Deformations of nodal surfaces

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This talk is based on the work done in my thesis. I shall introduce two related open problems in Hodge theory, the Torelli and the Schottsky problems, and examine them in the context of nodal surfaces.

The Torelli problem asks if varieties can be distinguished by their Hodge structures. I proved that this is true locally on families of nodal surfaces in $\mathbb{P}^3$, or more precisely, that non-trivial infinitesimal deformations of such nodal surfaces induce non-trivial variations of Hodge structures.

The Schottsky problem tackles the question of which Hodge structures can occur geometrically. It remains wide open, even in relatively simple cases. While K3 surfaces yield simple weight 2 Hodge structures of type $(1, r, 1)$ for all $r < 21$, there are few examples of varieties containing a simple sub-Hodge structure of type $(1, r, 1)$ with $r > 20$. In the thesis, we looked at examples of families of nodal surfaces with double covers branched on the nodes. We noted that the double covers of even 40-nodal surfaces have a sub-Hodge structure of type $(1, 26, 1)$. While we initially hoped to obtain some interesting new Hodge structures through this family, we showed instead that all such Hodge structures decompose and contain a simple part of K3 type. This was joint work with my thesis advisor, Bert van Geemen.