The derived category of a Fano variety sometimes contains an interesting admissible subcategory, known as the Kuznetsov component. Moduli spaces of Bridgeland semistable objects on the Kuznetsov component provide many examples to study, often generalizing constructions from classical algebraic geometry. In this talk, I will review several cases of Fano threefolds, and explain how moduli spaces on their Kuznetsov components behave similarly to moduli of sheaves on curves. Based on joint work with Chunyi Li, Yinbang Lin and Laura Pertusi.