The Non Abelian Hodge Theory (NAHT) of Simpson, Corlette, et al. yields canonical diffeomorphisms between the moduli spaces of Higgs bundles, flat connections, and representations of the fundamental group of a curve. In positive characteristic, there is a ``twisted" version of NAHT, but it is not clear how to extract geometric information from it.

I will report on joint work with graduating student Siqing Zhang, where we prove a cohomological version of NAHT, i.e., we exhibit a canonical isomorphism between the etale cohomology rings of the moduli of Higgs bundles and of flat connections.

I will explain how this works via vanishing cycle theory.

Time permitting, I will also discuss some perhaps unexpected corollaries relating cohomology rings of different moduli spaces, in equal and in mixed characteristic.