

In a 1977 paper Andre Weil introduced a 2-dimensional subspace of exceptional Hodge classes in the middle cohomology of abelian varieties admitting a suitable complex multiplication by an imaginary quadratic number field. These Hodge classes are known as Weil classes and these abelian varieties are said to be of Weil type. The construction was later generalized to complex multiplication by CM-fields. Weil himself was skeptical about the Hodge conjecture and thought that the Weil classes may provide counter examples, as did other skeptics more recently. Moonen and Zarhin seem to have held a more optimistic attitude and, following a suggestion of van Geemen, reduced the proof of the Hodge conjecture for abelian varieties of dimension at most 5 to the proof of the algebraicity of the Weil classes on abelian fourfolds. In these talks we will explain the general strategy for proving the algebraicity of the Weil classes, for all CM-fields and all dimensions, based on developments that were not available to Weil in 1977. These include 1) the Semi-regularity theorem of Buchweitz-Flenner, 2) the fact that spin groups arise as groups of auto-equivalences of the derived category of abelian varieties (Mukai, Polishchuk, and Orlov), and 3) Chevalley's theory of pure spinors. We will then specialize to the case of abelian varieties of dimension at most 6 and explain the proof of the algebraicity of the Weil classes using the above strategy.