Over the complex numbers period maps provide a measure of the non-isotriviality of families of varieties in terms of variations of Hodge structures. For example we get in this way a realization of the universal cover of the moduli of principally polarized abelian varieties of dimension g in terms of the Grassmanian of totally isotropic flags in a 2g-dimensional space.

In positive and mixed characteristics the work of the Fields medallist Peter Scholze provides new tools for the moduli of abelian varieties that resemble very closely the complex analytic approach. As an outcome one gets new exotic models of the moduli space of abelian varieties that deserve further study.

In this talk I will first explain the work of Scholze in this direction. I will then provide a rough comparison of his exotic models with the more familiar ones constructed as moduli spaces. Finally I will relate Scholze's period map to the more classical one of Ekedahl and Oort (and Moonen, Wedhorn, Pink, Ziegler, Chao Zhang etc.).