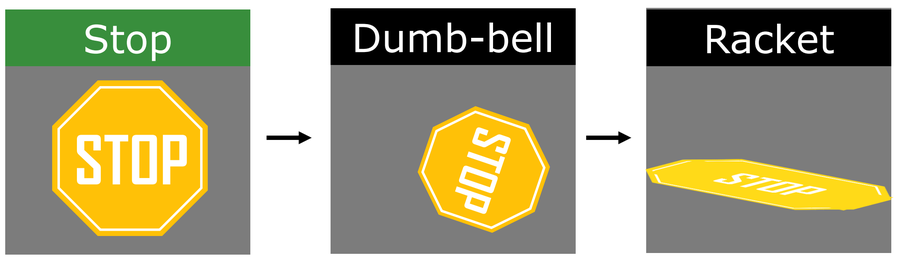
Group Equivariant Non-Expansive Operators: a mathematical tool to build Explainable Networks.

Giovanni Bocchi
Department of Environmental Science and Policy, University of Milan.

Motivation

Explainable Artificial Intelligence is rising mainly because traditional techniques of Deep Learning have proved to be deceivable, counterfaitable and hackable. Reason but also Public Authorities ask for trustworthy methods able to compete with their opaque adversaries.



GENEOs

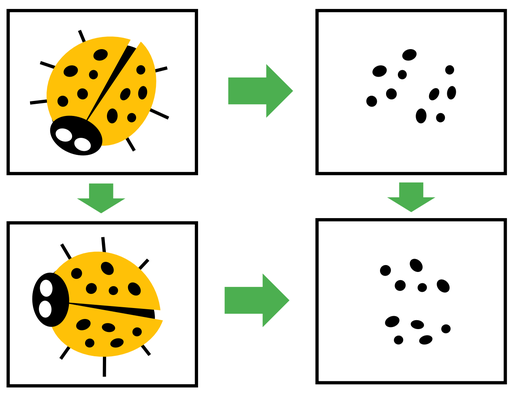
Group Equivariant Non-Expansive Operators are mathematical tools to process data with awareness of some important geometric properties. They can be regarded as observers that extract meaningful information from the data.

Given two functional spaces $\Phi = \{\varphi: X \to \mathbb{R}\}$ and $\Psi = \{\psi: Y \to \mathbb{R}\}$, G and H subgroups of $\mathrm{Homeo}_\Phi(X)$ and $\mathrm{Homeo}_\Psi(Y)$ and a group homomorphism $T: G \to H$, then a GENEO w.r.t T is a map $F: \Phi \to \Psi$ with the following properties:

1. **Equivariance:** $F(\varphi \circ g) = F(\varphi) \circ T(g)$ for all $\varphi \in \Phi$ and $g \in G$.
2. **Non Expansivity:** $||F(\varphi_1) - F(\varphi_2)||_\infty \leq ||\varphi_1 - \varphi_2||_\infty$ for all $\varphi_1, \varphi_2 \in \Phi$

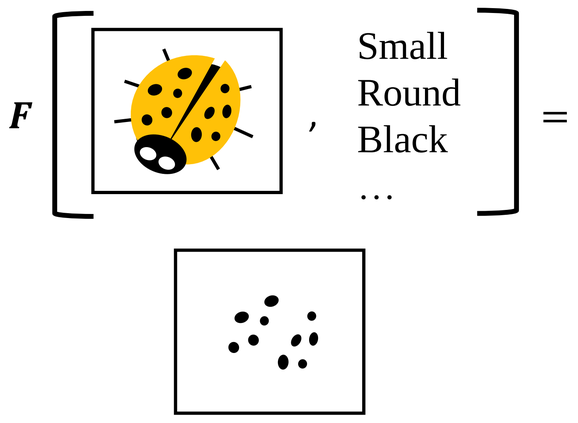
Equivariance

Equivariance allows to encode in the analysis significant geometrical properties of the data which are known in advance.



Prior Knowledge

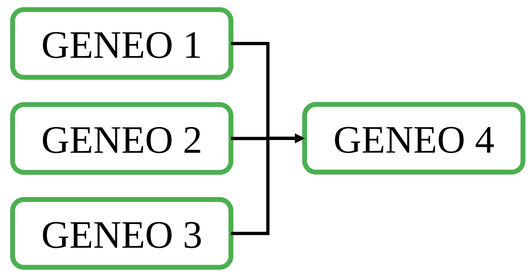
Apart from equivariance, the specific definition of a GENEO allows to take into account even more prior knowledge.



Networking

There is a list of admissible operations that can be applied to a set of GENEOs in order to get new operators (composition, convex combination…).

These operations are used to build networks of GENEOs.



GENEOnet

Proof of concept of a GENEO Network for Protein Pocket Detection. It exploits parametric families of GENEOs to build a model that incorporates prior knowledge and encodes equivariance w.r.t. rigid motions of the Euclidean space.

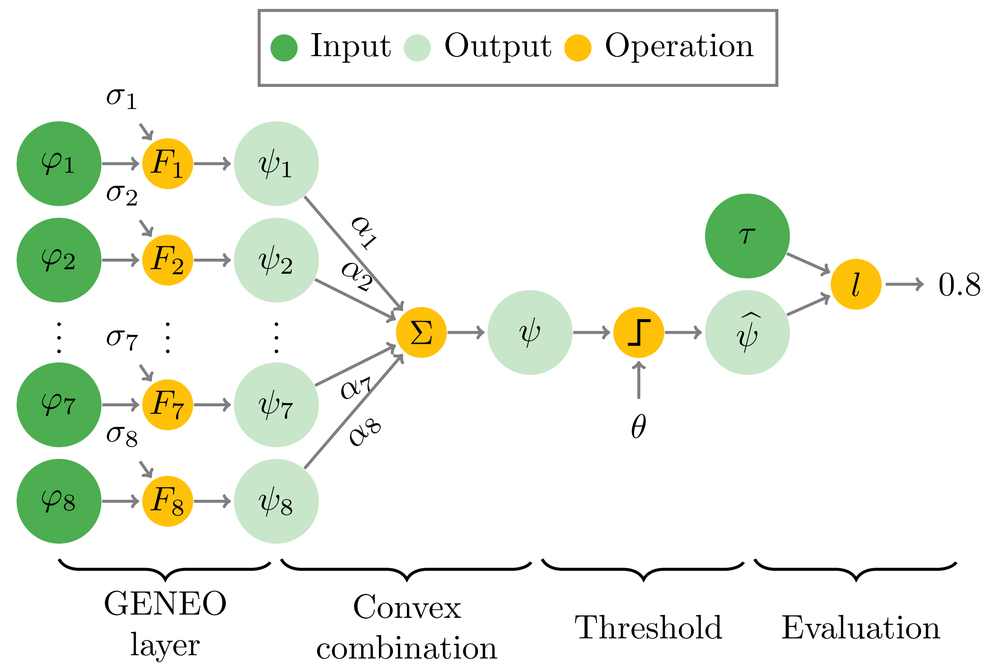
It is **Equivariant** by design

It allows to exploit **prior knowledge**

It has very few trainable parameters: **just 17**

Due to the architecture: we can assign a clear **meaning** to parameters' values

Equivariance + few parameters = **reduction of computational costs**



Joint work with:
Alessandra Micheletti, Patrizio Frosini, Alessandro Pedretti, Carmine Talarico, Filippo Lunghini, Andrea Beccari and Carmen Gratteri.

Reach me at:
giovanni.bocchi1@unimi.it
giovannibocchi6.wixsite.com/giovannibocchi

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