In this talk I will discuss about the unirationality of different moduli spaces of curves: the most famous M_g, the moduli space of smooth curves of genus g; M_g,n, the moduli space of n-pointed smooth curves; and the Hurwitz space H_{g,d}, parametrizing d-sheeted simply branched covers of the projective line by smooth curves of genus g. After introducing the main known results and open questions, I will report on some recent advances, based on works in collaboration with F.-O. Schreyer and H. Keneshlou.