The Republican primaries this year could easily shake your faith in democracy. The race whittled a field of 17 candidates down to a showdown between the two candidates with the highest unfavorability ratings in national polls.

In many respects, Donald Trump was less conservative than every other candidate; Ted Cruz was more so. Neither appeared to set mainstream Republicans in his sights.

You might think that democracy’s not supposed to work this way — that more broadly acceptable candidates should have risen to the top. Many Republicans I know worry that this outcome reflects a sickness in the Republican electorate. Perhaps. But maybe it is just an illustration of the inherent imperfection of democratic institutions.

The failure of democracy to provide a coherent ranking of political hopefuls is a central insight of the subfield of economics and political science known as social choice theory. The issue is neatly illustrated by Condorcet’s paradox, which shows that a shifting set of coalitions can make a collective body appear that it has no idea what it wants.

Here’s how. Think about the Republican Party as an alliance of moderates, conservatives and populists. A coalition of populists and moderates will vote to ensure a populist beats a conservative. Conservatives and populists will vote
together to ensure a conservative beats a moderate. And moderates and conservatives will join forces to help a moderate beat a populist.

The paradox is that these preferences form a cycle in which the populist is preferred to the conservative, who is preferred to the moderate, who is preferred to the populist, even though the populist was preferred to the conservative, who was preferred to the moderate.

It’s as if the voters prefer chocolate ice cream to strawberry, strawberry to vanilla, and vanilla to chocolate — a bizarre cycle that suggests that voters believe that each candidate is both better and worse than each of the alternatives.

It’s as if the will of the people is an incoherent concept.

The problem is not that individual voters are clueless; in this story, they’re not. Even if each individual voter is rational and knows what he or she wants, the electorate as a whole can act as if it were clueless and can’t decide. Individually rational choices don’t necessarily add up to collectively rational choices.

The depressing thing is that the problem is actually worse than this. The Marquis Condorcet, a French mathematician in the second half of the 18th century, showed how a majority-rule vote can lead to incoherent collective choices.

But Kenneth Arrow, the economics Nobel laureate, showed in his 1951 doctoral thesis that the problem runs far deeper than anyone had imagined. Mr. Arrow’s “impossibility theorem” says that there is no mechanism that can coherently speak for the will of the people.

Loosely speaking, this extraordinary result says that any mechanism that aims to speak for the will of the people — that is, not a dictatorship — will be susceptible to at least one of three defects.

The first possible defect is the problem the marquis illustrated — the problem of preference cycles. The second possible defect is that voters will make choices that suggest that the addition of irrelevant alternatives leads them to change their minds. This is the equivalent of choosing chocolate over vanilla, only to reverse course and choose vanilla instead, once strawberry has been added to the menu.
The third possibility is that even when each voter individually prefers chocolate ice cream to vanilla, somehow collectively the voters will choose vanilla instead. (I’m grateful to the economist Alex Tabarrok for this ice cream analogy.)

The news isn’t all bad. The fact that any voting rule is susceptible to one of these defects doesn’t mean that the voters must make incoherent choices, merely that it could happen. (And perhaps it just did.)

Mr. Arrow’s impossibility theorem suggests that maybe the Republican primary results say less about the desires of Republican voters than they do about tensions inherent in groups of people collectively deciding what to do.

Economic theorists have also pointed to a reason that the modern G.O.P. may be particularly susceptible to making strange choices. If disagreements between voters are simple enough — such as when some want more liberal policies and others more conservative policies — simple majority rule won’t result in any of the defects that concerned Mr. Arrow. Perhaps disagreements between Democrats are this simple.

But Republicans disagree both about the desirability of conservative versus moderate policies and about the need for an outsider or an establishment leader. This extra complexity again raises the possibility of collective irrationality even in the face of individual logic.

The point isn’t that democracy is bad, merely that it’s imperfect. And so even if this theorem points to the impossibility of a truly rational democracy, it doesn’t mean that the alternatives are any better. As Winston Churchill once said, “Democracy is the worst form of government, except for all the others.”

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