Let C be the moduli space of smooth cubic hypersurfaces of dimension 4. In this talk we will introduce the universal cubic above C, and we will study the birational geometry of its restriction to certain special loci contained in C. In particular, using recent results of Farkas-Verra and more classical ones of Mukai and Kollar, we will prove that the universal cubic above the Hassett divisors C_8, C_12, C_14, C_20, C_26 and C_42 is unirational. The result strongly depends on the existence of K3 surfaces associated to these cubics, and on the birational geometry of the moduli spaces of these surfaces. In general, we will also observe that, for d >> 0, there are infinitely many values of d for which the cubic above C_d cannot be unirational.