

This talk is motivated by the question of which cubic equations can be “solved,” a classical problem that goes back at least to the work of Euler, Clebsch, Severi, and others. I will explain how this perspective naturally leads to the notion of retract rational varieties. The main result is that very general cubic threefolds are not retract rational and hence not stably rational, providing the first examples of smooth cubic hypersurfaces of dimension at least two with this property. By a theorem of Voisin, this non–retract-rationality follows from disproving the integral Hodge conjecture for the associated intermediate Jacobian, a principally polarized abelian fivefold. Consequently, our result also yields the first examples of abelian varieties that fail the integral Hodge conjecture. I will explain the statements and, time permitting, discuss some aspects of the proof, which relies on ideas from the combinatorial theory of regular matroids. Joint work with Philip Engel and Olivier de Gaay Fortman.