We will start with Serre's old duality theorem. Grothendieck's duality theorem is a generalization, a relative version. Grothendieck duality
is a statement about finite-type morphisms $f:X\yo Y$ of noetherian schemes, and Serre duality is the special case where f is smooth and proper and Y is the one-point space.

Serre duality is about the cohomology of certain vector bundles on X. To even state Grothendiecks theorem one needs to introduce a slight technical homological improvement, remembering epsilon more about the cochain complexes that the cohomology came from - we will quickly recall the formalism of "derived categories".

The theorems are old, but the traditional approach to the proofs was complicated. The new progress we will describe allows one to prove everything much more simply and transparently.