Motivated by a conjecture of Kapranov-Ginzburg-Vasserot, we will explain how to reinterpret the classical Poisson structure on the affine Grassmannian using tools from derived symplectic (and Poisson) geometry. Our approach is based on the study of symplectic structures on derived stacks whose cotangent complex is not dualisable, and therefore uses in a crucial way the formalism of Tate modules. The talk is based on joint work with Mauro Porta and Pavel Safronov.