Triangulated categories, and in particular derived categories, are now a classical tool in homological algebra, with many relevant applications to algebraic geometry - typically, with derived categories of sheaves on a given scheme. It is well-known that, from a theoretical point of view, triangulated categories are far from being well-behaved: there is no sensible way to define a "triangulated category of triangulated functors between triangulated categories" or a tensor product. Problems arise essentially from the failure of functoriality of mapping cones. The solution to this issue is to consider enhancements of triangulated categories: namely, viewing them as shadows of more complicated structures - for instance, differential graded (dg) categories or stable \infty-categories. If we have such enhancements, another natural question we might ask is whether triangulated functors between triangulated categories can also be upgraded to "higher" functors between such enhancements. In the geometric framework, this problem is essentially the same as the problem of existence and uniqueness of Fourier-Mukai kernels of triangulated functors between derived categories of sheaves on schemes.

In this talk, I will show a new uniqueness result for lifts of triangulated functors coming from algebraic geometry: the derived pushforward and the derived pullback (of a flat morphism between suitable schemes). The strategy is completely algebraic-categorical and involves reconstructing such lifts uniquely from their restrictions to the subcategories of injective objects.