Stability of foliations induced by Lie group actions

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Let X be a smooth projective variety over the complex numbers and $S(d)$ the scheme parametrizing d-dimensional Lie subalgebras of $H^0(X, \mathcal{T}X)$. In this talk we will explore the geometry of the moduli space Inv of involutive distributions on X around the points $\mathcal{F} \in \text{Inv}$ which are induced by Lie group actions. For every $\mathfrak{g} \in S(d)$ one can consider the corresponding element $\mathcal{F}(\mathfrak{g}) \in \text{Inv}$, whose generic leaf coincides with an orbit of the action of $\exp(\mathfrak{g})$ on X. We show that under mild hypotheses, after taking a stratification $\coprod_i S(d)_i \to S(d)$ this assignment yields an isomorphism $\coprod_i S(d)_i \to \text{Inv}$ locally around $\mathfrak{g}$ and $\mathcal{F}(\mathfrak{g})$. This gives a common explanation for many results appearing independently in the literature. We also construct new stable families of foliations which are induced by Lie group actions.

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