The conjecture of Singer on the cohomology of aspherical manifolds plays an important role in modern Topology and Geometric Group theory. In this talk, I will give an overview of this problem in the context of Algebraic Geometry. The case of irregular complex surfaces will provide many concrete examples and interesting connections with classical algebraic geometry.