The multiplier ideal sheaves associated to a complex manifold X and a Q-effective divisor D is a family of ideal sheaves indexed by rational numbers. They are important tools in algebraic geometry and commutative algebra. In this talk, I will introduce a family of ideal sheaves associated to (X,D), called higher multiplier ideals, indexed by a rational number and an integer. When this integer index is zero, it recovers the usual multiplier ideal sheaves. They are closely related to, but different from, Hodge ideals introduced by Mustata and Popa. We study the local and global properties of higher multiplier ideals systematically using the notion of twisted Hodge modules, which builds on the language of twisted D-modules by Beilinson-Bernstein and the theory of complex Hodge modules by Sabbah-Schnell. If time permits, I will discuss new applications to conjectures of Casalaina-Martin, Gruschevsky and Debarre on singularities of theta divisors on principally polarized abelian varieties and the geometric Riemann-Schottky problem. This is based on the joint work with Christian Schnell.