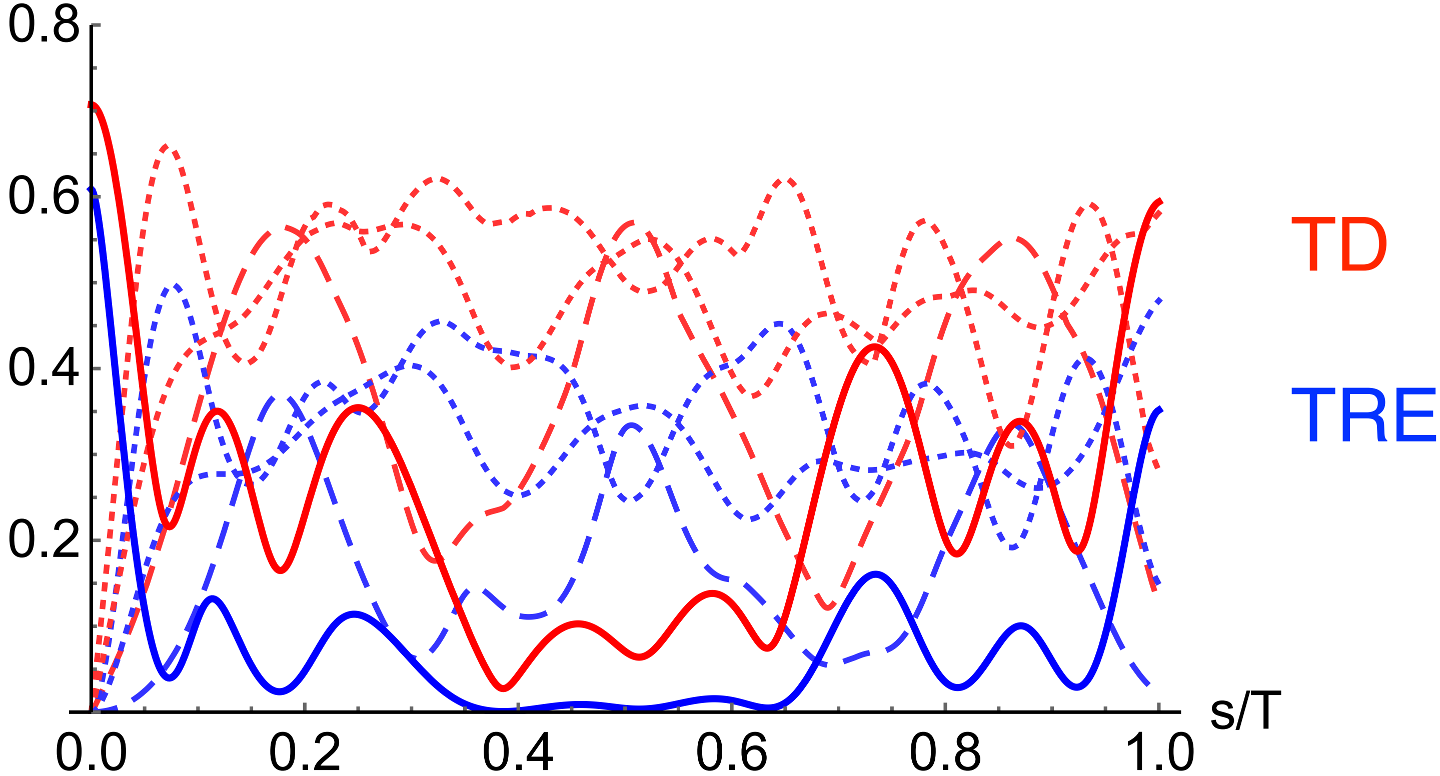
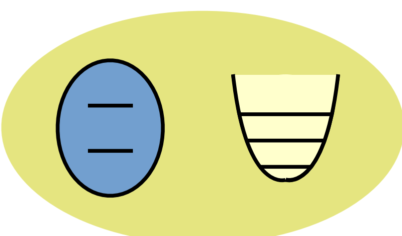
Telescopic relative entropy vs. trace distance





Jaynes-Cummings model

$$H = \omega_S \sigma_z \otimes \mathbb{I} + g(\sigma_+ \otimes b + \sigma_- \otimes b^\dagger) + \omega_E \mathbb{I} \otimes b^\dagger b$$

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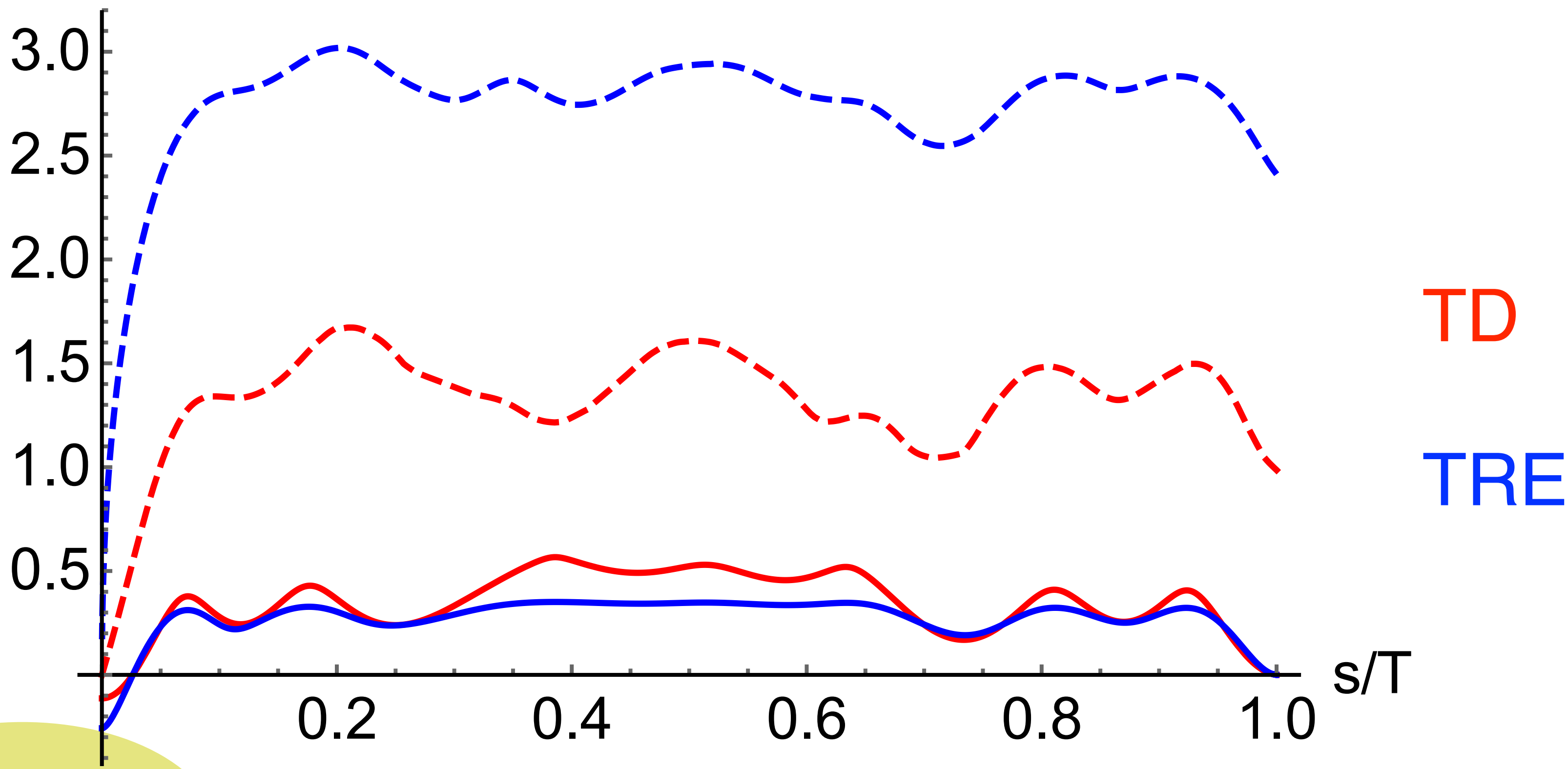

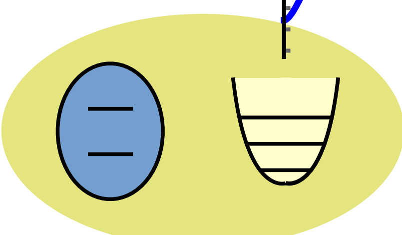

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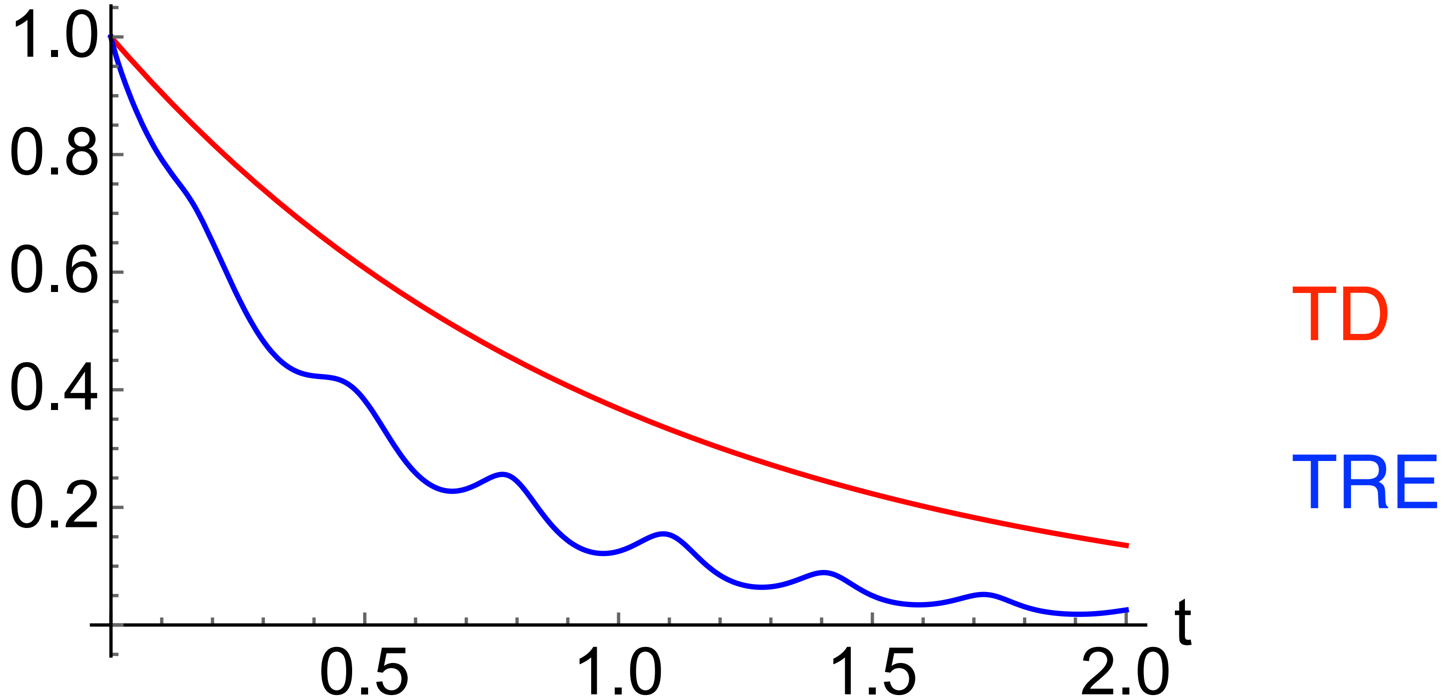
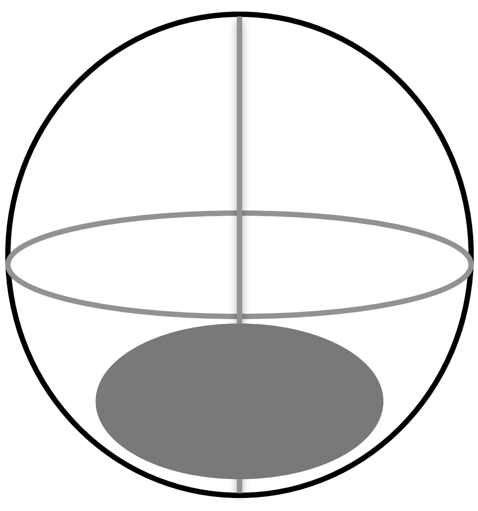


Nonunital qubit phase covariant dynamics



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Jensen-Shannon divergence

$$J(\varrho, \sigma) = \frac{1}{2}\left(\mathsf{S}_{1/2}(\varrho, \sigma) + \mathsf{S}_{1/2}(\sigma, \varrho)\right)$$

$\sqrt{J(\varrho, \sigma)}$ is a distance

D. Virosztek, Advances in Mathematics 380, 107595 (2021)
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$$\sqrt{\mathrm{J}(\varrho_S(t),\sigma_S(t))}-\sqrt{\mathrm{J}(\varrho_S(s),\sigma_S(s))}\leqslant\sqrt{\mathrm{J}(\varrho_E(s),\sigma_E(s))}+\sqrt{\mathrm{J}(\varrho(s),\varrho_S(s)\otimes\varrho_E(s))}$$
$$+\sqrt{\mathrm{J}(\sigma(s),\sigma_S(s)\otimes\sigma_E(s))}$$

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Outlook

- Measure of non-Markovianity: optimal states
- Use for detection of initial correlations

Thanks for your attention!