

One of the most celebrated instances of Koszul duality concerns the correspondence between the derived category of the algebra of cochains over a (sufficiently nice) topological space, and the derived category of the algebra of chains over its loop space. This can be interpreted as a vast generalization of the well-known Koszul duality between the symmetric algebra over a vector space and the exterior algebra over its dual. In this talk, I shall describe how the highly sophisticated formalism of homotopy theory and of enriched

∞ -categories allows us to categorify this Koszul duality principle.

First, I shall revisit the theory of local systems of categories over topological spaces, which naturally comes into picture in the framework of cobordism theory and of the geometric Langlands program. Then, I shall formulate a higher \mathbb{E}_n -Koszul duality principle between cochains over a space and chains over its loop space, which generalizes the classical one. Finally, I shall describe a wide class of topological spaces for which such \mathbb{E}_n -Koszul duality holds. This talk is based on joint works with J. Holstein and M. Porta, and with J. Pascaleff and N. Sibilla.