

A fundamental tool to study algebraic varieties is given by morphisms to projective space. A line bundle is called ample if, up to a multiple, it provides an embedding into projective space. Similarly, a line bundle is called semiample if some positive tensor multiple provides a morphism to projective space. Unlike the case of ample line bundles, there are no general numerical criteria to determine whether a line bundle is semiample. In particular, proving the semiampleness of line bundles can be a challenging task.

In this talk, we will report on recent developments regarding the semiampleness of line bundles coming from Hodge theory. As consequence, we will obtain functorial compactifications of general period images, generalizing classic work of Baily--Borel and thus confirming a conjecture of Griffiths. This talk is based on joint work with Bakker, Mauri, and Tsimerman.