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A critical remark on Planck's model of black–body

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We reexamine the model of matter–radiation interaction considered by Planck in his studies on the black–body problem, and point out that it is, in our opinion, inconsistent. The key point is that Planck's model deals in principle with a system of n material resonators interacting with the field, but in fact Planck actually deals with a single forced resonator, because he explicitly makes the assumption that the resonators act *independently of one another*, i.e. perform incoherent motions. We show that this assumption is inconsistent when the contribution of all the resonators is taken into account. Moreover we point out that, in view of the long range character of the electrodynamical forces, it would be more appropriate to deal from the start with the dynamics of the complete system, with the mutual interaction of matter and radiation taken into account, looking for solutions of a coherent or correlated type (normal modes).

Keywords: BLACK–BODY; COHERENCE

1. Introduction

Since a certain time we are involved in a research (for a review see 1) concerning the possibility of a consistent formulation of classical electrodynamics of point particles, a problem whose relevance was often stressed by F. Guerra, to whom the present paper is dedicated. A new input to such a line of research came recently to us from a critical reading of the two papers on the black–body theory (see 2, 3 and 4, particularly Chapters III and IV) that Planck wrote just before establishing his radiation law. The occasion for this was provided by the appearing of an italian translation[5] of a relevant selection of Planck's original papers on the black–body problem.

Through such a critical reading we happened to suddenly realize that the whole dynamical treatment of Planck appears to be inconsistent. The key problem is the following one. In dealing with radiation one is dealing with far fields, which produce a strong coupling between far–away objects. This produces a kind of nonextensivity which could have deep implications. Actually, this fact is very well known at the level of statistical mechanics, as illustrated for example in 6. Our point is that analogous deep implications might occur, before than in statistical mechanics, in the dynamics itself. Thus, a priori it is not at all clear that one might be allowed to treat a system of charged particles as if they were independent; on the other hand, as we will illustrate below, in the model of a black–body considered by Planck, it is explicitly assumed that his "resonators" act independently of one another.

In the present paper we show that Planck's model is indeed inconsistent, and indicate some perspectives which are thus suggested.

2. The weak equipartition principle

The first paper of Planck we refer to is the one in which he is credited to have proven a general relation between the spectral energy density of the field and the mean energy of a material "resonator", a relation which we like to call the "weak equipartition principle". In the familiar version that was later given to it by Rayleigh (and Jeans), such a relation asserts that at equilibrium (at a given temperature T) all the normal modes of the field in the frequency range ν, $\nu + d\nu$ have the same mean energy, which is just equal to the mean energy of any single material resonator of frequency ν. Namely, if $u_\nu \, d\nu$ represents the energy (per unit volume) of the field in that frequency range, and U_ν the mean energy of the resonator (u_ν and U_ν being functions of ν and T), the weak equipartition principle asserts that

$$u_\nu = \frac{8\pi\nu^2}{c^3} U_\nu \ , \tag{1}$$

where c is the speed of light. While according to the familiar equipartition principle all oscillators of any frequency should have the same mean energy, equal to kT with k the Boltzmann constant, according to its weak form (1) the equal share of energy is expected to occur only for all the oscillators (both material ones, or field oscillators) in the same frequency range ν, $\nu + d\nu$, leaving open the problem of which value should then be assigned to that mean energy. As everyone knows, on October 19, 1900 the formula for $U_\nu(\nu, T)$ was found by Planck through an astute interpolation of two limit formulas for the spectral density u_ν; conversely, in the year 1906, the weak equipartition principle was used by Einstein[7] in order to obtain, from the known Planck's formula for the spectral energy density u_ν of the field, the mean energy U_ν of a material resonator of a crystal.

Leaving aside any discussion concerning Planck's radiation formula itself, the problem of interest for us here is the consistency of the model which is assumed by Planck in his attempt at giving a dynamical foundation to the weak equipartition principle.

3. Planck's starting point: the interaction of a single resonator with the field

In his previous works on the black–body dynamics Planck had concentrated his attention on the extremely simplified problem of the interaction of a single material resonator with the electromagnetic field. The radiation of energy from the resonator was taken into account by Larmor formula and this leads to the equation which is now usually known under the name of Abraham–Lorentz, with its peculiar third order derivative. As an approximation, the third order derivative can be substituted by a first order derivative. In conclusion, the interaction of the material resonator with the electromagnetic field is dealt with in Planck's paper by the familiar elementary equation of a damped harmonic oscillator, with a forcing due to the field, and a suitable choice of the parameters. In his notations (apart from denoting by a dot the derivative with respect to time):

$$\ddot{f} + 2\sigma\nu\dot{f} + 4\pi^2\nu^2 f = \frac{3c^2\sigma}{4\pi^2\nu} Z \tag{2}$$

where f is the electric dipole moment of the resonator of proper frequency ν (Planck writes ν_0 in place of our ν) and damping constant σ, while Z is the component of the electric field in the direction of the resonator. By a very interesting reasoning entailing several subtleties, he then comes to his "fundamental equation" for the time evolution of the mean energy of the resonator, which we write in a slightly modified version involving the quantity u_ν instead of his "intensity" $\mathcal{J}_\nu$, namely

$$\dot{U}_\nu + 2\sigma\nu U_\nu = \frac{c^3\sigma}{4\pi\nu} u_\nu \ . \tag{3}$$

Then, relation (1) is obtained by just requiring equilibrium in (3), i.e. $\dot{U}_\nu = 0$. We do not enter here a discussion of the interesting way in which Planck takes into account macroscopic quantities described as "slowly variable ones", in contrast to microscopic fast quantities; see for example the quoted book of Kuhn. Particularly interesting is also the way in which Planck uses resonators of large damping constant as detectors of the field. Notice moreover that the deduction of the fundamental equation (3) is not obtained by Planck at a purely dynamical level, because a statistical hypothesis on the radiation (natural radiation hypothesis) is inserted.

The essential point for what follows is that from his analysis of the interaction of a single resonator with the field Planck establishes that at equilibrium the power emitted in the frequency range ν, $\nu + d\nu$ by ta single resonator is proportional to the spectral energy density u_ν, which is a specific quantity, namely independent of the volume.

4. Planck's black–body model with incoherent resonators

Then Planck comes to deal with the complete black–body model. In his words (Section 6 of the paper 3): "*Let us now imagine that in the stationary radiation field, instead of a single resonator, there be present an arbitrarily large number n of them, in all respects analogous to the one considered up to now, and that in such resonators there take place in the time interval dt exactly the same processes* independently *of one anoother*". The same point of view is taken in his book "Waermestrahlung". Indeed, in the english version of the second edition, of the year 1912[8], one finds:

"Let us now suppose that a large number n of similar oscillators with parallel axes, acting quite independently of one another, are distributed irregularly in a volume element".

Namely, he imagines that each of the n resonators absorbs and emits energy in a incoherent way, independently, as he explicitly says, of the other ones. One would have expected to see Planck work out some model of a black–body, with the mutual electromagnetic interaction between the resonators taken consistently into account, but this is not at all the case. The fact that he is dealing with a system of n bodies is used just in the following way. First, he admits that energy and entropy are both extensive. Then, by considering the equation for the increase of entropy of a single resonator displaced from equilibrium (that he had previously worked out), he deduces a functional relation between entropy and energy, which turns out to be equivalent to the Wien radiation law. This is actually his final result. No concrete model for the dynamics of the black–body as a whole (namely with the mutual interaction between field and resonators taken into account) is discussed.

5. Coherence versus incoherence: inconsistency of Planck's model

Now, it seems to us that, due to the long range of the electrodynamical forces, one is not a priori guaranteed that the system of resonators can be dealt with as if they were independent. On the contrary, we would be more inclined to believe that coherence effects might be dominating in the problem at hand, and even in any problem of matter–radiation interaction in bulk. Actually, in our opinion Planck's model is inconsistent.

A simple argument concerning matter in bulk is the following one. Let us consider the luminosity at a fixed point due to all resonators, which we assume to be uniformly distributed over a certain volume. By definition the luminosity (in the frequency range ν, $\nu + d\nu$) at a point due to a source (a given resonator, in our case) is proportional to the power emitted by the source in that frequency range, times $1/r^2$ where r is the distance of the resonator from the given point; on the other hand, as pointed out above, such a power, according to Planck's model, is independent of the volume. Furthermore, the luminosity due to several resonators just adds, if they are assumed to be incoherent; so, in particular, the luminosity due to the resonators lying in a spherical shell between r and $r + dr$ is constant, independent of r. Thus the total luminosity in that frequency range at a given point should be proportional to the volume of the cavity. On the other hand, by a standard argument going back to Kirchhoff, the luminosity at a point in that frequency range is proportional to specific energy u_ν, and is thus independent of the volume. This is a contradiction.

The paradox discussed above turns out to be analogous to the well known one of Olbers, which concerns the luminosity of the sky. Usually (see 9, Sec. 16.1) the Olbers paradox is cared by making recourse to cosmological considerations, such as taking into account that the universe had an origin at a certain time and is expanding, so that only the shells up to a maximal distance would contribute. Or, perhaps, the motions of the stars should not be assumed to be independent.

The latter possibility, i.e the assumption that one is dealing with coherent or correlated motions, is the relevant one in solving an analogous paradox in electrodynamics. This is discussed at the level of an exercise in the standard reference textbook of Jackson (see 10, Exercise 14.12 of the Chapter: Radiation by moving charges). In the previous Exercise 14.11 the calculation had been made of the energy radiated by a single particle performing a periodic motion, and in the following Exercise one is asked to calculate the radiation emitted by a set of n particles moving, with fixed uniformly distributed relative positions on a circle, moving with velocity v. The result is that "*the dependence on n of the total power radiated is dominantly as β^{2n}, so that in the limit $n \to \infty$ no radiation is emitted*" (here, $\beta = v/c$). This is indeed the explanation of the familiar fact that the radiation of a steady current in a loop is negligible, as is explicitly mentioned in item (e) of Exercise 14.12. The apparent paradox is that a steady current would instead be found to radiate considerable power if the particles were assumed to be performing incoherent motions. Thus the global system of particles constituting the current presents, in connection with radiation, a character of nonadditivity over the subsystems.

6. Perspectives

So, Planck's model of the black–body, with n resonators radiating in a incoherent way and $n > 1$, seems to be inconsistent. In our opinion, the whole subject matter–radiation interaction should be reconsidered, by looking at models of n resonators in which the mutual electromagnetic interaction interaction of the resonators is taken into account. Particular attention should be given to the possibility that there exist coherent, or correlated, motions. The model should be the one proposed long ago by Dirac[11], the lagrangian of which was however provided only quite recently[12]. For a system of resonators, dealt with in the usual dipole approximation, one should look for the normal modes of the system, which are indeed correlated motions. In a particularly simple model with infinitely many resonators located on a one–dimensional lattice, the existence of nonradiating normal modes has indeed been proved. The proof will be published elsewhere.

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