The geometrical Schottky problem is the problem of characterizing Jacobians of curves among principally polarized abelian varieties. I will present a Schottky type result based on the existence of points in special position with respect to twice the polarization. I will also show how this result reflects on the geometry of the punctual Hilbert scheme using non-reduced points. This is joint work with Martin G. Gulbrandsen.