

A basic question in the geometry of Riemann surfaces is to decide when a given divisor of degree 0 is the divisor of a rational function (is principal). In the 19th century Abel and Jacobi gave a beautiful solution: one writes the divisor as the boundary of a 1-cycle, and the divisor is principal if and only if every holomorphic differential integrates to zero against this cycle. This initiated the study of jacobians, abelian varieties, etc.

From a modern perspective it is natural to allow the curve and divisor to vary in a family, perhaps allowing the curve to degenerate to a singular (stable) curve so that the corresponding moduli space is compact. The double ramification cycle can be seen as describing the locus where the divisor becomes principal. We will explain how to define the double ramification cycle, and various ways to 'compute' it.