We study the problem of classifying compactified Jacobians of nodal curves that can arise as limits of Jacobians of smooth curves. The answer is given in terms of a new class of compactified Jacobians, that we call compactified Jacobians of vine type, and that is strictly larger than the class of classical compactified Jacobians, as constructed by Oda-Seshadri, Simpson, Caporaso and Esteves. We give several characterizations of compactified Jacobians of vine type. Furthermore, in the fine case, we show that most of the known properties of fine classical compactified Jacobians extend to fine compactified Jacobians of vine type: the relation to the Neron models of Jacobians, the autoduality property, the Fourier-Mukai equivalences, the perverse filtration of their cohomology, the relation to Mumford models of Jacobians. This is partly based on a joint work in progress with M. Fava and N. Pagani.