

Twisted cubics (and their flat degenerations) on a cubic fourfold form a variety which admits a contraction to a hyperkähler variety of dimension 8, the so-called LLSvS variety. It is deformation equivalent to the Hilbert scheme of 4 points on a K3 surface. We determine for which cubic fourfolds the LLSvS variety is actually birational to the punctual Hilbert scheme of some K3 surface. This is joint work with Nicolas Addington.