

It is well known that the category of representations of a group scheme over a field is an abelian tensor category. Moreover, if the field is algebraically closed, then the group scheme is completely determined by its category of representations. But which abelian tensor categories are equivalent to the representation category of an affine group scheme? If the field has characteristic zero, Deligne determined an internal characterisation of such categories: this is classical Tannaka duality.

Similarly, varieties, schemes, algebraic groups and various generalizations thereof are often studied via an associated symmetric monoidal Grothendieck category (the category of quasi-coherent sheaves, or the category of representations). Lurie has shown that this association gives an embedding of algebro-geometric objects into the 2-category of symmetric monoidal Grothendieck categories. Like before, it is natural to ask whether we can characterize the image of this embedding. Which symmetric monoidal Grothendieck categories are equivalent to the category of quasi-coherent sheaves on a stack?

In this talk, I will explore answers to these questions, both in positive characteristic and characteristic zero. If time permits, I will discuss implications for tensor-triangulated categories. Part of this is joint work with Kevin Coulembier.