A perfect obstruction theory for coherent systems

ABSTRACT. According to the definition introduced by Newstead and Le Potier, a coherent system on a curve is an algebraic vector bundle together with a linear subspace of prescribed dimension of its space of sections. Therefore, coherent systems are a natural generalization of the classical notion of linear series g^r_d's. It is well known that the variety which parametrizes g^r_d's on a fixed genus g curve has expected dimension given by the Brill-Noether number $\rho := g - (r+1)(g-d+r)$. An analogous result holds for moduli spaces of stable coherent systems: if α is a real number (which is needed in order to define the notion stability for coherent systems), the moduli space of α-stable coherent systems of rank n, degree d and dimension k has expected dimention given by $\beta := n^2(g-1) + 1 - k(k-d+m(g-1))$. Then, what is expected is the existence of a virtual fundamental class on these moduli spaces which justifies their expected dimensions.

In this talk I will show the construction of a perfect obstruction theory – which naturally provides a virtual fundamental class – for a generalization of this moduli problem. Indeed, I will consider couples of algebraic vector bundles on a curve, with a fixed morphism between the two bundles, and I will study the moduli stack/space attached to these objects. The existence of a perfect obstruction theory for coherent systems will follow from this construction.