ISOGENIES BETWEEN JACOBIANS

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In this series of two lectures we will report about the following Theorem: Let Z be a subvariety of codimension $k > 0$ of the moduli space $\mathcal{M}_g$ of curves of genus g with $g > 3k + 4$ and let $\chi : JC' \to JC$ an isogeny between two Jacobians, where C is generic in Z. Then C and C' are isomorphic and χ is the multiplication by an integer. The statement is also true for $k = 1$ and $g \geq 5$. This extends a result by Bardelli and Pirola for the case $Z = \mathcal{M}_g$ and $g \geq 4$.

There are two natural approaches to this problem. The first one, which will be the content of the first talk, uses infinitesimal variations of Hodge structures in order to translate the statement into a problem concerning the quadrics containing a canonical curve, probably of independent interest. The second one consists in to degenerate to some special (mainly singular stable) curves. This second approach, much more subtle, works for $g \geq 5$. However it needs a good control of the intersection of the closure of Z with the boundary of $\bar{\mathcal{M}}_g$, and for this we have to restrict to $k = 1$. This procedure will be explained in the second lecture.

This is a joint work of the speakers with V. Marcucci.