260.

REMARKS UPON THE LAW OF COMPLETE RADIATION.

[*Philosophical Magazine*, XLIX. pp. 539, 540, 1900.]

BY complete radiation I mean the radiation from an ideally black body, which according to Stewart* and Kirchhoff is a definite function of the absolute temperature θ and the wave-length λ. Arguments of (in my opinion†) considerable weight have been brought forward by Boltzmann and W. Wien leading to the conclusion that the function is of the form

$$\theta^5 \phi(\theta\lambda)\, d\lambda, \quad \text{.................................(1)}$$

expressive of the energy in that part of the spectrum which lies between λ and $\lambda + d\lambda$. A further specialization by determining the form of the function ϕ was attempted later‡. Wien concludes that the actual law is

$$c_1 \lambda^{-5} e^{-c_2/\lambda\theta}\, d\lambda, \quad \text{.............................(2)}$$

in which c_1 and c_2 are constants, but viewed from the theoretical side the result appears to me to be little more than a conjecture. It is, however, supported upon general thermodynamic grounds by Planck§.

Upon the experimental side, Wien's law (2) has met with important confirmation. Paschen finds that his observations are well represented, if he takes

$$c_2 = 14{,}455,$$

* Stewart's work appears to be insufficiently recognized upon the Continent. [See *Phil. Mag.* I. p. 98, 1901; p. 494 below.]

† *Phil. Mag.* Vol. XLV. p. 522 (1898).

‡ *Wied. Ann.* Vol. LVIII. p. 662 (1896).

§ *Wied. Ann.* Vol. I. p. 74 (1900).

θ being measured in centigrade degrees and λ in thousandths of a millimetre (μ). Nevertheless, the law seems rather difficult of acceptance, especially the implication that as the temperature is raised, the radiation of given wave-length approaches a limit. It is true that for visible rays the limit is out of range. But if we take $\lambda = 60\,\mu$, as (according to the remarkable researches of Rubens) for the rays selected by reflexion at surfaces of Sylvin, we see that for temperatures over 1000° (absolute) there would be but little further increase of radiation.

The question is one to be settled by experiment; but in the meantime I venture to suggest a modification of (2), which appears to me more probable *à priori*. Speculation upon this subject is hampered by the difficulties which attend the Boltzmann-Maxwell doctrine of the partition of energy. According to this doctrine every mode of vibration should be alike favoured; and although for some reason not yet explained the doctrine fails in general, it seems possible that it may apply to the graver modes. Let us consider in illustration the case of a stretched string vibrating transversely. According to the Boltzmann-Maxwell law the energy should be equally divided among all the modes, whose frequencies are as 1, 2, 3, ... Hence if k be the reciprocal of λ, representing the frequency, the energy between the limits k and $k + dk$ is (when k is large enough) represented by dk simply.

When we pass from one dimension to three dimensions, and consider for example the vibrations of a cubical mass of air, we have (*Theory of Sound*, § 267) as the equation for k^2,

$$k^2 = p^2 + q^2 + r^2,$$

where p, q, r are integers representing the number of subdivisions in the three directions. If we regard p, q, r as the coordinates of points forming a cubic array, k is the distance of any point from the origin. Accordingly the number of points for which k lies between k and $k + dk$, proportional to the volume of the corresponding spherical shell, may be represented by $k^2 dk$, and this expresses the distribution of energy according to the Boltzmann-Maxwell law, so far as regards the wave-length or frequency. If we apply this result to radiation, we shall have, since the energy in each mode is proportional to θ,

$$\theta k^2 dk, \qquad \text{.................................(3)}$$

or, if we prefer it,

$$\theta \lambda^{-4} d\lambda. \qquad \text{..............................(4)}$$

It may be regarded as some confirmation of the suitability of (4) that it is of the prescribed form (1).

The suggestion is that (4) rather than, as according to (2),

$$\lambda^{-5}\, d\lambda \qquad \text{.................................(5)}$$

may be the proper form when $\lambda\theta$ is great*. If we introduce the exponential factor, the complete expression will be

$$c_1 \theta \lambda^{-4} e^{-c_2/\lambda\theta} d\lambda. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

If, as is probably to be preferred, we make k the independent variable, (6) becomes

$$c_1 \theta k^2 e^{-c_2 k/\theta} dk. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Whether (6) represents the facts of observation as well as (2) I am not in a position to say. It is to be hoped that the question may soon receive an answer at the hands of the distinguished experimenters who have been occupied with this subject.

* [1902. This is what I intended to emphasize. Very shortly afterwards the anticipation above expressed was confirmed by the important researches of Rubens and Kurlbaum (*Drude Ann.* IV. p. 649, 1901), who operated with exceptionally long waves. The formula of Planck, given about the same time, seems best to meet the observations. According to this modification of Wien's formula, $e^{-c_2/\lambda\theta}$ in (2) is replaced by $1 \div (e^{c_2/\lambda\theta} - 1)$. When $\lambda\theta$ is great, this becomes $\lambda\theta/c_2$, and the complete expression reduces to (4).]