The past decade has witnessed a great advancement on the Tate conjecture for varieties with Hodge number $h^{2, 0} = 1$. Charles, Madapusi-Pera and Maulik completely settled the conjecture for K3 surfaces over finite fields, and Moonen proved the Mumford-Tate (and hence also Tate) conjecture for more or less arbitrary $h^{2, 0} = 1$ varieties in characteristic 0.
In this talk, I will explain that the Tate conjecture is true for mod $p$ reductions of complex projective $h^{2, 0} = 1$ varieties when p is big enough, under a mild assumption on moduli. By refining this general result, we prove that in characteristic p at least 5 the BSD conjecture holds for a height 1 elliptic curve E over a function field of genus 1, as long as E is subject to the generic condition that all singular fibers in its minimal compacification are irreducible. We also prove the Tate conjecture over finite fields for a class of surfaces of general type and a class of Fano varieties. The overall philosphy is that the connection between the Tate conjecture over finite fields and the Lefschetz (1, 1)-theorem over the complex numbers is very robust for $h^{2, 0} = 1$ varieties, and works well beyond the hyperkahler world.
This is based on joint work with Paul Hamacher and Ziquan Yang.