

# Reading seminar: Irreducible symplectic varieties and compactifications

The aim of this seminar is to give an introduction to the theory of irreducible symplectic varieties, which are a generalization of irreducible symplectic manifolds (or hyperkähler manifolds) in the singular setting. More precisely, we have the following goals:

1. Study the global moduli theory and Torelli theorem in the setting of primitive symplectic varieties proved by Bakker and Lehn in [1];
2. Investigate some known strategies to construct examples [3, 2].

**Time slot:** Each talk will be of 1 hour either on Monday or Tuesday at 12:30 in Aula 10, unless differently specified.

**19/11 - Talk 1: Introduction (Alice).** Following the survey [4], explain the different definitions in the singular case with some examples and motivations.

**25/11 - Talk 2: Preliminaries to global Torelli (Laura).** Introduce some preliminaries on primitive symplectic varieties as in [1, Sections 3, 2].

**02/12 - Talk 3: Primitive symplectic varieties and deformation theory (Laura).** Introduce the statement of [1, Theorem 1.1] and the projectivity criterion [1, Theorem 1.2]. Show that locally trivial deformations are unobstructed ([1, Theorem 4.7]).

**10/12 - Talk 4: Local Torelli Theorem (Ziqi).** Define the BBF form for psv, prove [1, Proposition 5.5] and discuss other properties like Fujiki relations, Riemann-Roch type formula and non-existence of subvarieties of odd dimension on a general deformation.

**17/12 - Talk 5: Proof of projectivity criterion (Roberto).** Explain the proof of [1, Theorem 1.2] in Section 6 and discuss the non-separatedness of the moduli space.

**13/01 - Talk 6: Proof of global Torelli (Benedetta).** Explain the proof of [1, Theorem 1.1] in Section 8.

**27/01 - Talk 7: Examples via compactification methods II (Luca).** State the criterion [3, Theorem 3.6] and explain how it follows from [3, Proposition 3.5] and [3, Theorem 3.1]. Proof [3, Theorem 3.1] about the existence of symplectic compactifications with  $\mathbb{Q}$ -factorial terminal singularities. **(13:30-14:30 Aula 5).**

**27/01 - Talk 8: Examples via compactification methods I (Luigi).** Explain [3, Proposition 3.5], showing that if a projective  $\mathbb{Q}$ -factorial terminal symplectic variety  $X$  admits a Lagrangian fibration whose general fiber is a simple abelian variety, then  $X$  is irreducible. **(14:30-15:30 Aula 5).**

**28/01 - Talk 9: Examples via compactification methods III (Chiara).** Discuss the application to the fibration in Jacobians of the family of smooth cubic fivefolds containing a fixed cubic fourfold in [3, Section 4].

**10/02 - Talk 10: Examples via singular symplectic surfaces (Alice).** Study the results of [2] about classification of singular symplectic surfaces and the results on associated Hilbert schemes of two points.

## References

- [1] B. T. Bakker and C. Lehn, The global moduli theory of symplectic varieties, *J. Reine Angew. Math.* **790** (2022), 223–265.
- [2] A. Garbagnati, M. Penegini, A. Perego, Singular symplectic surfaces, arXiv:2407.21173.
- [3] Y. Liu, Z. Liu, C. Xu, Irreducible symplectic varieties with a large second Betti number, arXiv:2410.01566.
- [4] A. Perego, Examples of irreducible symplectic varieties, in *Birational geometry and moduli spaces*, 151–172, Springer INdAM Ser., 39, Springer, Cham.