We will motivate and define the category of toric vector bundles. After highlighting some applications and potential applications, we will concentrate on extending various properties of line bundles on smooth toric varieties to higher-rank toric vector bundles. Klaychko's characterization of toric vector bundles intrinsically carries the definition of associated polytopes, extending the corresponding theory of line bundles. Positivity properties, as global generation, are visualised in terms of convex properties of the corresponding polytopes and provide useful criteria. I will illustrate this correspondence and show how these criteria lead to proving and disproving connections between various notions of positivity and cohomology vanishing. This is joint work with G. Smith and K. Jabbusch.