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Quantum jumps for general open system dynamics: A unified approach

Andrea Smirne

In collaboration with Jyrki Piilo, Dariusz Chruscinski, Kimmo Luoma & Matteo Caiaffa

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Outline

- Introduction and motivation
  - General (non-)Markovian open-system dynamics
  - Stochastic unraveling: the Monte Carlo Wave function method
- A novel quantum-jump method
  - Definition of the rate operator quantum jumps for P-divisible dynamics
  - Systematic continuous-measurement interpretation
- Extension to general open-system dynamics
  - Introduction of the reversed jumps
  - Markovianity, non-Markovianity and in-between
- Conclusion and outlook

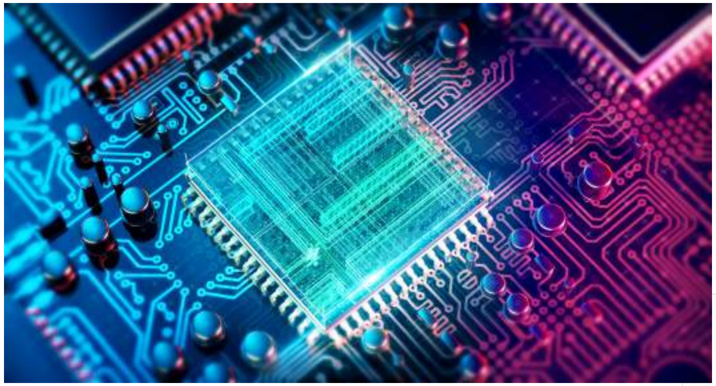
Outline

- Introduction and motivation
- A novel quantum-jump unraveling
- Extension to non-Markovian dynamics
- Conclusion and outlook

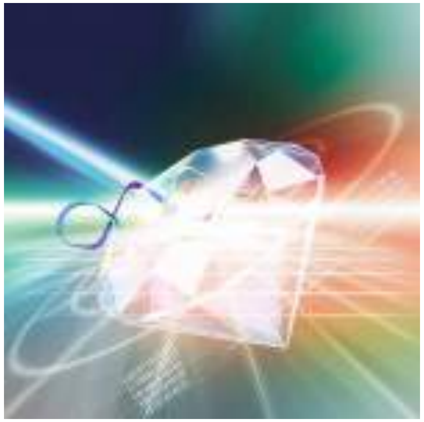
Open quantum systems: from foundations to applications

- The quantum systems we want to control and manipulate are subjected to interaction with the surrounding environment, i.e., they are open systems
- Quantum properties (such as entanglement and coherences) are particularly fragile under such an interaction

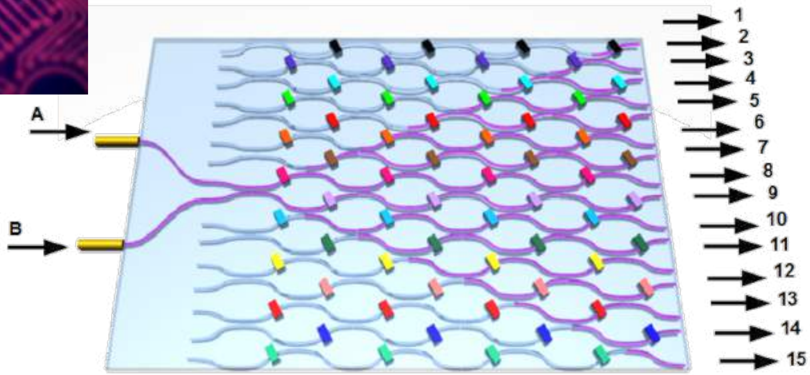
Quantum computation & communication



Quantum metrology and sensing



Quantum simulation



Open quantum systems: from foundations to applications

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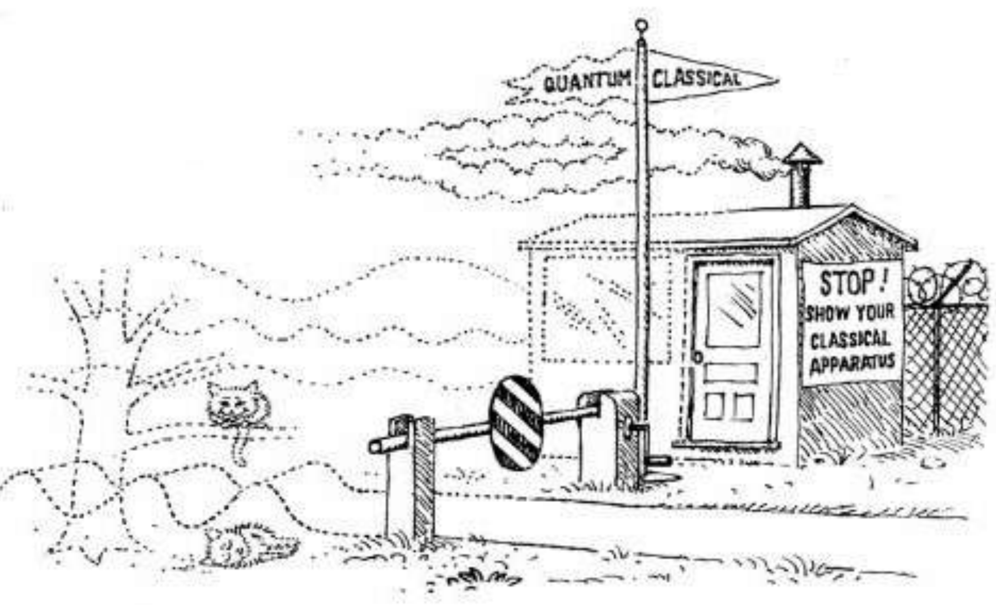
Foundations of Quantum Mechanics

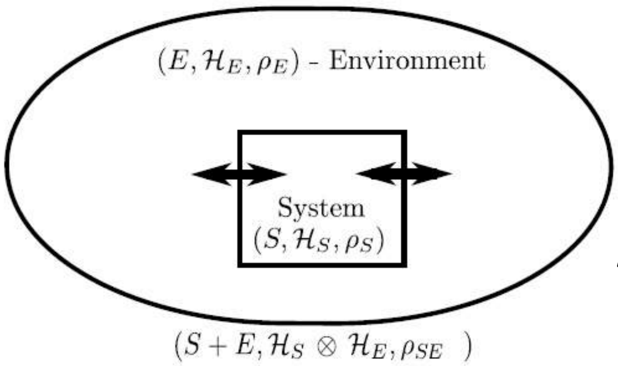


Zurek, Physics Today (1991)

✓ Border between classical and quantum (decoherence theory)

✓ A measurement is an interaction of the system of interest with an “environment”

Non-Markovianity – a physical picture



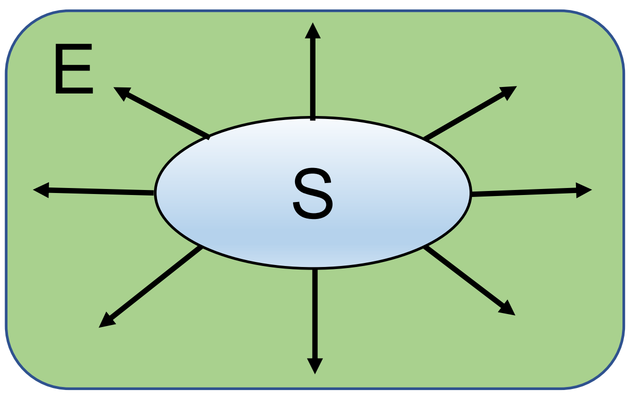
System: degrees of freedom we are interested in

Environment: usually very complex, it is <u>averaged out</u>

$$\rho_S(t) = \mathrm{tr}_E \left\{ U(t)[\rho_S(t_0) \otimes \rho_E(t_0)]U^\dagger(t) \right\} = \Lambda_S(t)[\rho_S(t_0)]$$

Completely positive evolution: $\Lambda_S(t) \otimes Id$ positive

- Markovian dynamics: <u>memoryless</u> scenario



$$\tau_E \ll \tau_R$$

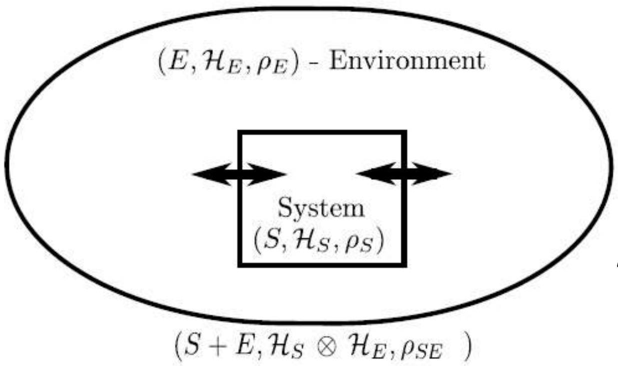
"Fast environment"

Decay time of E excitations

Relaxation time of S

<u>Unidirectional system</u> ⟶ <u>environment information flow</u>

Non-Markovianity – a physical picture



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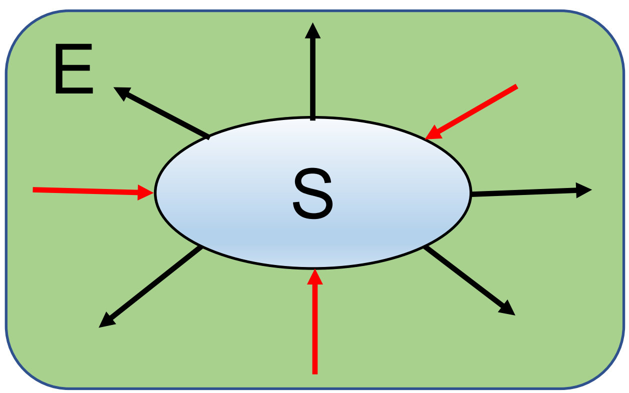
- Non-Markovian dynamics: memory effects are relevant



$$\tau_E \sim \tau_R$$

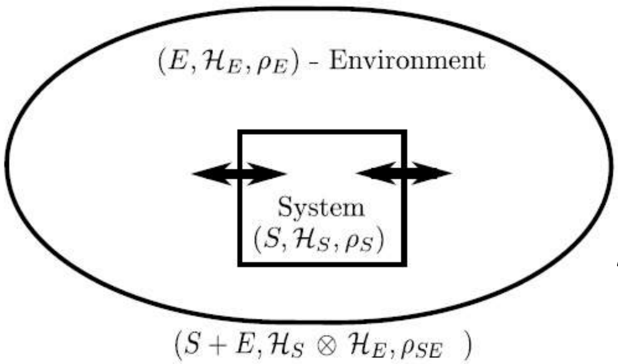
Decay time of E excitations

Relaxation time of S

Interplay of S relaxation and E internal dynamics ⇒ non-trivial role of SE correlations!

Bidirectional system ⟷ environment information flow

Non-Markovianity – a physical picture



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Environment: usually very complex, it is averaged out

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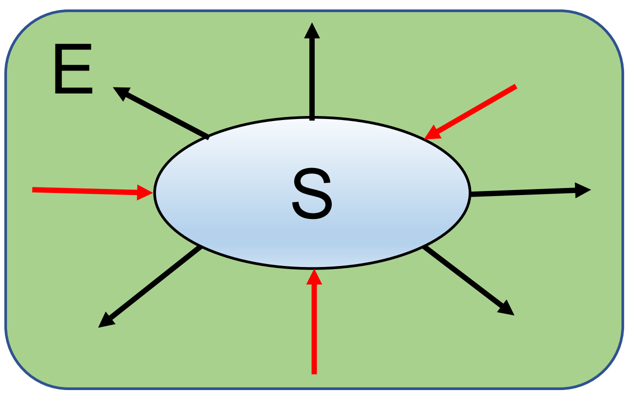


$\tau_E \sim \tau_R$

Talk by Nina Megier on April 27th :

Information Flow in open quantum system dynamics

Interplay of S relaxation and E internal dynamics

⇒ non-trivial role of SE correlations!

Bidirectional system ⟷ environment information flow

Non-Markovianity – a multifaceted topic

- Different <u>non-equivalent definitions</u> of quantum non-Markovianity

information flow, divisibility, multi-time probabilities, …

REVIEWS OF MODERN PHYSICS, VOLUME 88, APRIL–JUNE 2016

***Colloquium*: Non-Markovian dynamics in open quantum systems**

Heinz-Peter Breuer*

Physikalisches Institut, Universität Freiburg, Hermann-Herder-Straße 3, D-79104 Freiburg, Germany

Elsi-Mari Laine

QCD Labs, COMP Centre of Excellence, Department of Applied Physics, Aalto University, P.O. Box 13500, FI-00076 AALTO, Finland and Turku Centre for Quantum Physics, Department of Physics and Astronomy, University of Turku, FI-20014 Turun yliopisto, Finland

Jyrki Piilo

Turku Centre for Quantum Physics, Department of Physics and Astronomy, University of Turku, FI-20014 Turun yliopisto, Finland

Bassano Vacchini

Dipartimento di Fisica, Università degli Studi di Milano, Via Celoria 16, I-20133 Milan, Italy and INFN, Sezione di Milano, Via Celoria 16, I-20133 Milan, Italy

IOP Publishing

Rep. Prog. Phys. **77** (2014) 094001 (26pp)

Review Article

Quantum non-Markovianity: characterization, quantification and detection

Ángel Rivas[1], Susana F Huelga[2,3] and Martin B Plenio[2,3]

[1] Departamento de Física Teórica I, Facultad de Ciencias Físicas, Universidad Complutense, 28040 Madrid, Spain
[2] Institut für Theoretische Physik, Universität Ulm, Albert-Einstein-Allee 11, 89073 Ulm, Germany
[3] Center for Integrated Quantum Science and Technologies, Albert-Einstein-Allee 11, 89073 Ulm, Germany

- Variety of <u>analytical and numerical</u> methods to treat general dynamics
  - Master equations
  - Path integrals
  - Perturbative expansions
  - Numerical ab-initio techniques
  - Stochastic methods

The Theory of Open Quantum Systems
Breuer & Petruccione (2002)

Quantum Noise
Gardiner & Zoller (2004)

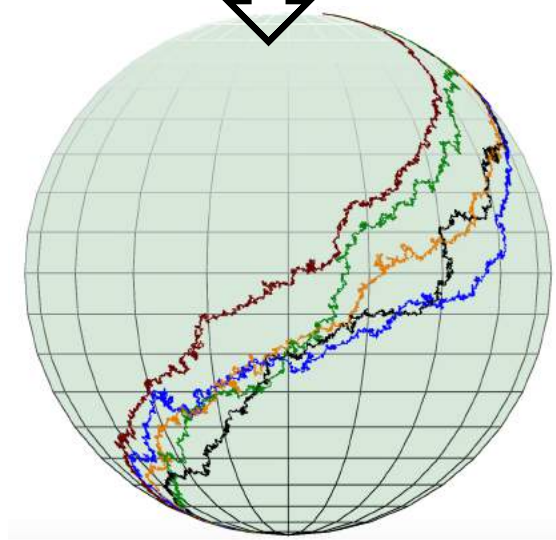
Stochastic unraveling – general idea

Starting point: time-local master equation for the system state $\rho(t)$

$$\frac{d}{dt}\rho(t) = -i[H_S(t), \rho(t)] + \sum_{\alpha=1}^{n^2-1} c_\alpha(t) \left(L_\alpha(t)\rho(t)L_\alpha(t)^\dagger - \frac{1}{2}\left\{L_\alpha^\dagger(t)L_\alpha(t), \rho(t)\right\}\right)$$

Infinitely many possible mappings

Stochastic trajectories
on the set of pure states
(fixed by a SDE)



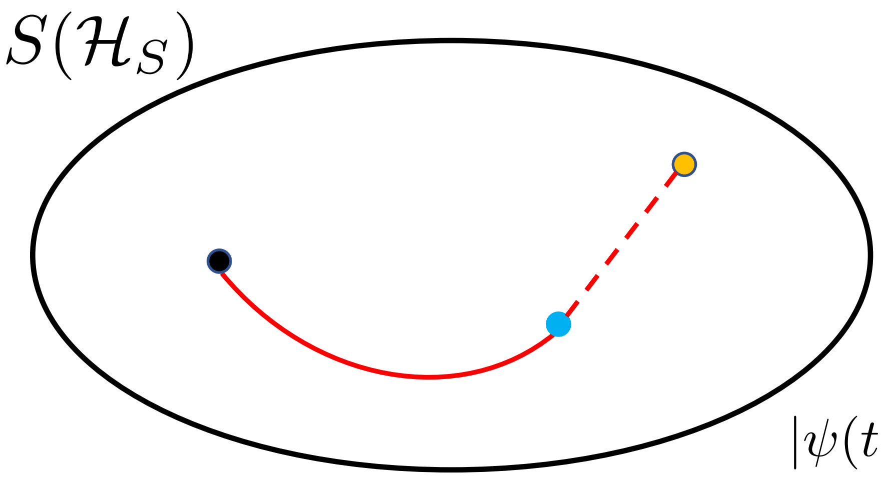
Average of the trajectories

$$\rho(t) = \frac{1}{N}\sum_{i=1}^{N} |\psi_i(t)\rangle\langle\psi_i(t)|$$

- Diffusive unraveling Gisin & Percival JPA **25**-1992, Diosi & Strunz PLA **235**-1997
- Jump unraveling Dalibard, Castin & Molmer PRL **68**-1992

Monte Carlo wave function (MCWF) method

Carmichael *An Open System Approach to Quantum Optics* (1993)



- Deterministic evolution fixed by

$$H_{eff} = H_S - \frac{i}{2} \sum_{\alpha=1}^{n^2-1} c_\alpha L_\alpha^\dagger L_\alpha$$

- Interrupted by random jumps

$$|\psi(t)\rangle \mapsto \frac{L_\alpha |\psi(t)\rangle}{\| L_\alpha |\psi(t)\rangle \|} \qquad p_\alpha(t) = c_\alpha \| L_\alpha |\psi(t)\rangle \|^2 dt$$

rare events!

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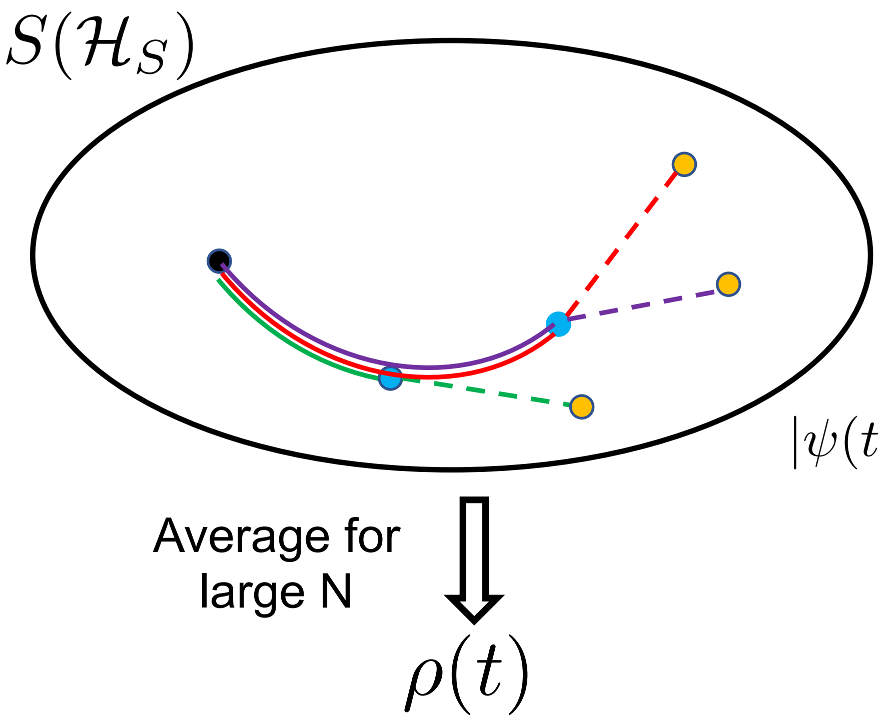

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rare events!

○ Quantum jumps observed experimentally in several platforms

Basche, Kummer & Brauchle Nature **373** (1995) Jelezko et al APL **81** (2002)

Peil & Gabrielse PRL **83** (1999) Gleyzes et al Nature **446** (2007)

MCWF: advantages and limits

- Pure states: the simulation cost scales <u>linearly</u> with the system dimension

  Routinely used for complex, many body OQS   Daley Adv. Phys. **63** (2014)

- Physical interpretation in terms of <u>continuous measurements</u>

$$|\psi(t)\rangle\langle\psi(t)| \mapsto \frac{L_\alpha|\psi(t)\rangle\langle\psi(t)|L_\alpha^\dagger}{\| L_\alpha|\psi(t)\rangle\|^2}$$

State transformation due to a measurement with outcome α

$$p_\alpha(t) = c_\alpha \| L_\alpha |\psi(t)\rangle\|^2 dt$$

Probability that the measurement gives outcome α

MCWF: advantages and limits

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$$|\psi(t)\rangle\langle\psi(t)| \mapsto \frac{L_\alpha|\psi(t)\rangle\langle\psi(t)|L_\alpha^\dagger}{\| L_\alpha|\psi(t)\rangle\|^2} \qquad p_\alpha(t) = c_\alpha \| L_\alpha|\psi(t)\rangle\|^2 dt$$

For the <u>definition</u> of the MCWF we need positive coefficients! $c_\alpha(t) \geq 0$

Positivity of the coefficients corresponds to <u>CP-divisibility</u> of the dynamics

Definition of Markovianity in Rivas, Huelga & Plenio PRL **105** (2010)

Can we extend the continuous-measurement interpretation?

Interplay of <u>measurement backaction</u> and <u>backflow of information</u>!

Diosi PRL **100** (2008)   Wiseman & Gambetta PRL **101** (2008)

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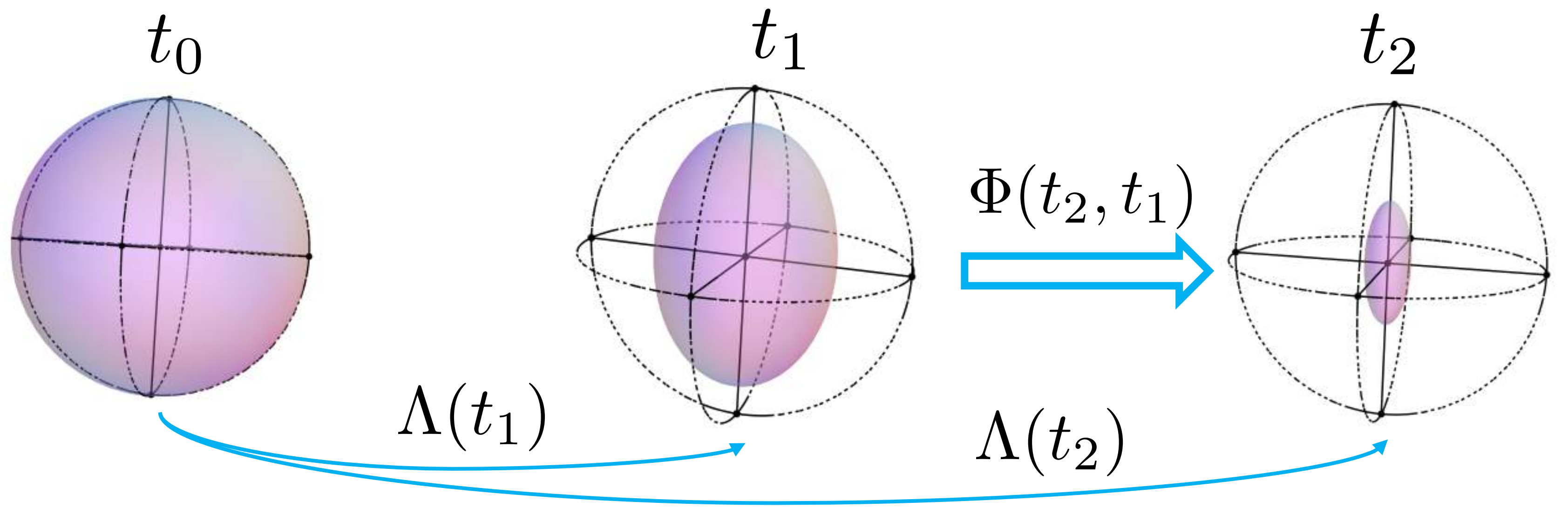
(C)P-divisible dynamics



The evolution of $\rho(t)$ can be split into (completely-)positive maps

$$\Lambda(t_2) = \Phi(t_2, t_1) \circ \Lambda(t_1)$$

$\Phi(t_2, t_1)$ is (C)P

- P-divisibility definition of Markovianity in Breuer, Laine, Piilo & Vacchini RMP **88** (2016)
- It allows for negative coefficients $c_\alpha(t)$: MCWF does not apply!

$$\begin{pmatrix} \rho_{11} & \rho_{10} \\ \rho_{01} & \rho_{00} \end{pmatrix} \mapsto \begin{pmatrix} \rho_{11} e^{-\lambda t} & \rho_{10} e^{-\eta t} \\ \rho_{01} e^{-\eta t} & \rho_{00} e^{-\lambda t} \end{pmatrix} + (1 - e^{-\lambda t}) \frac{Id}{2}$$

P: any $\lambda, \eta \geq 0$

CP: $\eta \geq \lambda/2 \geq 0$ **!**

Rate-operator quantum jumps

Rate Operator (RO)

$$W^J_{\psi(t)} = \sum_{\alpha=1}^{n^2-1} c_\alpha(t)(L_\alpha(t) - \ell_{\psi(t),\alpha})|\psi(t)\rangle\langle\psi(t)|(L_\alpha(t) - \ell_{\psi(t),\alpha})^\dagger$$

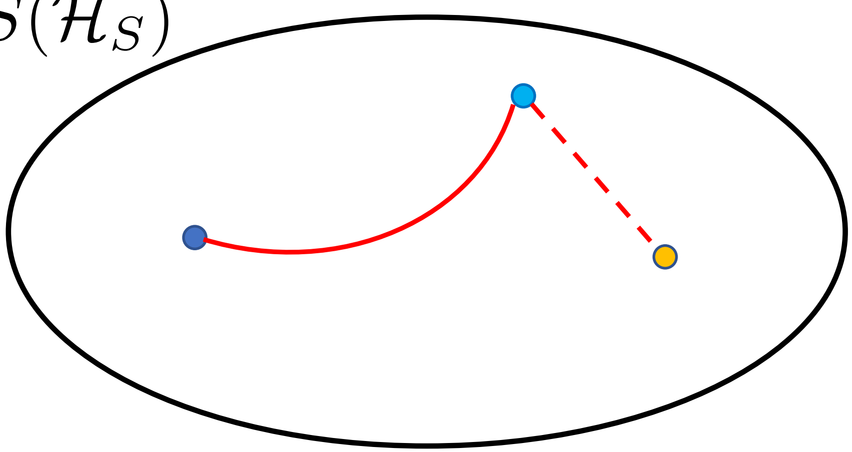
Diosi PLA **114** (1986)

$$\ell_{\psi(t),\alpha} = \langle\psi(t)|L_\alpha(t)|\psi(t)\rangle$$

Kossakowski Rep Math Phys **3** (1972) Wißman, Breuer & Vacchini PRA **92** (2015)

$$W^J_{\psi(t)} \geq 0 \quad \forall|\psi(t)\rangle \in \mathcal{H}_S \iff \Lambda(t) \ \text{P-divisible}$$

$S(\mathcal{H}_S)$



- Deterministic evolution fixed by

$$H_{eff} = H_S - \frac{i}{2}\sum_{\alpha=1}^{n^2-1} c_\alpha \left(L^\dagger_\alpha L_\alpha - 2\ell^*_{\psi(t),\alpha} L_\alpha + |\ell_{\psi(t),\alpha}|^2\right)$$

- Interrupted by jumps

$$|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j}\rangle \qquad p_j(t) = \lambda_{\psi(t),j}dt$$

$$W^J_{\psi(t)} = \sum_{j=1}^{n} \lambda_{\psi(t),j}|\varphi_{\psi(t),j}\rangle\langle\varphi_{\psi(t),j}|$$

Eigenvectors and **positive** eigenvalues

Sketch of the proof

- Given the current state $|\psi(t)\rangle$ and jump operators $V_{\psi(t),j} = \sqrt{\lambda_{\psi(t),j}}|\varphi_{\psi(t),j}\rangle\langle\psi(t)|$

n possible jumps between *t* and *t+dt*

$$|\psi(t)\rangle \to \frac{V_{\psi(t),j}|\psi(t)\rangle}{\|V_{\psi(t),j}|\psi(t)\rangle\|} \qquad p_j(t) = \|V_{\psi(t),j}|\psi(t)\rangle\|^2 dt$$

deterministic evolution

$$|\psi(t)\rangle \mapsto \frac{|\phi(t+dt)\rangle}{\||\phi(t+dt)\rangle\|} \qquad 1 - \sum_{j=1}^{n} p_j(t) = \||\phi(t+dt)\rangle\|^2$$

$$|\phi(t+dt)\rangle = \left[1 - iH_S dt - \frac{dt}{2}\sum_{\alpha=1}^{n^2-1} c_\alpha(t)\left(L_\alpha^\dagger(t)L_\alpha(t) - 2\ell^*_{\psi(t),\alpha}L_\alpha(t) + |\ell_{\psi(t),\alpha}|^2\right)\right]|\psi(t)\rangle$$

- On average

$$|\phi(t+dt)\rangle\langle\phi(t+dt) + \sum_{j=1}^{n} p_j(t)\frac{V_{\psi(t),j}|\psi(t)\rangle\langle\psi(t)|V^\dagger_{\psi(t),j}}{\|V_{\psi(t),j}|\psi(t)\rangle\|^2}$$

Spectral decomposition of the RO

$$\sum_{j=1}^{n} V_{\psi(t),j}|\psi(t)\rangle\langle\psi(t)|V^\dagger_{\psi(t),j}dt = \sum_{j=1}^{n}\lambda_{\psi(t),j}|\varphi_{\psi(t),j}\rangle\langle\varphi_{\psi(t),j}| = W^J_{\psi(t)}dt$$

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- On average $|\phi(t+dt)\rangle\langle\phi(t+dt) + \sum_{j=1}^{n} p_j(t)\frac{V_{\psi(t),j}|\psi(t)\rangle\langle\psi(t)|V^\dagger_{\psi(t),j}}{\|V_{\psi(t),j}|\psi(t)\rangle\|^2}$

$$\sum_{j=1}^{n} V_{\psi(t),j}|\psi(t)\rangle\langle\psi(t)|V^\dagger_{\psi(t),j}dt = \sum_{j=1}^{n}\lambda_{\psi(t),j}|\varphi_{\psi(t),j}\rangle\langle\varphi_{\psi(t),j}| = W^J_{\psi(t)}dt$$

- Master equation from $W^J_{\psi(t)} = \sum_{\alpha=1}^{n^2-1} c_\alpha(t)(L_\alpha(t) - \ell_{\psi(t),\alpha})|\psi(t)\rangle\langle\psi(t)|(L_\alpha(t) - \ell_{\psi(t),\alpha})^\dagger$

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- On average

$$|\phi(t+dt)\rangle\langle\phi(t+dt) + \sum_{j=1}^{n} p_j(t)\frac{V_{\psi(t),j}|\psi(t)\rangle\langle\psi(t)|V^\dagger_{\psi(t),j}}{\|V_{\psi(t),j}|\psi(t)\rangle\|^2}$$

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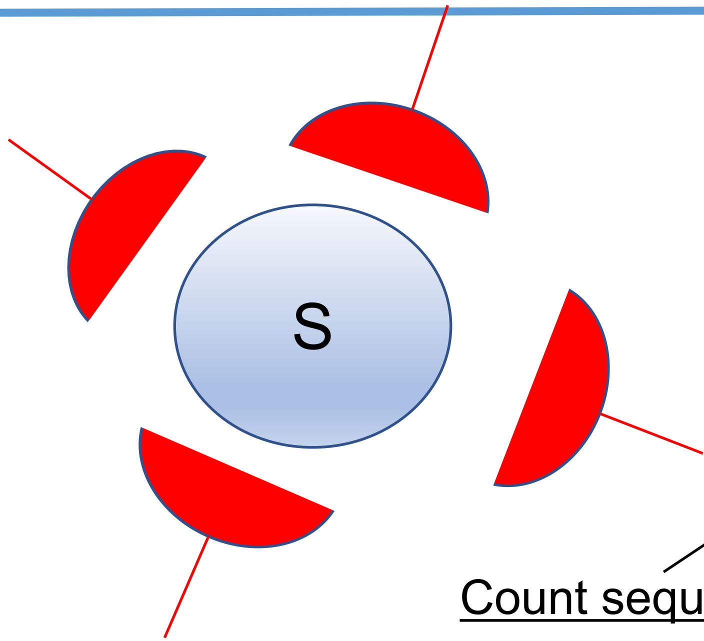
! The role of L_α and c_α in the MCWF is replaced by eigenvalues and eigenvectors of the RO ⇒ $p_j > 0$ also if some $c_\alpha < 0$

Continuous-measurement framework



The system is <u>continuously</u> monitored by *n* detectors: <u>every dt</u> two kinds of events

- No detector clicks: *null count* $\emptyset$
- The j-th detector clicks

$$\omega_t = (t_1, j_1; t_2, j_2; \ldots t_m, j_m)$$

<u>Count sequence</u>: Instants and types of the (non-null) counts

It fixes the system state $|\psi(\omega_t)\rangle$

<u>Quantum instrument</u>: map from the set of outcomes to CP maps on $S(\mathcal{H}_S)$

$$\mathcal{I}_{\omega_t,\emptyset}\rho = F_{\omega_t,\emptyset}\rho F^\dagger_{\omega_t,\emptyset} \qquad F_{\omega_t,\emptyset} = \left(Id - iH^{eff}_{\omega_t} dt\right)|\psi(\omega_t)\rangle\langle\psi(\omega_t)|$$

$$\mathcal{I}_{\omega_t,j}\rho = V_{\omega_t,j}\rho V^\dagger_{\omega_t,j} dt \qquad V_{\omega_t,j} = \sqrt{\lambda_{\psi(\omega_t),j}}|\varphi_{\psi(\omega_t),j}\rangle\langle\psi(\omega_t)|$$

Trajectories of the continuous measurement

- State transformations and probabilities by postulates of quantum mechanics

$$\rho \mapsto \frac{\mathcal{I}_{\omega_t, j(\emptyset)}\rho}{\mathrm{Tr}\left\{\mathcal{I}_{\omega_t, j(\emptyset)}\rho\right\}} \qquad p_{j(\emptyset)}(t) = \mathrm{Tr}\left\{\mathcal{I}_{\omega_t, j(\emptyset)}\rho\right\}$$

- When applied to the state $|\psi(\omega_t)\rangle\langle\psi(\omega_t)|$ fixed by the count sequence ω_t

$$\mathcal{I}_{\omega_t,j}|\psi(\omega_t)\rangle\langle\psi(\omega_t)| = V_{\omega_t,j}|\psi(\omega_t)\rangle\langle\psi(\omega_t)|V^\dagger_{\omega_t,j}dt \quad \begin{cases} |\psi(\omega_t)\rangle\langle\psi(\omega_t)| \mapsto \dfrac{V_{\omega_t,j}|\psi(\omega_t)\rangle\langle\psi(\omega_t)|V^\dagger_{\omega_t,j}}{\|V_{\omega_t,j}|\psi(\omega_t)\rangle\|^2} = |\varphi_{\psi(\omega_t),j}\rangle\langle\varphi_{\psi(\omega_t),j}| \\ p_j(t) = \|V_{\omega_t,j}|\psi(\omega_t)\rangle\|^2 dt = \lambda_{\psi(\omega_t),j}dt \end{cases}$$

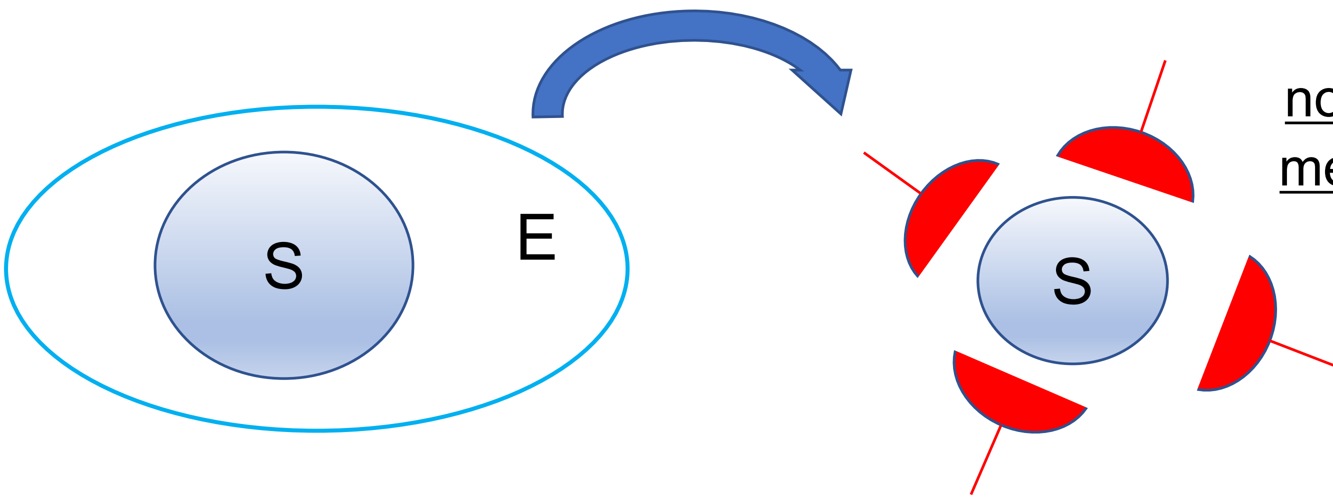
$$\mathcal{I}_{\omega_t,\emptyset}|\psi(\omega_t)\rangle\langle\psi(\omega_t)| = F_{\omega_t,\emptyset}|\psi(\omega_t)\rangle\langle\psi(\omega_t)|F^\dagger_{\omega_t,\emptyset} \quad \begin{cases} |\psi(\omega_t)\rangle\langle\psi(\omega_t)| \mapsto \dfrac{\left(1 - iH^{eff}_{\omega_t}dt\right)|\psi(\omega_t)\rangle\langle\psi(\omega_t)|\left(1 + iH^{eff,\dagger}_{\omega_t}dt\right)}{\|\left(1 - iH^{eff}_{\omega_t}dt\right)|\psi(\omega_t)\rangle\|^2} \\ P(t) = \|\left(1 - iH^{eff}_{\omega_t}dt\right)|\psi(\omega_t)\rangle\|^2 = 1 - \sum_{j=1}^{n} p_j(t) \end{cases}$$

If we identify the count sequence with the sequence of jumps, we get the <u>same trajectories and associated probabilities</u> as the unraveling!!

Environment as a non-selective observer

- By construction, the <u>average</u> over the trajectories gives us

$$\frac{d}{dt}\rho(t) = -i[H_S(t),\rho(t)] + \sum_{\alpha=1}^{n^2-1} c_\alpha(t)\left(L_\alpha(t)\rho(t)L_\alpha(t)^\dagger - \frac{1}{2}\left\{L_\alpha^\dagger(t)L_\alpha(t),\rho(t)\right\}\right)$$



<u>non-selective measurement</u>

Consistent continuous-measurement picture Barchielli & Belavkin JPA **24** (1991)

! $\mathcal{I}_{\omega_t,j}(\emptyset)$ The instrument itself depends on the sequence of jumps

Dynamics' complete positivity <u>not</u> needed: positivity is enough

Case study 1: Qubit evolution

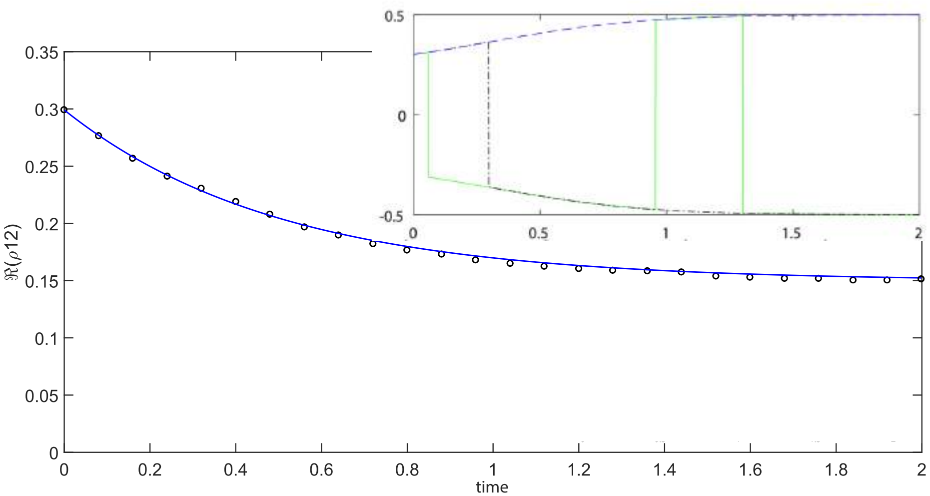
Hall, Cresser, Li & Andersson PRA **89** (2014)

$$\frac{d}{dt}\rho(t) = \frac{1}{2}\sum_{k=1}^{3}\gamma_k(t)\left[\sigma_k\rho(t)\sigma_k - \rho(t)\right]$$

$$\begin{aligned}\gamma_1(t) &= 1\\ \gamma_2(t) &= 1\\ \gamma_3(t) &= -\tanh(t)\end{aligned}$$

- The dynamics is completely positive and P-divisible
- One negative coefficient for any t>0

⇒ MCWF can never be applied (Eternal non-Markovianity)



- Solid line: solution of the master equation
- Circles: average over 10^4 trajectories
- $dt = 2 \times 10^{-3}$
- Inset: example of 3 trajectories
- Error bars within the circles

Effective unraveling for all t>0; jumps associated with detectors “clicks”

Outline

- Introduction and motivation
- The rate operator quantum method
- Extension to general open-system dynamics
- Conclusion and outlook

Beyond P-divisible dynamics

$$W^J_{\psi(t)} = \sum_{j^+} \lambda_{\psi(t),j^+} |\varphi_{\psi(t),j^+}\rangle\langle\varphi_{\psi(t),j^+}| - \sum_{j^-} \left|\lambda_{\psi(t),j^-}\right| |\varphi_{\psi(t),j^-}\rangle\langle\varphi_{\psi(t),j^-}|$$

Positive eigenvalues: as before $|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j^+}\rangle$ $\quad p_{j^+}(t) = \lambda_{\psi(t),j^+} dt$

Negative eigenvalues: we define reversed jumps, to avoid $p_{j^-}(t) < 0$

Piilo, Maniscalco, Harkonen, Suominen PRL **100** (2008)

$$|\psi_k(t)\rangle \mapsto |\psi_{k'}(t)\rangle$$

$$|\psi_k(t)\rangle = |\varphi_{\psi_{k'}(t),j^-}\rangle$$

Constraint on the source&target states

$$p_{j^-}^{(k\to k')}(t) = \frac{N_{k'}(t)}{N_k(t)} \left|\lambda_{\psi_{k'}(t),j^-}\right| dt$$

Probabilities relate different trajectories

Beyond P-divisible dynamics

$$W^J_{\psi(t)} = \sum_{j^+} \lambda_{\psi(t),j^+} |\varphi_{\psi(t),j^+}\rangle\langle\varphi_{\psi(t),j^+}| - \sum_{j^-} |\lambda_{\psi(t),j^-}| \, |\varphi_{\psi(t),j^-}\rangle\langle\varphi_{\psi(t),j^-}|$$

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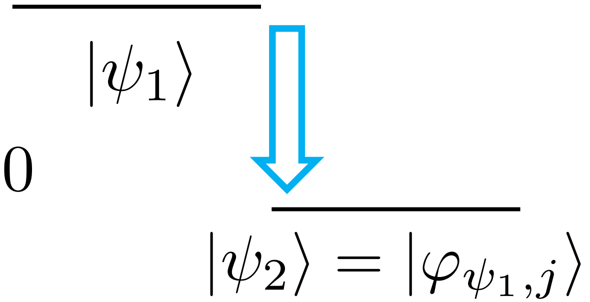
Constraint on the source&target states

$$p^{(k\to k')}_{j^-}(t) = \frac{N_{k'}(t)}{N_k(t)} |\lambda_{\psi_{k'}(t),j^-}| \, dt$$

Probabilities relate different trajectories

$\lambda_{\psi_1,j} > 0$



Beyond P-divisible dynamics

$$W^{J}_{\psi(t)} = \sum_{j^{+}} \lambda_{\psi(t),j^{+}} |\varphi_{\psi(t),j^{+}}\rangle\langle\varphi_{\psi(t),j^{+}}| - \sum_{j^{-}} |\lambda_{\psi(t),j^{-}}|\, |\varphi_{\psi(t),j^{-}}\rangle\langle\varphi_{\psi(t),j^{-}}|$$

Positive eigenvalues: as before $|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j^{+}}\rangle \qquad p_{j^{+}}(t) = \lambda_{\psi(t),j^{+}} dt$

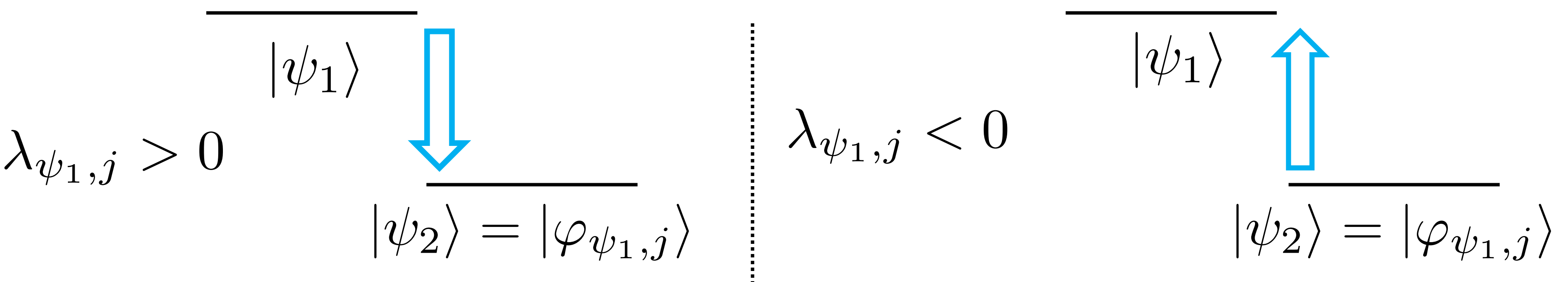
Negative eigenvalues: we define reversed jumps, to avoid $p_{j^{-}}(t) < 0$

Piilo, Maniscalco, Harkonen, Suominen PRL **100** (2008)

$|\psi_k(t)\rangle \mapsto |\psi_{k'}(t)\rangle$

$|\psi_k(t)\rangle = |\varphi_{\psi_{k'}(t),j^{-}}\rangle$ — Constraint on the source&target states

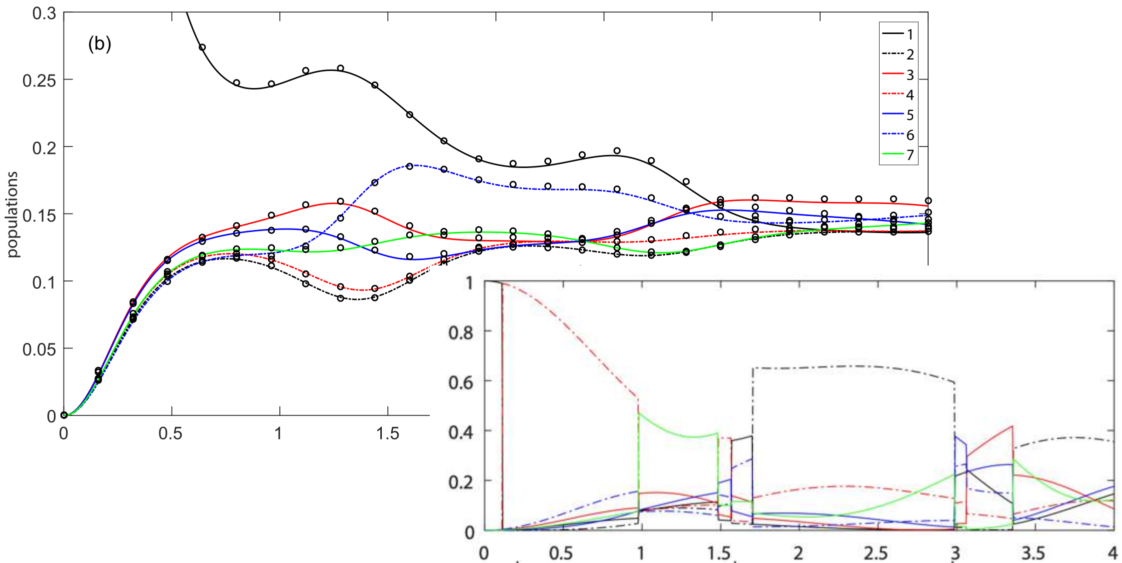
$p^{(k\to k')}_{j^{-}}(t) = \frac{N_{k'}(t)}{N_k(t)} |\lambda_{\psi_{k'}(t),j^{-}}|\, dt$ — Probabilities relate different trajectories



Case study 2: 7-coupled-site system

Dissipative network of coupled sites (transport phenomena, …)

- Coherent coupling among the sites $H_S = \sum_{i \neq j} \Omega_{i,j} |i\rangle\langle j|$ $0 \leqslant \Omega_{i,j} \leqslant 0.6$ uniformly random
- 49 decay channels: transitions+dephasing $c(t)|i\rangle\langle j|$ $c(t) < 0$ at some *t*



- Different site populations
- Trajectory of populations with reversed jumps
- dt = 2×10^{-3}
- average 3×10^{4} trajectories

Excellent agreement during the whole evolution; only 7 jump operators

Markovian, non-Markovian and in-between

- Non-Markovianity: memory effects influence the evolution

Strength of memory effects

Qualitative scale!

- All coefficients are positive (CP-divisible dynamics)

  MCWF and standard measurement interpretation apply

  ! The jump probabilities at t depend on the previous sequence of jumps $p_\alpha(t) = c_\alpha \| L_\alpha |\psi(t)\rangle \|^2 dt$

- P-divisible dynamics

  ROQJ and novel measurement interpretation apply

  The jump probabilities <u>and the kind of jumps</u> depend on the previous sequence $|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j}\rangle$ $p_j(t) = \lambda_{\psi(t),j} dt$

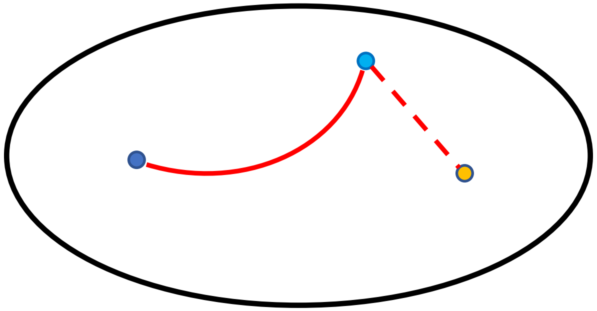
- Not P-divisible dynamics: reversed jumps

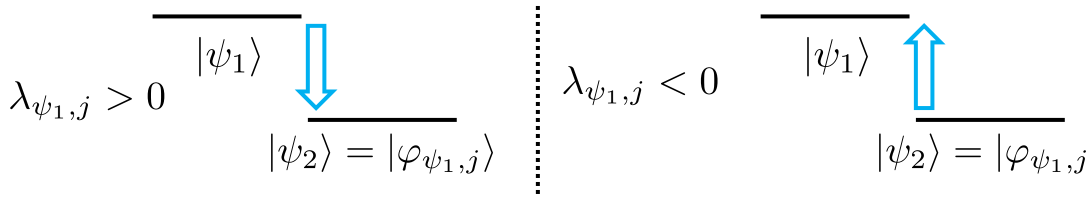
  The future on a trajectory depends on the <u>past of all trajectories</u>

  No continuous measurement interpretation !!

Conclusion

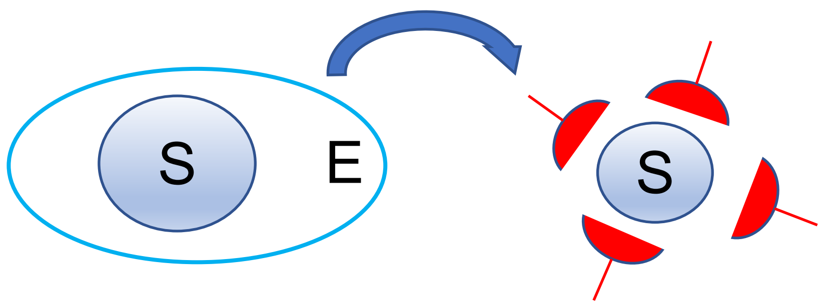
✓ Unified framework to quantum-jump unraveling based on rate operator



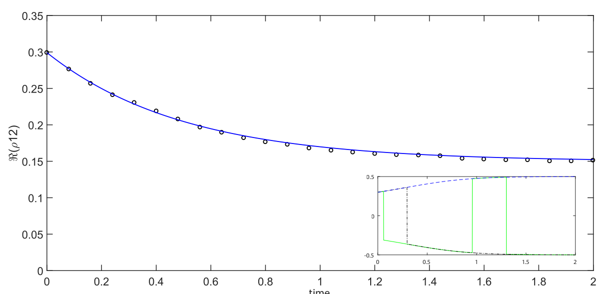


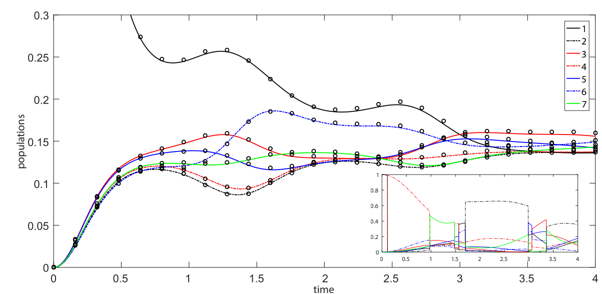
$|\psi(t)\rangle \mapsto |\varphi_{\psi(t),j}\rangle$ $\qquad$ $p_j(t) = \lambda_{\psi(t),j} dt$

✓ Continuous-measurement interpretation: positivity is enough! No need of CP



✓ Unraveling of dynamics for which other methods do not apply

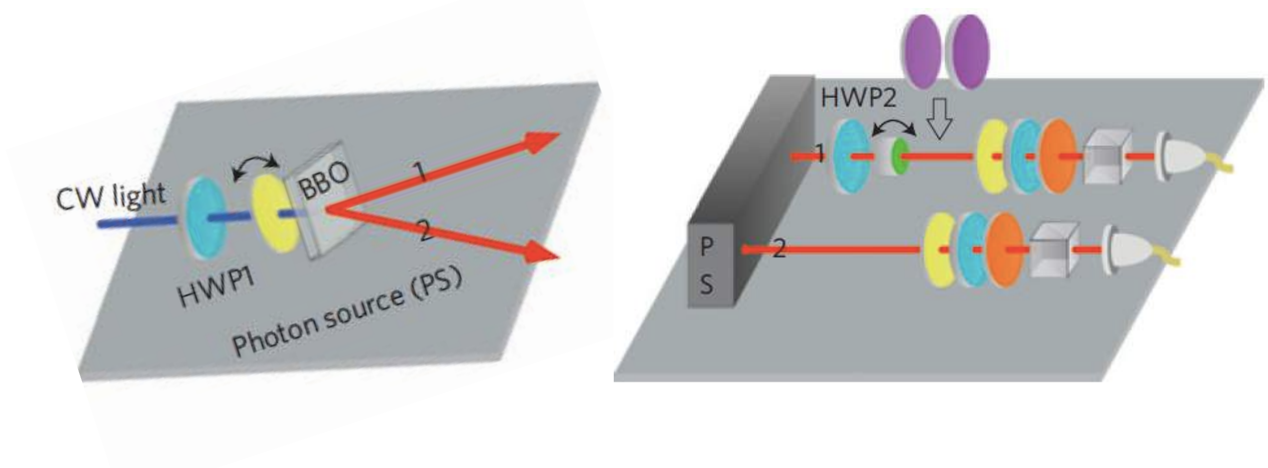




Outlook

- Experimental realization: adaptive measurement apparatus

Example: polarization and spatial d.o.f. of a photon produced in SPDC



- Family of rate-operator unravelings: design of the trajectories

$$R_{\psi(t)} = \sum_{\alpha=1}^{N^2-1} c_\alpha(t) L_\alpha(t)(|\psi(t)\rangle\langle\psi(t)|) L_\alpha^\dagger(t)$$

$$H_{eff} = H - \frac{i}{2}\sum_{\alpha=1}^{n^2-1} c_\alpha L_\alpha^\dagger L_\alpha$$

arxiv:2009.11312

$R_{\psi(t)} > 0$ if the dynamics is dissipative


Thank
you

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A simple example

- Two-level system coupled to a laser field and undergoing <u>spontaneous emission</u>

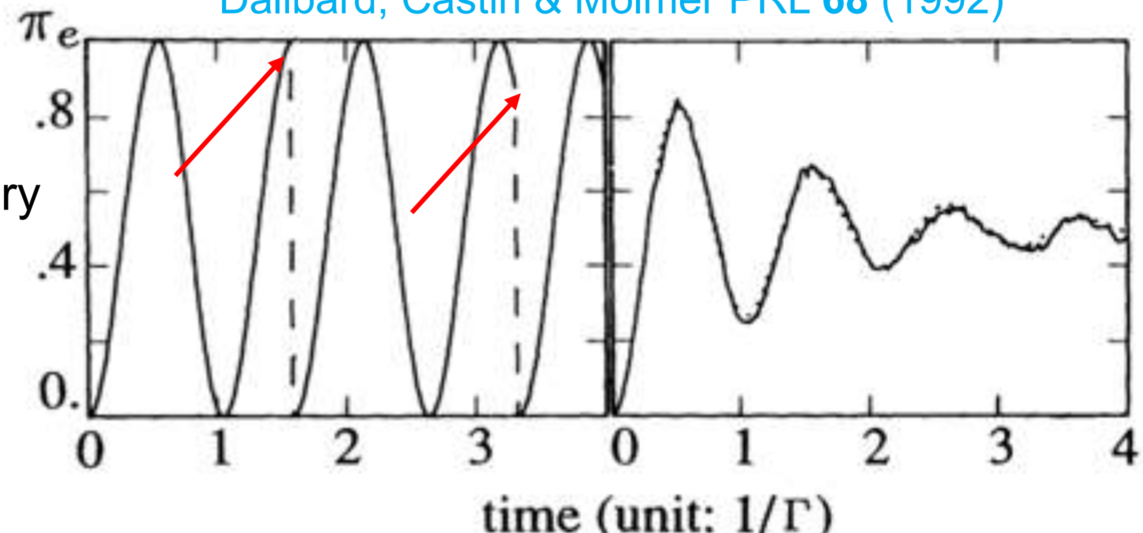
$$H_S = \Delta\sigma_z + \Omega\sigma_x \qquad L = \sigma_- \qquad c = \Gamma > 0$$

- Deterministic evolution $\quad H_{\text{eff}} = H_S - \frac{i\Gamma}{2}|e\rangle\langle e|$
- One kind of jump: $\quad \alpha(t)|g\rangle + \beta(t)|e\rangle \mapsto |g\rangle$

  with probability $\quad p(t) = \Gamma|\beta(t)|^2 dt$

Dalibard, Castin & Molmer PRL **68** (1992)

Single trajectory



Average over 100 trajectories