



Universiteit Leiden

# Independent Oscillator Models in System-Bath Quantum Dynamics

modeling energy dissipation in H scattering on surfaces

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# Outline

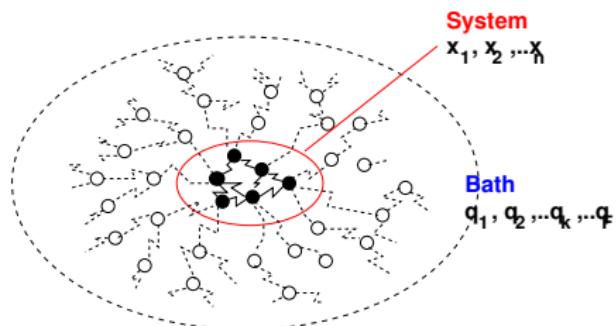
1 Introduction

2 Relaxation and Sticking Models

3 Conclusions

# System-bath dynamics

- **System:** relevant part, experimentally probed  
⇒ Few, important DOFs
- **Bath:** irrelevant part, but responsible for energy transfer  
⇒ Large number of DOFs of non-direct relevance



Quantum description is mandatory for inherently quantum systems and/or low-temperature baths..

# Vibrational Relaxation and Atomic Sticking

- Energy transfer to the surrounding
- Challenging problems: large set of degrees of freedom

## Vibrational Relaxation

- Morse Potential for Graphite-H
- System Energy in time

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<sup>a</sup>G. A. Worth, H.-D. Meyer and L. S. Cederbaum, J.Chem.Phys. **109**, 3518 (1998)  
<sup>b</sup>M. Nest and H.-D. Meyer, J.Chem.Phys. **119**, 24 (2003)

# Vibrational Relaxation and Atomic Sticking

- Energy transfer to the surrounding
- Challenging problems: large set of degrees of freedom

## Sticking

- Morse Potential for Graphite-H
- **Sticking Probability**,  
*i.e.* fraction of trapped norm

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<sup>a</sup>G. A. Worth, H.-D. Meyer and L. S. Cederbaum, J.Chem.Phys. **109**, 3518 (1998)

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# Independent Oscillator Model

- Quantum Model of dissipation
- System exchanges energy with a thermal bath of HO

$$H_{CL} = H_{System} + \sum_n \left\{ \frac{p_n}{2\mu_n} + \frac{1}{2}\mu_n\omega_n^2 q_n^2 \right\} + coupling$$

# Independent Oscillator Hamiltonian

$$H = \frac{p^2}{2M} + V(s) + \sum_k \left\{ \frac{p_k^2}{2} + \frac{\omega_k^2}{2} \left( x_k - \frac{c_k s}{\omega_k^2} \right)^2 \right\}$$

$$H \equiv H^{\text{sys}} + \Delta V(s) + H^{\text{int}} + H^{\text{bath}}$$

$H^{\text{sys}} = \frac{p^2}{2M} + V(s)$ : system Hamiltonian

$\Delta V(s) = \frac{1}{2} \left( \sum_k \frac{c_k^2}{\omega_k^2} \right) s^2 = \frac{1}{2} M \delta \Omega^2 s^2$ : "renormalization" potential

$H^{\text{int}} = - \sum_k c_k x_k s$ : interaction term

$H^{\text{bath}} = \sum_k \frac{p_k^2}{2} + \frac{\omega_k^2}{2} x_k^2$ : "bath" Hamiltonian

<sup>a</sup>A.O. Caldeira and A.J. Leggett, Phys.Rev.A **31**, 1059 (1985)

<sup>b</sup>G.W. Ford, J. T. Lewis and R. F. O'Connell Phys.Rev.A **37**, 4419 (1988)

# Generalized Langevin Equation

Solving the classical IO Hamiltonian, the system experience a force due to the environment, given by

$$F^{\text{env}} = \xi(t) - M \int_{t_0}^{+\infty} \gamma(t-t') \dot{s}(t') dt'$$

where

$$\xi(t) = \sum_k \left\{ \left[ x_k(t_0) - \frac{c_k}{\omega_k^2} s(t_0) \right] \cos(\omega_k t) + \frac{\dot{x}_k(t_0)}{\omega_k} \sin(\omega_k t) \right\} c_k$$

$$M \kappa(t) = \sum_k \frac{c_k^2}{\omega_k^2} \cos(\omega_k t)$$

$$\gamma(t) = \Theta(t) \kappa(t)$$

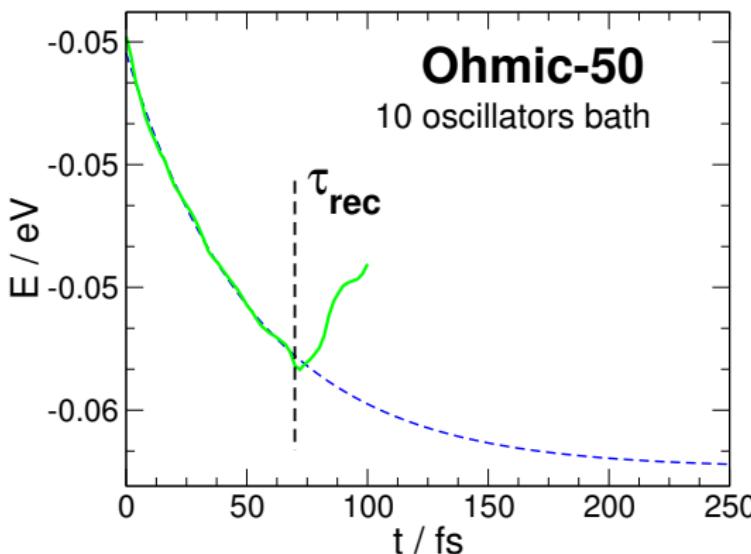
# Recurrence Time

In simulations, we adopt a **finite set of HO**



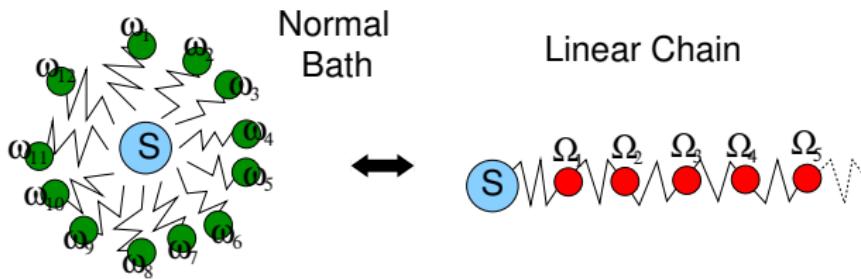
**Poincaré's recurrence**

After  $\tau_{rec}$  the energy gets back to the system

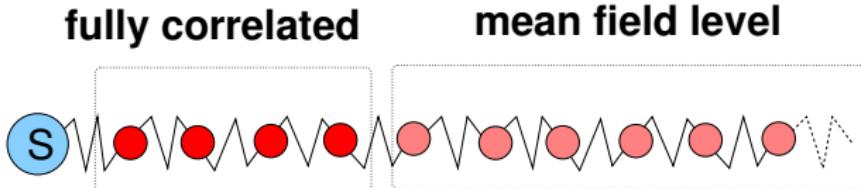


# Linear Chain Representation of the Bath

Performing a normal model transformation...



Advantage: stronger approximation on the oscillators "far" from the system



<sup>a</sup>K. H. Hughes, C. D. Christ and I. Burghardt, J.Chem.Phys. **131**, 024109 (2009)

<sup>b</sup>R. Martinazzo, B. Vacchini, K. H. Hughes and I. Burghardt, J.Chem.Phys. **134**, 011101 (2011)

<sup>c</sup>R. Martinazzo, K. H. Hughes and I. Burghardt, Phys.Rev.E **84**, 030102 (2011)

# Our project

## Aim of the work

Check the accuracy and the computational performances of the linear chain representation

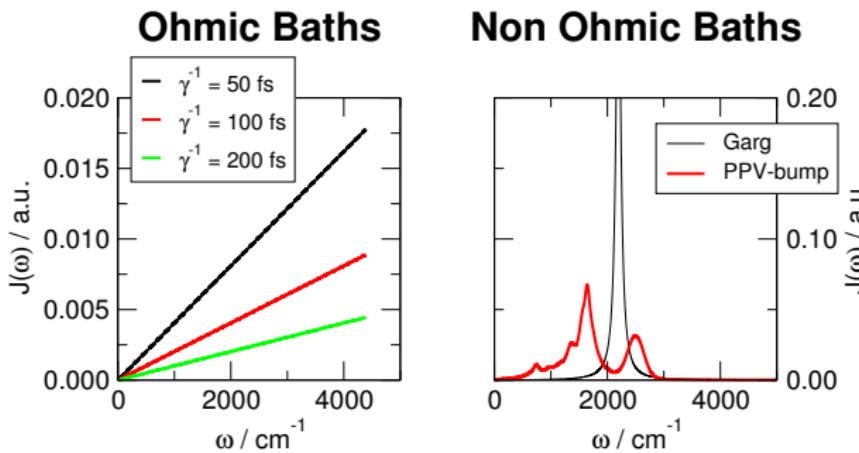
- Problems: vibrational relaxation and sticking
- Bath properties: different bath models (analytical / from literature)
- Linear chain: different degree of approximation (no. of correlated oscillators)
- Methodology: MCTDH<sup>a</sup> (Heidelberg's MCTDH Package<sup>b</sup>)

<sup>a</sup>H.-D. Meyer, U. Manthe and L. S. Cederbaum, Chem.Phys.Lett. **165**, 73 (1990)  
M. H. Beck, A. Jäckle, G. A. Worth and H.-D. Meyer, Phys.Rep. **324**, 1 (2000)

<sup>b</sup>G. A. Worth, M. H. Beck, A. Jäckle and H.-D. Meyer, The MCTDH Package, Version 8.4 (2007)

# Bath models

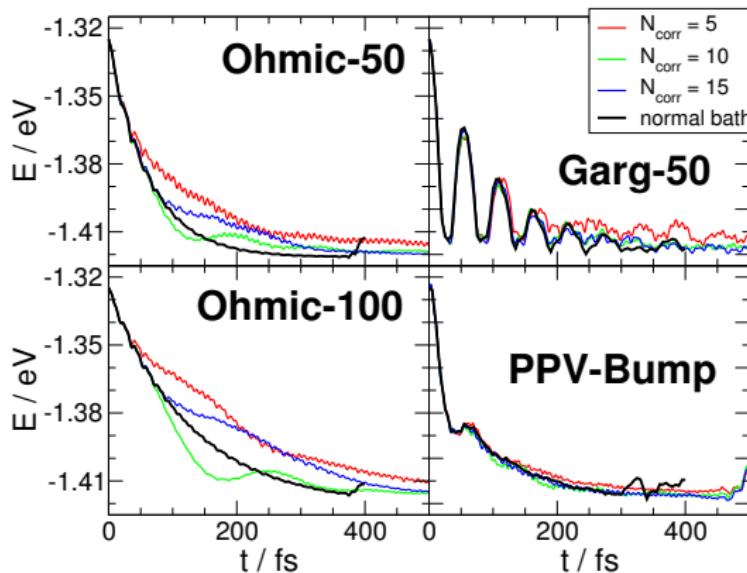
- The properties of the bath are defined by  $\omega_n$  and the coupling
- Sampling of the **spectral density** of the bath, *i.e.* dissipation in the frequency-domain
- We considered different model spectral densities



# Example of Chain Dynamics

- WP Relaxation
- PPV Bath in chain form, 100 oscillators (15 fully correlated)

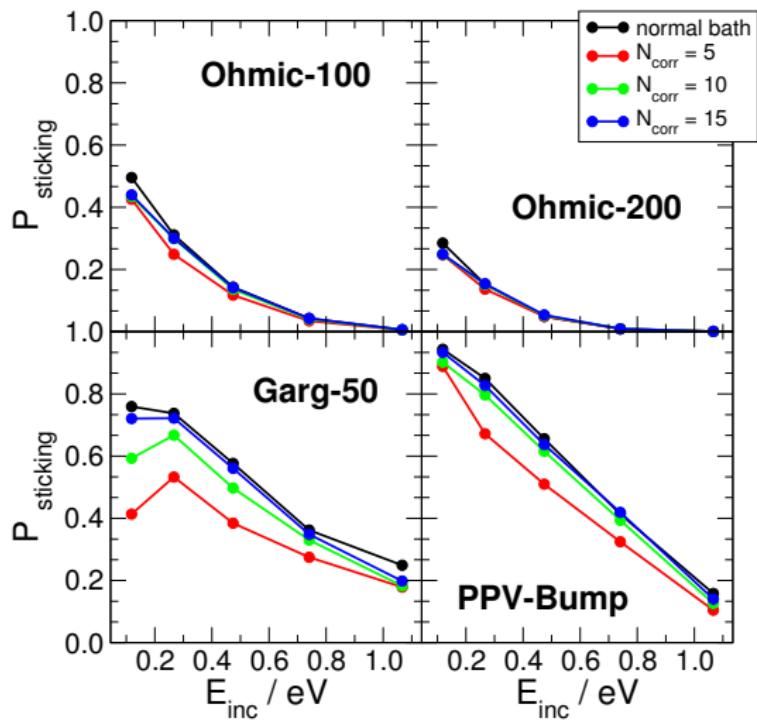
## Results - Energy Relaxation



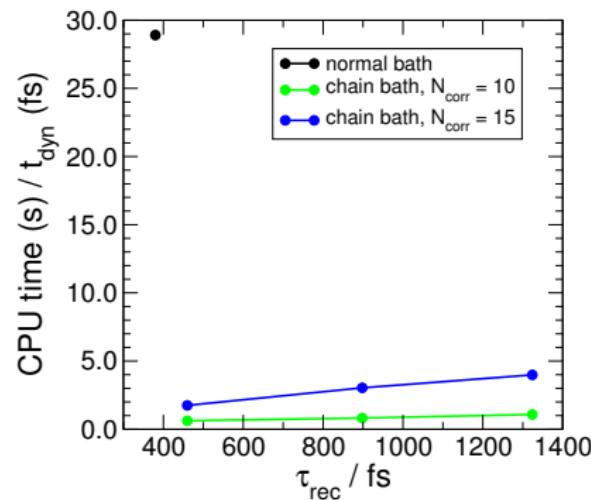
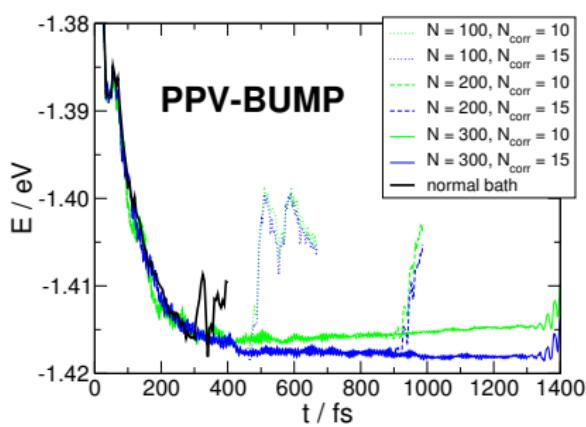
- Exact results at short time
- At longer time good qualitative behaviour
- Excellent agreement for non Ohmic Baths

# Results - Sticking

- $P_{\text{stick}}$  decreases with increasing  $E_{\text{inc}}$ , as expected
- Excellent results with the chains
- For  $E_{\text{inc}} = 0.12 \text{ eV}$  small error



# Results - Long Time Relaxation



- For the same recurrence time, great reduction of CPU time
- The CPU time scales linearly with the recurrence time

# Conclusions

- Very good results: even in the worse case qualitatively consistent behaviour
- Excellent agreement for realistic bath (including memory effects)
- Achieved a reduction of the computational cost of the simulation
- The recurrence time of the bath can be increased at a reasonable computational cost

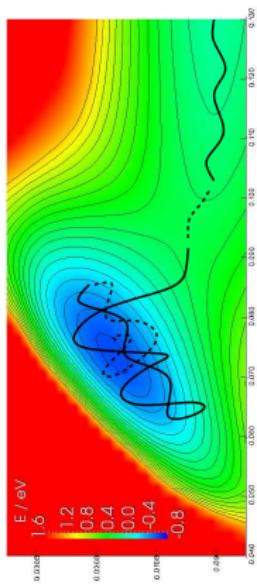
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<sup>a</sup>M.Bonfanti, R.Martinazzo, K.H.Hughes, I.Burghardt and G.F.Tantardini  
*Effective mode based wavefunctions for system bath quantum dynamics,*  
in preparation (2011)

# Further developments

In future...

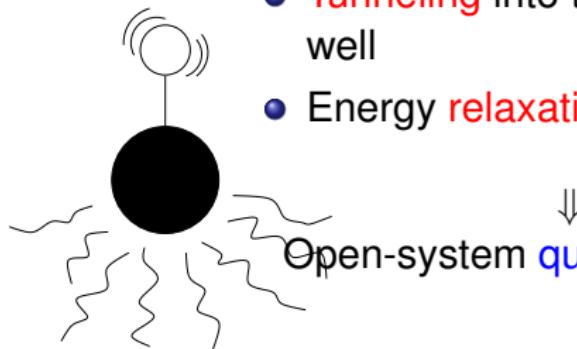
Application to a realistic sticking problem



## H Sticking on Graphene

Low energy H atoms:

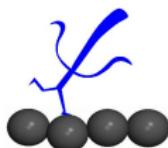
- Tunneling into the chemisorption well
- Energy relaxation



Open-system quantum problem

## Acknowledgments

- Gian Franco Tantardini, Dr. Rocco Martinazzo,  
Università degli Studi di Milano, Italy



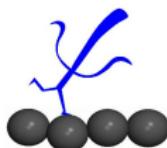
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- Irene Burghardt, Institute of Physics and Theoretical Chemistry, Goethe University Frankfurt, Germany
- Keith Hughes, School of Chemistry, Bangor University, United Kingdom

... and thank you, for your  
attention!

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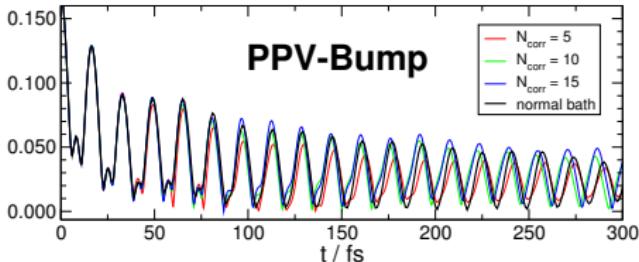
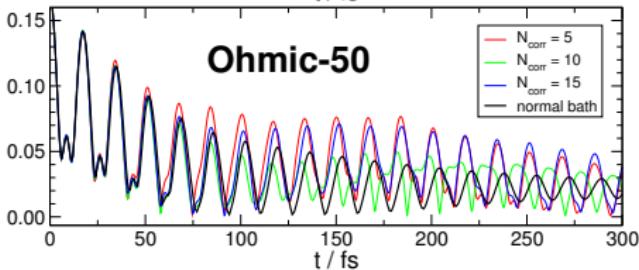
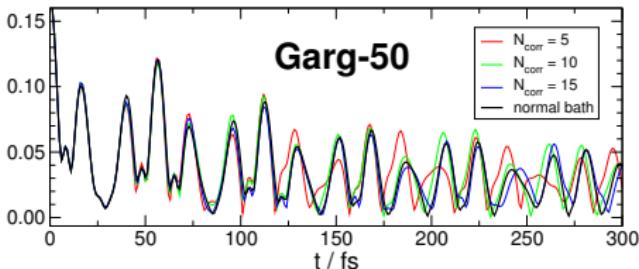


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# Position correlation function



# Bound State Probability

