Due to Torelli theorems, moduli spaces of K3 surfaces are orthogonal Shimura varieties. In the 60's-80's, compactifications of such varieties were constructed by Baily-Borel, Ash-Mumford-Rapaport-Tai, and Looijenga. But are any of these "semitoroidal" compactifications distinguished, in the sense that they parameterize some stable K3 or Enriques surfaces?

Work on the Minimal Model Program from the 80's-00's by Kollar-Shepherd-Barron-Alexeev proved that an ample divisor on a Calabi-Yau variety defines a notion of stability, leading to compact moduli spaces. I will overview joint work with Alexeev, relating the Hodge-theoretic and MMP approaches to compactification.

The first lecture will overview moduli spaces of smooth polarized K3 surfaces, with examples. We will discuss the period map, Torelli theorem, and the Baily-Borel and (semi)toroidal compactifications of Type IV arithmetic quotients.

The second lecture will introduce nice models of degenerations, such as Kulikov and stable models, the notion of a "recognizable divisor", and KSBA compactifications. We will outline the proof that for a recognizable divisor, one gets a semitoroidal compactification of the period domain.

The third lecture will focus on examples, such as degree 2 and elliptic K3 surfaces. We will construct the fan in these cases and prove, via integral-affine structures on the two-sphere, that it gives the KSBA compactification.

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