A couple of years back, Ambro, Cascini, Shokurov and Spicer observed that the moduli divisor of a "nice" fibration is linearly equivalent to a log canonical divisor of the foliation induced by it. Inspired by this observation, they provided the first proof of the positivity of the moduli divisor of an Ic trivial fibration using minimal model program which does not rely on Hodge theory.

This talk covers joint work with Omprokash Das. We develop the minimal model program for generalized foliated pairs (in the setting of algebraically integrable foliations) and use it to prove a special case of the semiampleness conjecture of the moduli divisor. In particular, we prove a basepoint free theorem for foliated dlt pairs, thus solving a conjecture of Cascini and Spicer.