Markman and O'Grady uncovered a deep relation between abelian fourfolds of Weil type with discriminant 1 and hyper-Kähler varieties of generalized Kummer type, at the level of Hodge theory and period domains. This was used by Markman to prove the Hodge conjecture for these fourfolds. He then found a different proof of this result, which in fact works for Weil fourfolds with arbitrary discriminant and implies the Hodge conjecture for all abelian varieties of dimension 4.
In my talk I will explain how Weil fourfolds with discriminant 1 are also closely related to certain hyper-Kähler varieties of OG6-type in a direct and geometric way. As a consequence, we obtain yet another proof of the Hodge conjecture for Weil fourfolds with discriminant 1, as well as for many families of hyper-Kähler varieties of OG6-type which form loci of codimension 1 in their moduli spaces. The results that I will discuss are joint work with Lie Fu.