

Let S be a K3 surface and C a genus g curve lying on it. The Donagi-Morrison conjecture concerns complete linear series A of type gr_d on C such that $d \leq g - 1$ and the Brill-Noether number is negative. It predicts that such an A is contained in another linear series which is cut out from a line bundle on S and satisfies some numerical conditions, meaningful from a Brill-Noether theory viewpoint. We provide counterexamples to the conjecture already for $r = 2$. We then study a slight modification of the conjecture, which is known to hold for $r = 1, 2$. By using coherent systems and generalized Lazarsfeld-Mukai bundles, we prove the modified conjecture under some hypotheses on secant divisors to the curve C and its deformations. We show that these hypotheses cannot be avoided by exhibiting counterexamples obtained jointly with A.L.Knutsen.