

# You better play 7: mutual versus common knowledge of advice in a weak-link experiment

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**Abstract** This paper presents the results of an experiment on mutual versus common knowledge of advice in a two-player weak-link game with random matching. Our experimental subjects play in pairs for thirteen rounds. After a brief learning phase common to all treatments, we vary the knowledge levels associated with external advice given in the form of a suggestion to pick the strategy supporting the payoff-dominant equilibrium. Our results are somewhat surprising and can be summarized as follows: in all our treatments both the choice of the efficiency-inducing action and the percentage of efficient equilibrium play are higher with respect to the control treatment, revealing that even a condition as weak as mutual knowledge of level 1 is sufficient to significantly increase the salience of the efficient equilibrium with respect to the absence of advice. Furthermore, and contrary to our hypothesis, mutual knowledge of level 2 induces, under suitable conditions, successful coordination more frequently than common knowledge.

**Keywords** Coordination games · Experimental philosophy · Epistemic attitudes · Weak-link game · Conventions

## 1 Epistemic attitudes: the formal and the experimental

The purpose of this paper is to discuss—from the point of view of formal epistemology—some experimental results on the role of common knowledge as a coordination device in a weak-link coordination game.

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Common knowledge has long been regarded as a fundamental component in the modeling of rational interaction. As such it features prominently, though informally, in Nash's characterization of equilibrium. As game theory evolved towards capturing increasingly more refined solution concepts, common knowledge was there to stay as a fundamental modeling assumption. Harsanyi and Selten, to mention a particularly relevant example from the point of view of the present paper, put forward a view according to which the assumption of common knowledge of the players' rationality is sufficient to ensure the joint selection of efficient equilibria in (non-pure) coordination games.

In a line of research which for a long time has been developed almost independently of the game-theoretic one, epistemic logicians have advanced, over the past two decades, rigorous characterizations of the epistemic attitudes of both individual and group knowledge and belief. The emergence of multi-agent epistemic logic thus enabled, around mid 1990s, the development of rigorous analyses of the epistemic conditions underlying game theoretic solution concepts.

The ensuing investigations on the connection between rational agents' epistemic attitudes and their strategic behavior tended to fall into two seldom communicating categories, the formal and the experimental. This paper constitutes an attempt at fostering the cross-fertilization between those lines of research. It aims at so doing by taking full advantage of the mutual feedback that experimental economics and formal epistemology can give one another. Formal epistemology lends methodological clarity to experimental analysis, especially concerning modeling assumptions. In turn, experimental analysis gives empirical substance to the formal analysis of epistemic attitudes of group knowledge. This results in a virtuous circle, which we expect to be a key feature of the emerging field of experimental formal epistemology. From this vantage point, our contribution speaks to the community of experimental economists, to game theorists interested in the refinement of epistemic conditions for solution concepts, and to the relatively younger community of logicians and philosophers of logic interested in empirical evidence on modes of reasoning.<sup>1</sup>

The paper is organized as follows. Section 2 describes the weak-link coordination game used in our experiment and Sect. 2.1 discusses the relevant related literature. The experimental design is then illustrated in Sect. 3. Section 4 is devoted to the analysis of the experimental results regarding choice behavior (Sect. 4.2) and the subjects' beliefs (Sect. 4.1). Section 6 draws some general conclusions about our rather surprising findings and delineates what we envisage to be promising future lines of development of the research reported in this paper. The remainder of this section aims at illustrating the motivation of this work by presenting an outline of how the recent research on epistemic attitudes has tended to fall under the "formal" and the "experimental" headings, with very little overlap.

### 1.1 The formal

Robert Aumann opens his classic paper on correlated equilibrium by reviewing the concept of Nash Equilibrium and asks

<sup>1</sup> See van Benthem (2008) and Verbrugge (2009) for two influential position papers, and D'Agostino (2010) for an example of a logic for realistic agents.

why should any player assume that the other players will play their components [of a Nash Equilibrium] and indeed why should they? [...] In a two-person game, for example, Player 1 would play his component only if he believes that Player 2 will play his; this in turn would be justified only by 2's belief that 1 will play his component; and so on. (Aumann 1987)

The von Neumann–Nash analysis of game-theoretic equilibrium relies essentially on the concept of best-response which is normatively prescribed to a rational player only under the assumption of the players' *common knowledge* of each others' rationality (and, clearly, of the game). It is certainly beyond the scope of the present paper to discuss which solution concept is appropriate for a justified formal characterisation of rational interaction. Yet what Aumann's remark clearly illustrates is the centrality of *epistemic attitudes* in the game-theoretic analysis of rational interaction.

In hindsight two things appear to be rather surprising. The first is that game theorists, and economists in general, have been rather happy with an intuitive (as opposed to mathematically rigorous) notion of "knowledge" for their modeling until Aumann (1976) put forward what have become known as *Aumann structures*. In an nutshell, the construction of the Aumann structure begins, in continuity with the (single-agent) decision theoretic tradition, by postulating the existence of a *state space*, which is assumed to provide a complete description of the "world". To each individual agent (in a finite set, interpreted as "player") is associated a specific partition of the state space. As in Savage's model, events—the object of the agents' knowledge—are subsets of the set of states. The operations of individual, mutual and common knowledge are defined by suitably constrained operators on the algebra generated by the partitions of the state space. Aumann's formalization has effectively given rise the actively researched area of epistemic game theory.<sup>2</sup>

This leads us to the second surprising fact (in hindsight), namely that it took a relatively long time for game theorists and formal epistemologists to realize that they were working on essentially similar problems. Indeed, philosophers have long been interested in epistemic attitudes as specific instances of the wider class of *intentional* attitudes. Put very crudely, epistemic attitudes capture the relation between an agent's reasoning (and disposition to act) and the information they possess. The first systematic attempt at giving this relation a rigorous logical formalization appeared in Hintikka (1962). This work paved the way for the syntactic development of epistemic logic which was soon to be complemented by the introduction of relational semantics (or Kripke frames).<sup>3</sup> By the mid-1990s epistemic logic was a standard analytical tool in artificial intelligence (Meyer and van der Hoek 1995).

Since Hintikka's pioneering work, epistemic logicians have tended to focus on normative models of individual *knowing* and *believing*.<sup>4</sup> These notions were extended to

<sup>2</sup> See Aumann and Brandenburger (2008) for a terse overview of the field.

<sup>3</sup> See e.g. Blackburn et al. (2001) for a state-of-the-art introduction to Modal logic and Blackburn et al. (2007) for a comprehensive account of its development.

<sup>4</sup> Recent attention has been devoted to the logic of "being informed" and that of "being aware". See Allo (2011) and Halpern and Rêgo (2008) for recent overviews. On awareness, the interested reader can consult Burkhard Schipper's bibliography (Schipper 2012).

their multi-agent counterpart in the context of artificial intelligence.<sup>5</sup> In an attempt to make this paper as self-contained as possible, we now briefly recall the central epistemic attitudes captured in multi-agent epistemic logic, namely individual, mutual and common knowledge.<sup>6</sup>

Let  $N$  be a set of agents,  $S$  be a set of states (or “possible worlds”) and  $\theta, \phi$  etc. be sentences of some propositional (modal) logic. The key intuition captured by epistemic logics is that when being in epistemic state, say,  $s$ , agent  $i$  may *access* other epistemic states, say  $t$ . This motivates the introduction of an *accessibility* relation  $R_i \subseteq S^2$  (one for each agent  $i \in N$ ). We say that an agent  $i \in N$  in epistemic state  $s \in S$  *knows*  $\theta$  just if  $\theta$  is true (in the classical logic sense) *in all epistemic states* which are accessible to  $i$  from  $s$ . Distinct formalizations of “knowledge” (and “belief”) arise from distinct constraints that we impose on the relation  $R$ . In the most widely studied (multi-agent) epistemic logic, known as  $S5$ ,  $R$  is an equivalence relation, thus making the resulting logical characterization of knowledge effectively equivalent to that provided by Aumann’s structures.

This individual knowledge operator naturally extends to *groups* of agents as follows.<sup>7</sup> Suppose, for definitiveness, that  $i$  and  $j$  are the only members of group  $G \subseteq N$ . If both  $i$  and  $j$  know  $\theta$ , we say that “ $\theta$  is *mutual knowledge* among group  $G$ ”. Since this latter can be expressed as a well-formed formula of epistemic logic, the construction can be iterated, so we can have that “ $i$  and  $j$  know that  $\theta$  is *mutual knowledge* among group  $G$ ”. This sentence thus expresses *second-level mutual knowledge*, because  $i$  knows that  $j$  knows that they both know  $\theta$ , and so on. When the mutual knowledge operator is iterated infinitely many times, we say that  $\theta$  is *common knowledge* between  $i$  and  $j$ , a concept introduced<sup>8</sup> by Lewis (1969).

Over the past two decades or so, the formal analysis of (group) epistemic attitudes has led the momentous development of both formal epistemology (see, e.g. Hendricks 2005) and epistemic game theory.<sup>9</sup> As to this latter, it is well worth recalling how the rigorous characterization of epistemic attitudes has resulted in a much sharper analysis of the “epistemic conditions” yielding various kinds of solution concepts. A particularly interesting case in point, from the point of view of this paper, is the demotion of the common knowledge condition for Nash equilibrium put forward by Aumann and Brandenburger (1995), where it is shown that decidedly weaker conditions (some involving mutual knowledge) suffice for the justification of the classical solution concept. The main result reported in this paper, namely that in the weak-link game of our experimental setting *mutual knowledge* leads to the selection of efficient equilibria more frequently than common knowledge, certainly adds experimental support to such a demotion of common knowledge in rational interaction.

<sup>5</sup> Fagin et al. (1996) collects various works providing the first systematic formalization of the epistemic interaction of logical agents.

<sup>6</sup> Self-contained formal presentations can be found in Fagin et al. (1996) and van Ditmarsch et al. (2007).

<sup>7</sup> Cf. what Aumann dubs “syntactical structures” in Aumann (1999).

<sup>8</sup> For an overview on the notion of common knowledge, see Vanderschraaf and Sillari (2007).

<sup>9</sup> In turn, both areas contributed essentially to the development of the *social software* research programme (see, e.g. Parikh 2002).

In addition to making rigorous the otherwise rather subtle distinction between mutual and common knowledge, the general framework of formal epistemology caters for another conceptual distinction playing a fundamental role in the present investigation, namely the distinction between normative and descriptive accounts of epistemic attitudes. Whilst the former prescribe how idealized agents *should* behave in order to qualify as rational, the latter aims at describing the behavior of actual agents (e.g. our experimental subjects) facing choice problems which involve reasoning about other agents' epistemic attitudes.

It is commonly accepted that normative models of rationality play a crucial role in helping real decision makers correcting their mistakes. This role of normative models is grounded on the assumption that whilst their cognitive limitations may never allow them to fully attain the recommendations of the model, real decision makers should aim at the standard of rationality captured by the norm. Now, the formal work on multiagent epistemic logics which we have briefly surveyed in this section undoubtedly suggests that common-knowledge provides the most favourable condition for coordination in strategic interaction. The experimental results discussed in this paper, however, tell another story.

## 1.2 The experimental

With the present contribution we relate to two connected strands of experimental and behavioral literature,<sup>10</sup> namely the extensive experimental literature on coordination games and the philosophical and experimental literature on social conventions and social norms.

Among the early advocates of the empirical investigation of coordination games is Thomas Schelling, who, at the beginning of the 1960s pointed out how

One cannot, without empirical evidence, deduce what understandings can be perceived in a non-zero sum game of maneuver any more than one can prove, by purely formal deduction, that a particular joke is bound to be funny (Schelling 1980, p. 164)

The power of “focal points” to facilitate coordination, which Schelling himself tested in a series of informal experiments with his students, was first identified empirically by Metha et al. (1994), who introduced the notion of “Schelling salience” to refer to features—such as action labels or others (see, e.g. Bacharach and Bernasconi 1997; Cubitt and Sugden 2003)—able to confer unambiguous distinctiveness to one specific equilibrium. The power of “Schelling salience” to focus players' expectations on some unique outcome and hence to solve coordination problems is most evident in so-called games of pure coordination, i.e., games in which all equilibria are payoff-equivalent. As a result, the essence of this kind of coordination problems consists only in avoiding a disequilibrium outcome. A different, but closely related class of coordination games contains games with multiple equilibria, some of which are more rewarding than others

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<sup>10</sup> For a primer on the use of the experimental method in economics, see Friedman et al. (1994) for a critical survey of experiments in game theory see Camerer (2003).

for all players. Typically, these games embody a tension between efficiency and security (or risk-dominance), as players choosing strategies that support payoff-superior equilibria incur in greater losses if their choice is not matched by all other players. As a consequence, two types of coordination failure may occur: a disequilibrium outcome and coordination on a suboptimal equilibrium.

Harsanyi and Selten (1988) formalized the tradeoff implicit in these games and stated that rational players, in the presence of common knowledge of rationality, should select the efficient (i.e., payoff-dominant) equilibrium. However, a plethora of experimental studies on the stag hunt and weak-link games (see, e.g. Cooper et al. 1990, 1992; Van Huyck et al. 1990) has convincingly demonstrated that Harsanyi and Selten's prediction holds true only under very special conditions, while in all remaining cases coordination failure occurs almost invariably. Starting from these results, most of the subsequent experimental studies on coordination have been aimed at identifying the efficiency-enhancing properties of several features of the strategic interaction at hand. Our study contributes to this strand of literature by identifying the *epistemic conditions* that, in a specific experimental setup highly conducive to coordination failure, render external interventions in the form of third party announcements effective in directing the players' choices towards superior and more risky equilibria.

The second strand of research to which our investigation is related includes the philosophical and experimental literature on social conventions and social norms pioneered by the game theoretically informed work of Lewis (1969) on conventions. David Lewis's account of social convention is based on coordination games. When players' interests are aligned (as in pure coordination games) and interactions are recurrent, players may succeed in coordinating their behavior by selecting a salient equilibrium. If they recurrently succeed in coordinating, the regular solution becomes salient by virtue of precedent, and, according to Lewis's definition, is a *social convention*. When there is a trade-off between security and efficiency (as in the kind of coordination game our experimental work focuses on), payoff considerations make the choice of salient equilibrium particularly relevant. Lewis's account requires that conventions be common knowledge and the importance of such an epistemic requirement, especially with reference to salience, has been a debated issue in the literature (see Sect. 2.1 below). The present contribution investigates the effectiveness of various levels of epistemic access to the salience-inducing coordination device (i.e. a suggestion to play the strategy supporting the efficient equilibrium), thus contributing to the discussion on the role of common knowledge in coordination and convention.

Lewis's work on social conventions has spun a vast literature in philosophy of social sciences. Particularly relevant are accounts of social norms based, to varying degrees, on Lewis's idea of convention as coordination (cf. for instance Ullman-Margalit 1977; Sugden 1986, 2004; Young 1998; Vanderschraaf 1995), culminating in Cristina Bicchieri's theory of social norms (Bicchieri 2006). In Bicchieri's account, a social norm transforms a mixed-motive game into a coordination game: when a social norm is in place and agents hold the relevant expectations about others' preferences, norm-abidance behavior is the preferred course of action, while the same course of action would have been dispreferred in the absence of a social norm. The expectations involved in Bicchieri's account of social norms are of two kinds: (i) *empirical expectations* (first-order beliefs that others will conform) and (ii) *normative expectations*

(second-order beliefs that other will expect one to conform). Extensive experimental work by Bicchieri and colleagues (cf., e.g., Bicchieri and Xiao 2009; Bicchieri and Chavez 2010) show that empirical expectations by themselves may not be sufficient to motivate compliance to social norms, and that the presence of aligned second-order, normative expectations, can trigger compliance. While the present experiment does not deal directly with social norms, it does look at how first and second-order expectations as elicited by different experimental conditions affect coordination outcomes, thus offering insights on the role and nature of coordinating expectations.

## 2 The game

Arguably, the most prominent testbeds of the risk-efficiency tradeoff in the domain of coordination games are the stag hunt and minimum effort (or weak link) games. We limit ourselves to describe the latter.<sup>11</sup> On-time aircraft departures are prototypical examples of coordination problems of the weak-link type (see, e.g., Knez and Simester 2001) since the airplane cannot take off before all operations (e.g., fueling, security checks, loading of luggage, boarding of passengers, etc.) have been completed. Other examples include relationships between different branches of a bank, the writing of a grant proposal involving several participants, an edited volume involving several authors, and many others (Camerer and Knez 1996, 1997). Common to all aforementioned examples is the fact that output is determined by the agent exerting the lowest level of effort (the “weak link”), and any effort above the minimum is wasted.

A weak-link game is defined by  $N$  players who must simultaneously choose a natural number  $x$  in  $\{1, 2, \dots, X\}$ . The payoff function for the normal form game is defined as follows:

$$\pi(x_i) = a + b * \min(x_1, x_2, \dots, x_n) - c[x_i - \min(x_1, x_2, \dots, x_n)] \quad (1)$$

where  $x_i$  is the number chosen by player  $i$ , and  $a$ ,  $b$  and  $c$  are positive parameters.  $x_i$  is intuitively interpreted as the “effort level chosen by player  $i$ ”. Provided that the payoff function is common knowledge, the game has  $X$  strict Nash equilibria, corresponding to the  $X$  action combinations in which all players select the same effort level. Furthermore, the Nash equilibria can be Pareto-ranked, with the combination  $\{x_1 = x_2 = \dots = x_n = X\}$  being the *efficient*, or payoff-dominant equilibrium. Viceversa, the  $\{x_1 = x_2 = \dots = x_n = 1\}$  corresponds to the *secure* equilibrium.<sup>12</sup>

In our experiment we chose the following parameter values:  $N = 2$ ,  $X = 7$ ,  $a = 6$ ,  $b = c = 1$ . For these parameters, the game presents seven Pareto-ranked Nash equilibria, with the combinations  $\{x_1 = x_2 = 7\}$  and  $\{x_1 = x_2 = 1\}$  representing the payoff-dominant equilibrium and the secure equilibrium, respectively. Each player

<sup>11</sup> In fact the minimum effort game is an extension of the stag hunt game to the case of  $n$  strategies with  $n > 2$ . See, e.g., Camerer (2003, Chap. 7).

<sup>12</sup> “Security” is an extension of Harsanyi and Selten’s risk-dominance principle to games with more than two strategies. A “secure” equilibrium is selected when all players choose in accordance with the maximin criterion.

**Table 1** Payoff of the weak-link game

| My choice | Minimum number chosen |   |   |    |    |    |    |
|-----------|-----------------------|---|---|----|----|----|----|
|           | 1                     | 2 | 3 | 4  | 5  | 6  | 7  |
| 1         | 7                     | – | – | –  | –  | –  | –  |
| 2         | 6                     | 8 | – | –  | –  | –  | –  |
| 3         | 5                     | 7 | 9 | –  | –  | –  | –  |
| 4         | 4                     | 6 | 8 | 10 | –  | –  | –  |
| 5         | 3                     | 5 | 7 | 9  | 11 | –  | –  |
| 6         | 2                     | 4 | 6 | 8  | 10 | 12 | –  |
| 7         | 1                     | 3 | 5 | 7  | 9  | 11 | 13 |

is penalized the further her choice is from the minimum in the group. The resulting payoff matrix<sup>13</sup> is shown in Table 1.

## 2.1 Related literature

The first comprehensive experiment on the weak link game was conducted by Van Huyck et al. (1990), varying group size, parameter values and matching protocol: unravelling to the lowest minimum over time always occurred with large groups (i.e.,  $N = 914$ ) interacting repeatedly. Pairs did slightly better but never converged when randomly rematched in each round: only when interacting repeatedly in fixed pairs were players able to coordinate on the payoff-dominant equilibrium. The results in Van Huyck et al. (1990) have been extensively replicated *ceteris paribus*. Taking these findings as point of departure, subsequent studies have investigated different ways to engineer coordination success in the lab: pre-play communication, inter-group competition, increased financial incentives, gradual group growth, and leadership are all mechanisms that have been shown to increase the likelihood of efficient coordination; others, such as increased feedback, have been tested with inconclusive results (see Devetag and Ortmann 2007) for a review of experiments on both the weak link and stag hunt coordination games). Of particular relevance for our study are those experiments that tested the effectiveness of advice, either from a third party or from players themselves. Van Huyck et al. (1992) tested the role of non-binding, publicly announced external assignments in two-person coordination games; their results indicate that assignments are not considered credible when they conflict with payoff-dominance or symmetry. It has to be pointed out, however, that in their games the two principles of payoff-dominance and risk-dominance do not conflict. Chaudhuri et al. (2009) tested intergenerational advice in the minimum effort game with large groups: they found out that the payoff-dominant equilibrium was selected only when advice not only was public and publicly shared (i.e., when all advice from previous

<sup>13</sup> While in a two-person game the natural choice would that of depicting one's own choice against the other player's choice instead of own against minimum of the two, we opted to use the latter representation in order to make our results fully comparable with the previous literature.



generation members was made accessible to all new generation members, and this fact was common knowledge), but was also common knowledge in the sense of being read aloud. The authors call the first condition “almost common knowledge” as in Rubinstein (1989), and in their experiments it leads groups to select the Pareto-worst outcome. Only the common knowledge condition leads to Pareto efficiency. Bangun et al. (2006) replicate the design by Van Huyck et al. (1992) using a stag-hunt game embedding a clear conflict between payoff-dominance and risk-dominance. They show that, unlike the Van Huyck et al. (1992) experiment, an external assignment is sufficient to produce coordination on the payoff-dominant outcome, even when the assignment is “almost common knowledge”. Finally, Chaudhuri and Paichayontvijit (2010) test the power of an external assignment (in the form of a recommendation to choose the strategy consistent with the efficient equilibrium) in the weak link game with groups of five players: their  $2 \times 2$  between-subject design tests fixed versus random matching, and “common knowledge” versus “almost common knowledge” of the assignment. A further treatment tests the power of an increased bonus for coordination. As in the Chaudhuri et al. (2009) study, “common knowledge” refers to the recommendation being read aloud, while “almost common knowledge” implies that the recommendation is handed out to each subject to be read privately, and the fact that everybody is given the same recommendation is common knowledge. Their findings show behavior that is largely consistent with behavior in Chaudhuri et al. (2009): with random re-matching of groups in every round, only the increased financial incentive (publicly announced) can induce efficient coordination; neither “common knowledge” nor “almost common knowledge” of the recommendation are sufficient; fixed groups are generally able to reach successful coordination even in absence of external interventions. This last finding is at odds with the finding of Chaudhuri et al. (2009), in which players interacting in fixed groups failed to coordinate successfully even in the presence of a credible assignment; the difference is most likely due to the different content of the advice (in the study done by Chaudhuri and colleagues, many players advised their successors to actually select the secure equilibrium), and to the larger group size of the Chaudhuri et al. (2009) design, a parameter which has been shown to be crucial in determining the probability of efficient coordination in the weak-link game.

While in Chaudhuri and Paichayontvijit (2010) a distinction is made between *public* and *common* knowledge of advice (their conditions 1 and 2, respectively), it is clear from the discussion above that the two conditions are both common knowledge conditions. The former condition (in which subjects receive a sheet of paper stating the advice *and* that every other player receives the exact same advice and message) induces first-order knowledge of the advice (because of the text on the paper), second-order knowledge that everyone has knowledge of the advice (because the fact that everyone receives the same text is publicly known), third-order knowledge that everyone knows that everyone has knowledge of the advice (because everyone knows that everyone knows everyone has received the same text), and so on. Of course, subjects could be doubtful about whether others have read the message, paid attention to all of its contents, etc., hence the notion that this is an *almost* common knowledge condition.

None of the previous studies on the efficiency-enhancing properties of external advice has tested epistemic conditions weaker than common or at least almost common knowledge. Our paper fills this gap in the literature by examining, along with

common knowledge, first- and second-order mutual knowledge of advice. First-order mutual knowledge is obtained by an experimental setup that induces knowledge of advice in each player, without offering subjects the opportunity to draw inferences beyond first-order, i.e. the experimenter does not give them enough information to infer that others have knowledge of advice. Second-order mutual knowledge is similarly obtained by a setup in which knowledge that one's opponent has knowledge of advice can be inferred, but knowing that the opponent knows that one knows that cannot.

Our experimental work is linked to a further strand of literature, namely that on game-theoretic accounts of social conventions. [Cubitt and Sugden \(2003\)](#) have advanced a formal reconstruction of Lewis's account of social convention that stresses the importance of inductive reasoning as based on common knowledge of precedent. On the other hand, evolutionary accounts of social conventions (cf. [Binmore 2008](#); [Skyrms 1996](#)) deny the relevance of common knowledge for sustaining a social convention (cf. [Sillari 2008](#) and [Rescorla 2011](#) for overviews of the debate). Our experimental study provides a contribution to this debate, as our experimental conditions allow us to observe the impact of different kinds of group knowledge on the formation of conventions. To be sure, our experiment does not deal with convention formation, as iterations of the interactive choice are numerous enough to speak of an emerging convention. However, our experiment does yield insights on a crucial mechanism of convention formation, i.e. that of salience building. In Lewis's account of convention as it is discussed in the recent debate, a specific kind of salience (precedent) is at work in perpetuating a convention. Instead, we focus on plausible mechanisms other than precedent for the emergence of salience, in particular we observe how subjects react to advice. Notice that the experimenters' advice does not add any factual knowledge to what subjects are supposed to (commonly) know, i.e. the structure of the game and payoffs. Rather than providing knowledge, the suggestion received by the subjects makes them aware of a relevant item of knowledge or, in other words, makes such an item salient to them. Assuming, with Lewis, that subjects coordinate by virtue of salience, our experiment then discriminates the effectiveness of salience-building mechanisms in (weak-link) coordination games.

### 3 Experimental design

Ninety participants were recruited from Luiss Guido Carli in Rome. Participants were undergraduate students enrolled in the various programs of the University, economics, law and political science. All experiments were conducted at the CESARE experimental economics Lab at Luiss, by using a dedicated software. The payoff function was the one shown in [Table 1](#) above.

Most sessions were conducted with 10 participants, with a few containing 8 participants. Each session consisted of a total of 13 rounds of play of the stage game. Participants were randomly assigned to computer terminals isolated from one another by glass separators. Initial instructions (equal for all treatments) explaining general rules were distributed in paper copy and were also read aloud by the experimenter. At this stage, subjects were instructed that the experiment was divided in two parts,

and that further instructions would appear on everybody's monitor as the experiment progressed. Control questions were administered to assure that the rules of the game had been correctly understood by all. A paper copy of the payoff matrix was given to each subject, and the same matrix also appeared on each participant's screen when submitting their choice for the round. Subjects were told that they would be randomly paired with another anonymous participant from the room in each round. When everybody had entered their choice, each subject received private feedback regarding chosen number, minimum number chosen in the pair and payoff earned in the round. In order to enhance independence between rounds, the final payment was based on a randomly selected round, and this was explicitly stated in the instructions.

The first three rounds were equal for all treatments and were played with no external intervention, to increase comparability with previous experiments (Chaudhuri and Paichayontvijit 2010) and to allow adequate learning of the payoff function. In fact, we wanted to avoid subjects following the experimenter's suggestion simply as a result of inexperience and being unaware of all its possible implications.

After the third round had been completed and before subjects could enter their choice for the fourth round, the experiment in all treatments was interrupted and subjects were asked to state the following two forecasts on a sheet of paper:

- the average number chosen in period 4 by all participants in the session;
- the average forecast expressed by all participants in the session.

Sincere belief elicitation was incentivized in the following way: for each forecast, subjects earned 1 ECU minus the difference between the stated average and the actual average, up to 0. After all participants had stated their beliefs, the experiment was resumed, and subjects could make their choices for round 4 and subsequent rounds.

The following four treatments were conducted. In the *control* treatment (2 sessions,  $N = 16$ ), subject were asked to state their beliefs at the end of the third round, and then asked to proceed with making their choices for round 4 and the remaining rounds, with no further instructions.

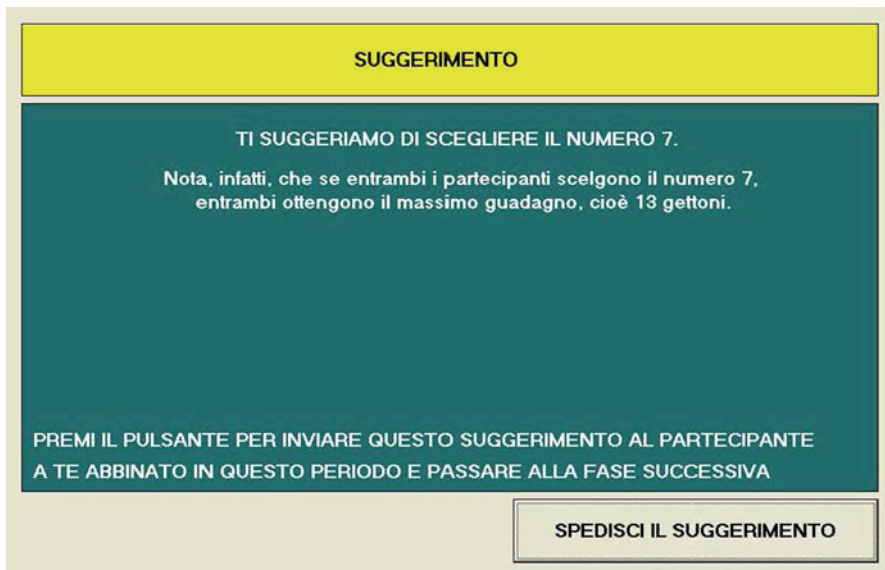
In the *mutual knowledge of level 1* treatment (3 sessions,  $N = 30$ ), before subjects were asked to make their predictions and choice for round 4 and at the beginning of all subsequent rounds of play, the following statement appeared on every participant's monitor:

WE SUGGEST YOU TO PICK NUMBER 7  
IN FACT, NOTE THAT, IF BOTH YOU AND THE OTHER PARTICIPANT PICK  
7, YOU BOTH EARN THE MAXIMUM POSSIBLE PAYOFF, 13 ECUS

The fact that the same suggestion appeared on every participant's monitor was *not* common knowledge.<sup>14</sup> Subjects had to click on "continue" to move from the suggestion screen to the choice screen, to assure that the suggestion was not overlooked.

In the *mutual knowledge of level 2* treatment (3 sessions,  $N = 26$ ), before subjects were asked to make their forecasts and choice for period 4, and at the beginning of

<sup>14</sup> This design feature marks an important difference in our setup with respect to that in Chaudhuri and Paichayontvijit (2010).



**Fig. 1** The suggestion screen

all subsequent rounds of play, the same statement shown above appeared on every participant's monitor. In addition, the lower part of the screen reported the following:

CLICK ON "SEND" TO SEND THIS SUGGESTION TO THE PARTICIPANT WITH WHOM YOU ARE PAIRED IN THIS ROUND.

Subjects had to click "send" to move to the following screen. The following screen reported the sentence shown below:

THE PARTICIPANT WITH WHOM YOU ARE PAIRED IN THIS ROUND IS SENDING YOU THIS MESSAGE:

followed by the suggestion text.

The subject was asked to click "read" to send the other player a confirmation that the message had been read before moving to the next screen. The last screen reported:

THE PARTICIPANT WITH WHOM YOU ARE PAIRED NOTIFIES YOU THAT THEY HAVE RECEIVED THE MESSAGE THAT YOU SENT THEM.

Then the choice screen appeared. Neither the suggestion appearance nor the message exchange was common knowledge among participants.<sup>15</sup>

Finally, in the *common knowledge* treatment (2 sessions,  $N = 18$ ), the suggestion text was read aloud by the experimenter before subjects were asked to state their forecast for period 4. The suggestion was read aloud at the beginning of round 4 only, and

<sup>15</sup> Notice that while this protocol can legitimately generate knowledge of the suggestion up to level 2, it also legitimately generates a third-order *belief*, as each player can believe (but not know) that the other player has seen the acknowledgment sent by the former player.

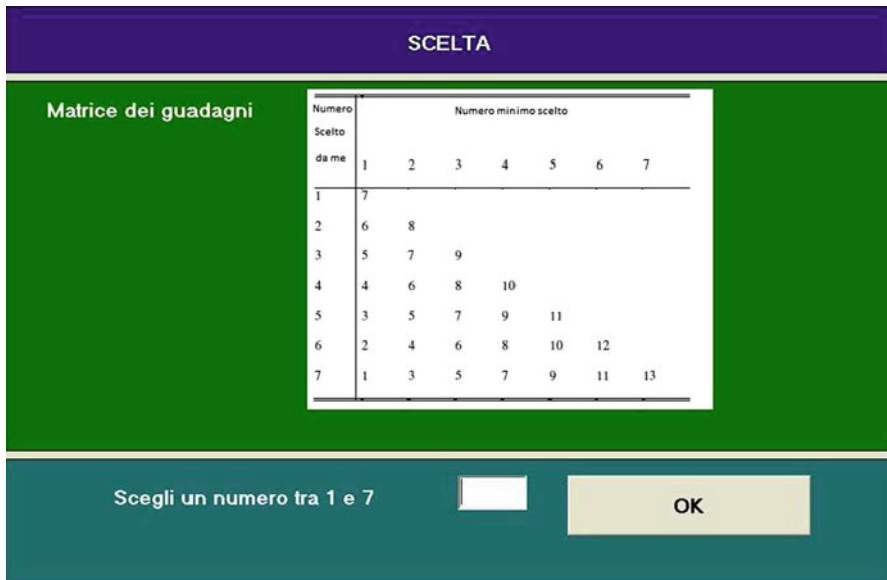


Fig. 2 The choice screen

not at the beginning of every subsequent round. Figures 1 and 2 illustrate two samples of the computer interface used in the experiment.

Before moving on to stating our hypotheses and analyzing the data, we would like to address the issue of what in fact is the *content* of the proposition on which the present experiment hinges. The suggestion that the experimenters send to the subjects contains two elements: one is the suggestion itself (“we suggest you pick number 7”) and the other is a factual explanation of the rationale behind the suggestion (“note that if both you and the other participant pick 7, you both earn the maximum possible payoff, 13 ECU’s.”) As the message read by subjects contains a factual explanation of the rationale behind the suggestion, it is reasonable to believe that subjects are led to an *indicative* reading of the suggestion, so that the content of, e.g., the proposition commonly known in the third treatments is “experimenters suggest to pick 7,” or perhaps “experimenters suggest that I pick 7.”<sup>16</sup>

To facilitate the exposition of the results, we will refer to our four treatments according to the following stipulation:

- *control*—*C* = control;
- *mutual knowledge*—*MK* = mutual knowledge of level 1;
- *message*—*Message* = mutual knowledge of level 2;
- *common knowledge*—*CK* = common knowledge.

<sup>16</sup> The indication of a rationale for the suggestion is thus important to avoid that subjects understand the suggestion in imperative terms, as for instance in “experimenters suggest that I should pick 7.” We are grateful to an anonymous referee for prompting us to make this clarification.

The control treatment is necessary to provide us with a benchmark for the frequency of coordination, as well as to control for learning and restart effects. The other three treatments allow us to test the effect that adding one level of knowledge has on the probability of coordination on the efficient equilibrium.

The choice to use the payoff function of Van Huyck et al. (1990) is motivated by the fact that this payoff function has already been tested for a group size of two players with random matching. Results have shown high variability and no convergence over time in this condition. To our knowledge, this is the worst coordination outcome that has been reported in a minimum effort game with group size of two. As a high level of coordination failure in our baseline treatment is a necessary condition for our experimental treatments to be of interest, we chose to implement the same payoff function used by Van Huyck and co-authors.

Experiments lasted forty minutes on average, including instruction time. Average total earnings were equal to 13.5 euro, including the 2 euro show up fee.

We formulated the following hypotheses:

- coordination failure is the modal outcome in the control treatment;
- both the share of choices equal to 7 and the frequency of efficient coordination outcomes increase as more knowledge levels are added.

Hence, we expected to observe widespread coordination failure in the control treatment, and higher frequencies of coordination success as moving from the *control* to the *common knowledge* treatment.

## 4 Results

### 4.1 Analysis of choice behavior

Figure 3 reports the average choices, whilst average minima and relative frequency of choices equal to 7 over time in the four treatments are reported in Figs. 4 and

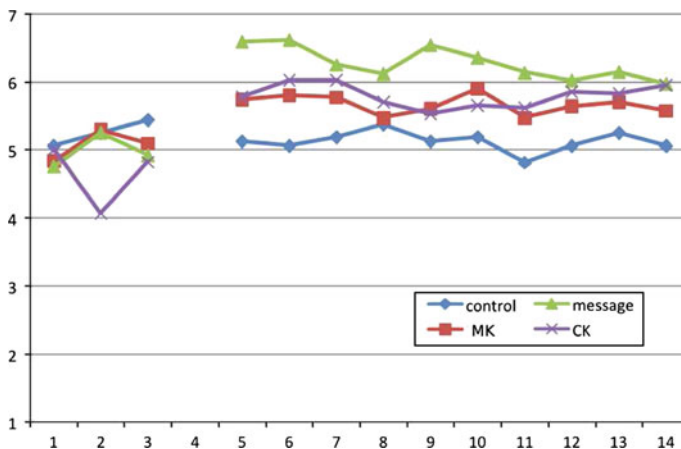


Fig. 3 Average choices over time, all sessions pooled

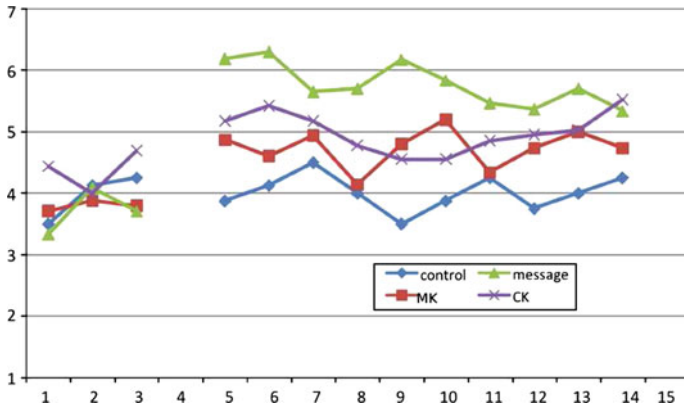


Fig. 4 Average minima over time, all sessions pooled

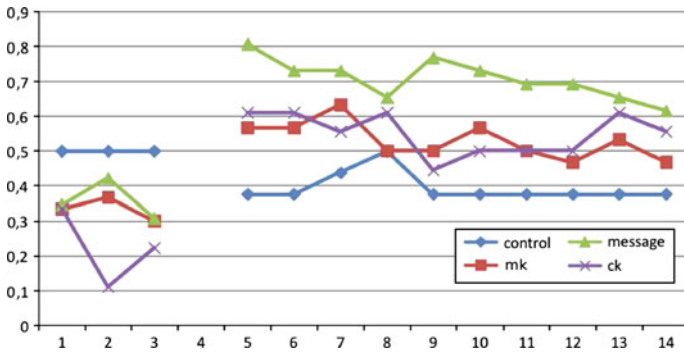


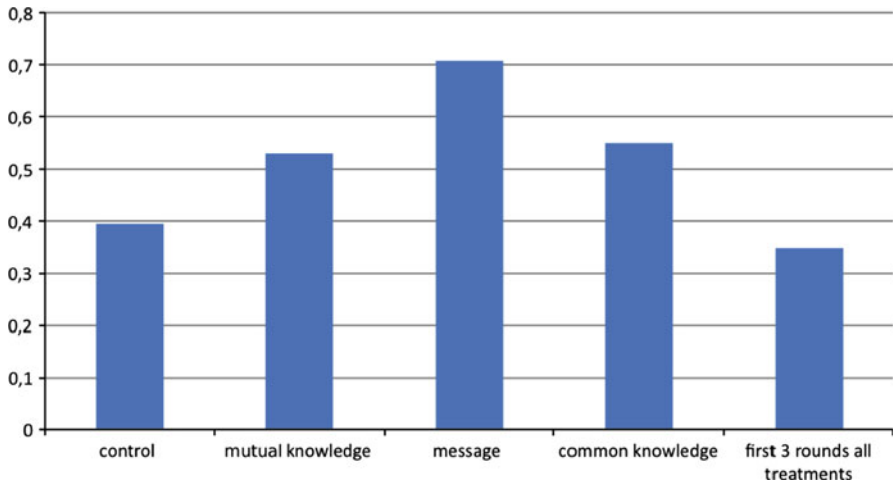
Fig. 5 Relative frequency of choices equal to 7 over time, all sessions pooled

5, respectively. All values are pooled across sessions. The following facts are worth noticing: first, both minima and 7 choices in the control treatment indicate widespread coordination failure with no detectable trend over time. Minima stabilize around 4 and choices of 7 around 40%. Hence, the control treatment by and large replicates previous findings obtained with the same group size and matching protocol in Van Huyck et al. (1990). Secondly, all experimental treatments reveal an improvement in coordination with respect to the control, as hypothesized.

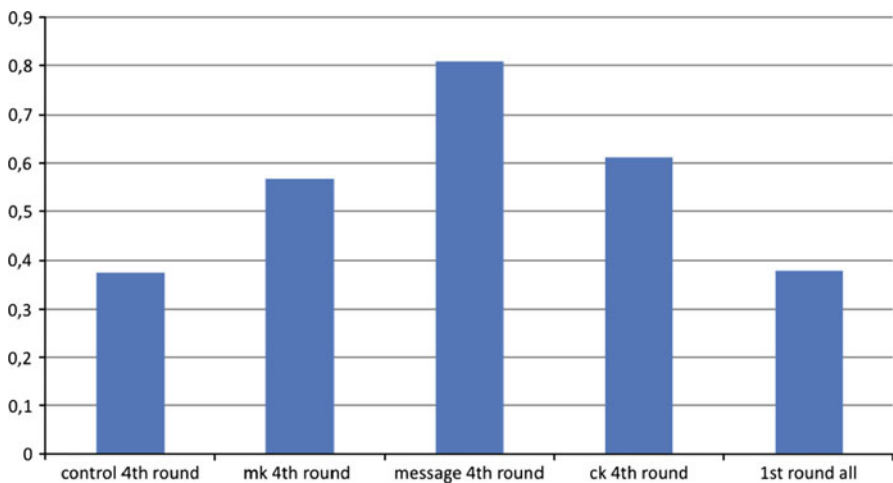
However, somewhat surprisingly and contrary to our hypothesis, the increase is not monotonic in knowledge levels. More specifically, the highest values are attained in the message treatment, while the values for the mutual knowledge and common knowledge treatments lie between the two extremes of “control” and “message”.

Figure 6 reports the frequency of the choice of 7 in rounds 4–13 separately for each treatment, and in rounds 1–3 for all treatments combined. Figure 7 reports the same information, respectively, for round 4 only and for round 1 only.

Figure 6 clearly shows that, overall, the message treatment was by far the most effective in fostering choice of the strategy supporting the efficient equilibrium. The second finding worth noticing is that mutual knowledge of level 1 is able to induce the choice of 7 in a proportion comparable to that attained in the common knowledge treatment.



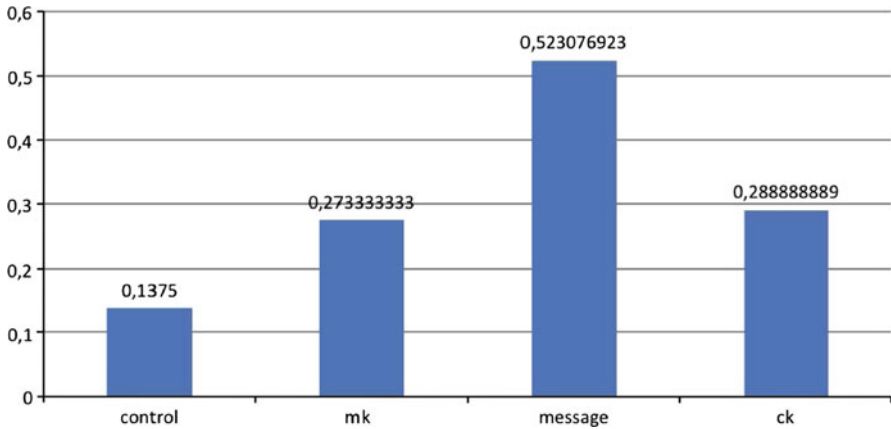
**Fig. 6** Frequency of choices equal to 7 in rounds 4–13, disaggregated by treatment, and in rounds 1–3, all treatments pooled



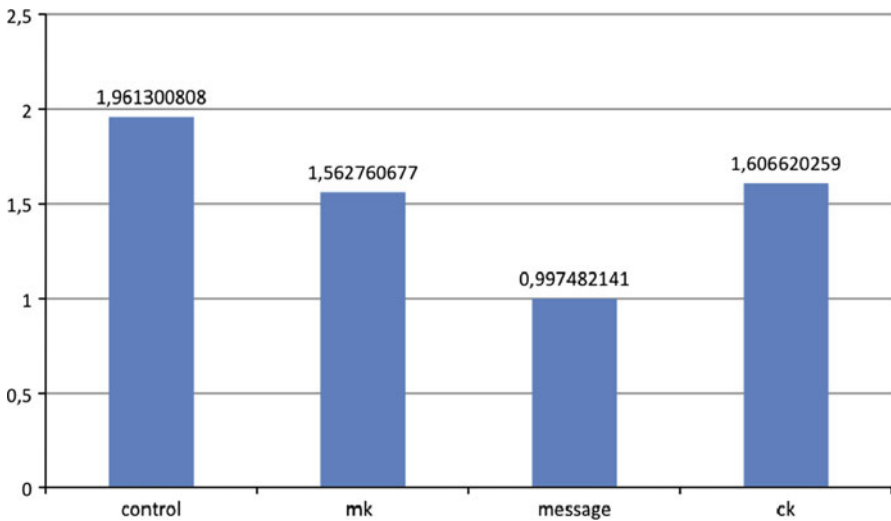
**Fig. 7** Frequency of choices equal to 7 in round 4, disaggregated by treatment, and in round 1, all treatments pooled

By comparing the same data in round 4 only, as reported in Fig. 7, we can measure the effect of different levels of knowledge excluding the potentially confounding factor given by message repetition. Moreover, the salience of the publicly read announcement in the CK treatment is supposed to be highest in round 4 as compared to subsequent rounds. Hence, the CK treatment is given its best chance to produce efficient coordination immediately after the public announcement. Data from round 4 suggest that the message treatment is the most effective even without the aid of message repetition; in fact, choices of 7 amount to 80% of all choices in that treatment. The CK treatment is the second best, with 60% of 7 choices. MK, however, does almost as well as the CK





**Fig. 8** Percentage of equilibrium play in rounds 4–13



**Fig. 9** Standard deviation of choices in rounds 4–13, disaggregated by treatment

treatment, whereas the percentage in the control treatment does not differ from that observed in the first three rounds of play.

The relative ranking of the four different knowledge levels on goodness of coordination is further supported by data on equilibrium play. Figure 8 reports the percentage of play consistent with the efficient equilibrium (i.e., both players in the pair picking 7) in the four treatments in rounds 4–13. In the message treatment more than half of play (52 %) resulted in coordination on the efficient equilibrium, whereas data for the CK treatment and for the MK treatment amount to 28.8 and 27.3 %, respectively. Hence, again, mutual knowledge of level 1 is comparable to common knowledge in its efficiency-enhancing effects, and mutual knowledge of level 2 fares substantially better than common knowledge. In accordance with our hypothesis and with previous results, the control treatment results in the highest inefficiency, with only 13.8 % of

**Table 2** Fisher's exact test on the frequency of choices 7 in the different treatments in rounds 4–13 (all  $p$ -values are two-tailed)

|                  | Control | Mutual knowledge | Message     | Common knowledge |
|------------------|---------|------------------|-------------|------------------|
| Control          |         | $p = .0061$      | $p = .0001$ | $p = .0047$      |
| Mutual knowledge |         |                  | $p = .0001$ | $p = .7057$      |
| Message          |         |                  |             | $p = .0008$      |
| Common knowledge |         |                  |             |                  |

**Table 3** Fisher's exact test on the frequency of equilibrium play in the different treatments in rounds 4–13 (all  $p$ -values are two-tailed)

|                  | Control | Mutual knowledge | Message     | Common knowledge |
|------------------|---------|------------------|-------------|------------------|
| Control          |         | $p = .0010$      | $p = .0001$ | $p = .0009$      |
| Mutual knowledge |         |                  | $p = .0001$ | $p = .7528$      |
| Message          |         |                  |             | $p = .0001$      |
| Common knowledge |         |                  |             |                  |

efficient equilibrium play. Consistently, data on the standard deviation of choices in the four treatments in round 4–13 (Fig. 9) reveal that the message treatment reduces strategic uncertainty the most with respect to the control, while the MK and CK treatment values are indistinguishable.

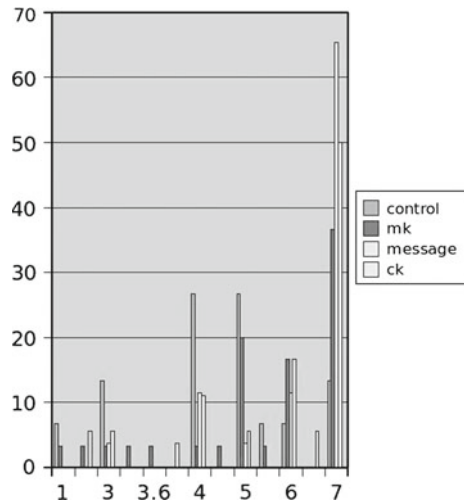
Pairwise comparisons between choices in the first round in the different treatments reveal that there are no statistically significant differences in choice distributions in period 1 (Mann–Whitney  $U$  test; control-mutual knowledge  $p = .65$ ; control-message  $p = .499$ ; control-common knowledge  $p = .95$ ; mutual knowledge-message  $p = .83$ ; mutual knowledge-common knowledge  $p = .51$ ; message-common knowledge  $p = .44$ . All  $p$ -values are two-tailed).

Table 2 reports results of the Fisher's exact test on the differences in the share of 7 choices in rounds 4–13, and Table 3 reports results of the same test on the frequency of efficient equilibrium play. All differences are highly significant *except* that between MK and CK.

## 4.2 Analysis of beliefs

Figure 10 shows the distribution of first order beliefs (i.e., subjects' forecasts about the average of all participants' choices in round 4) by treatment. It is noteworthy that beliefs behavior is highly consistent with choice behavior, in that a shift of the distribution in the direction of higher numbers can be detected when moving from the control treatment to the MK, CK and message treatments. In the message treatment, roughly 65% of forecasts are concentrated on the value 7.

**Fig. 10** Distribution of beliefs by treatment



**Table 4** Descriptive statistics on first order, second order beliefs and choices in round 4 by treatment

|                           | <i>N</i> | Minimum | Maximum | Mean | SD. Deviation |
|---------------------------|----------|---------|---------|------|---------------|
| 1st order belief CONTROL  | 15       | 1       | 7       | 4.57 | 1.568         |
| 2nd order belief CONTROL  | 16       | 1       | 7       | 4.31 | 1.740         |
| Choice in round 4 CONTROL | 16       | 1       | 7       | 5.13 | 1.962         |
| 1st order belief MK       | 30       | 1       | 7       | 5.47 | 1.630         |
| 2nd order belief MK       | 30       | 1       | 7       | 5.58 | 1.602         |
| Choice in round MK        | 30       | 1       | 7       | 5.73 | 1.999         |
| 1st order belief MESSAGE  | 26       | 3       | 7       | 6.18 | 1.311         |
| 2nd order belief MESSAGE  | 25       | 4       | 7       | 6.37 | 1.026         |
| Choice in round MESSAGE   | 26       | 3       | 7       | 6.58 | 1.027         |
| 1st order belief CK       | 18       | 2       | 7       | 5.88 | 1.597         |
| 2nd order belief CK       | 17       | 4       | 7       | 6.00 | 1.225         |
| Choice in round CK        | 18       | 2       | 7       | 5.83 | 1.724         |

Table 4 reports some descriptive statistics on first order, second order beliefs and choices in round 4 by treatment.<sup>17</sup>

High consistency between beliefs and choices is revealed by looking at the column of mean values. Data on beliefs reveal important insights: first, they support the hypothesis that the suggestion (and its related knowledge levels) rendered sub-

<sup>17</sup> One subject in the control treatment stated a first order belief equal to 10, one subject in the message treatment stated a second order belief equal to 42, and one subject in the common knowledge treatment stated a second order belief equal to 8. We eliminated these outliers from the computation of the average values and of the subsequent correlation indexes. The results of the correlation do not change if we include the outliers. We did not observe any anomalous behavior in these subjects at the choice level, hence they may have not understood the prediction task correctly.

jects on average more optimistic, and greater optimism in turn induced the choice of the risky action significantly more often with respect to the absence of an external suggestion.<sup>18</sup> In other words, an experimenter demand effect (Zizzo 2010) as an explanation of behavior in our experimental treatments can be safely excluded. Rather, the most plausible explanation of our subjects' behavior involves increased optimism and trust. Moreover, data on beliefs strongly point at the fact that subjects in the MK and message treatments believed that other participants in the session received the same type of information that they themselves received.

The positive effect of *any* knowledge level can be detected by looking at minimum values in column 3. Unlike the control treatment, where a minimum of 1 is observed both at the belief and at the choice level, in all experimental treatments the external announcement is able to eliminate the secure action (i.e., the choice of 1) from both participants choices and forecasts. Finally, Table 5 reports the correlation coefficients between first order beliefs, second order beliefs and choices in round 4 separately for each treatment. As the table shows, all relevant correlation coefficients are positive and highly significant.

## 5 Refined *message* treatment

The rationale for the next treatment is to assess the potential impact of the change in propositional content caused by the linguistic form of the *message* treatment.<sup>19</sup> More specifically, the act of sending a message to the other player, combined with the ambiguous message content (“We suggest you to pick 7...”) might have created normative pressure that is absent in the other two conditions, deriving from a possible interpretation of the suggestion as coming from the other player as well as from the experimenter. To address this issue, we designed and conducted a further “level-2” treatment (MK2 henceforth) in which any possible source of normative pressure was eliminated. In this further treatment, the message exchange phase was left unaltered, but the whole procedure was rephrased so as to make it explicit that the suggestion was only from the experimenter, and even the minimal interaction between players was eliminated.

Hence, the subject's monitor showed the experimenter's suggestion, as in the previous treatment. The lower part of the screen reported now the following sentence:

CLICK HERE TO HAVE THE EXPERIMENTER'S SUGGESTION VISUALIZED ON  
THE SCREEN OF THE PARTICIPANT WITH WHOM YOU ARE PAIRED IN THIS  
ROUND

Subjects had to click to move to the following screen, which reported the sentence shown below:

---

<sup>18</sup> Notice also that, since the elicited beliefs are beliefs in the average choice of action of all other participants in the session, choosing an action higher than one's expressed prediction denotes optimist risk-seeking behavior.

<sup>19</sup> We most warmly thank one anonymous referee for pointing out this.

**Table 5** Spearman’s rho correlation coefficients (with relative *p*-values, two-tailed) between subjects’ 1st order, 2nd order predictions, and 4th round choices, divided by treatment

|                           | 1st order pred. C  | 2nd order pred. C  | 4th round choice C  |
|---------------------------|--------------------|--------------------|---------------------|
| <b>1st order pred. C</b>  |                    |                    |                     |
| Corr. Coeff.              | 1.00               | 0.86               | 0.61                |
| Sig.                      |                    | <b>0.00</b>        | <b>0.02</b>         |
| <i>N</i>                  | 15.00              | 15.00              | 15.00               |
| <b>2nd order pred. C</b>  |                    |                    |                     |
| Corr. Coeff.              | 0.86               | 1.00               | 0.61                |
| Sig.                      | 0.00               |                    | <b>0.01</b>         |
| <i>N</i>                  | 15.00              | 16.00              | 16.00               |
|                           | 1st order pred. MK | 2nd order pred. MK | 4th round choice MK |
| <b>1st order pred. MK</b> |                    |                    |                     |
| Corr. Coeff.              | 1.00               | 0.80               | 0.38                |
| Sig.                      | –                  | <b>0.00</b>        | <b>0.04</b>         |
| <i>N</i>                  | 30.00              | 30.00              | 30.00               |
| <b>2nd order pred. MK</b> |                    |                    |                     |
| Corr. Coeff.              | 0.80               | 1.00               | 0.59                |
| Sig.                      | <b>0.00</b>        | –                  | <b>0.00</b>         |
| <i>N</i>                  | 30.00              | 30.00              | 30.00               |
|                           | 1st order pred. M  | 2nd order pred. M  | 4th round choice M  |
| <b>1st order pred. M</b>  |                    |                    |                     |
| Corr. Coeff.              | 1.00               | 0.92               | 0.69                |
| Sig.                      |                    | <b>0.00</b>        | <b>0.00</b>         |
| <i>N</i>                  | 26.00              | 25.00              | 26.00               |
| <b>2nd order pred. M</b>  |                    |                    |                     |
| Corr. Coeff.              | 0.92               | 1.00               | 0.51                |
| Sig.                      | <b>0.00</b>        | –                  | <b>0.01</b>         |
| <i>N</i>                  | 25.00              | 25.00              | 25.00               |
|                           | 1st order pred. CK | 2nd order pred. CK | 4th round choice CK |
| <b>1st order pred. CK</b> |                    |                    |                     |
| Corr. Coeff.              | 1.00               | 0.74               | 0.76                |
| Sig.                      | –                  | <b>0.00</b>        | <b>0.00</b>         |
| <i>N</i>                  | 18.00              | 17.00              | 18.00               |
| <b>2nd order pred. CK</b> |                    |                    |                     |
| Corr. Coeff.              | 0.74               | 1.00               | 0.72                |
| Sig.                      | <b>0.00</b>        |                    | <b>0.00</b>         |
| <i>N</i>                  | 17.00              | 17.00              | 17.00               |

The relevant significance values are reported in bold face

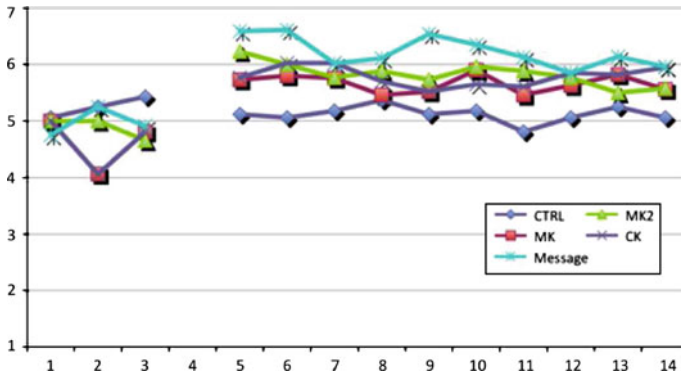


Fig. 11 Average choices over time for the five treatments combined

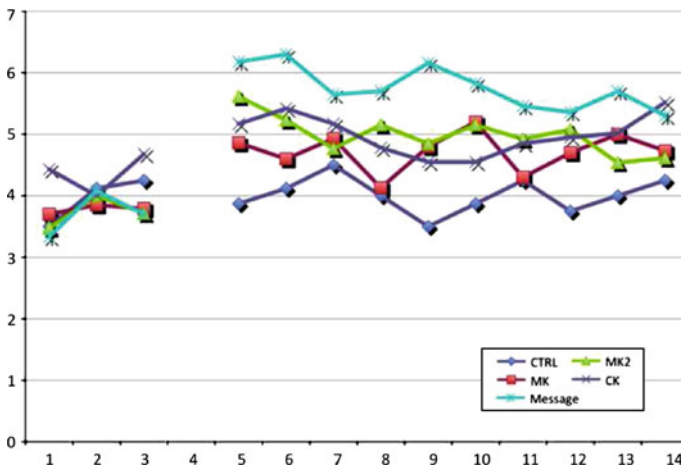


Fig. 12 Average minima

THE PARTICIPANT WITH WHOM YOU ARE PAIRED IN THIS ROUND IS  
FORWARDING YOU THIS SUGGESTION FROM THE EXPERIMENTER

followed by the suggestion text.

The subject was asked to click “read” to send the other player a confirmation. The last screen reported:

WE CONFIRM THAT THE EXPERIMENTER’S SUGGESTION HAS BEEN  
VISUALIZED ON THE SCREEN OF THE PARTICIPANT WITH WHOM YOU ARE  
PAIRED

Then the choice screen appeared. As before, neither the suggestion appearance nor the message exchange were common knowledge among participants.

Three sessions of the new treatment were conducted (two sessions with 8 players and one session with 10 players) for a total of 18 subjects. Figure 11 shows average choices over time for the five treatments combined, Fig. 12 reports average minima

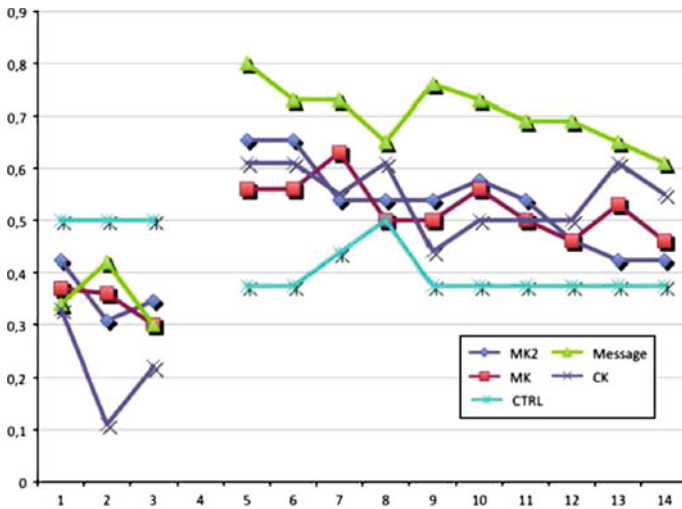


Fig. 13 Relative frequency of “7”

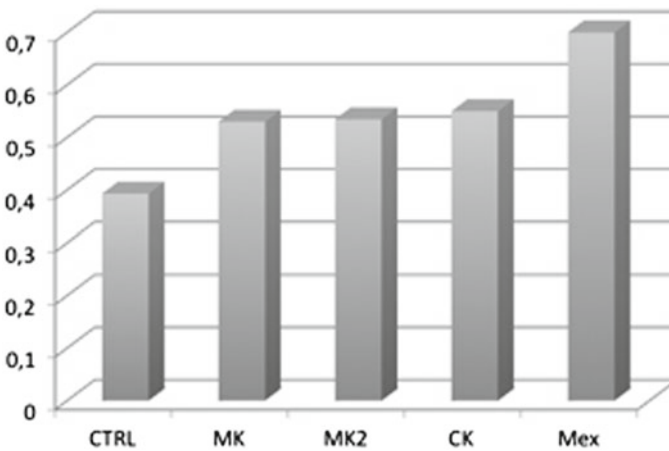
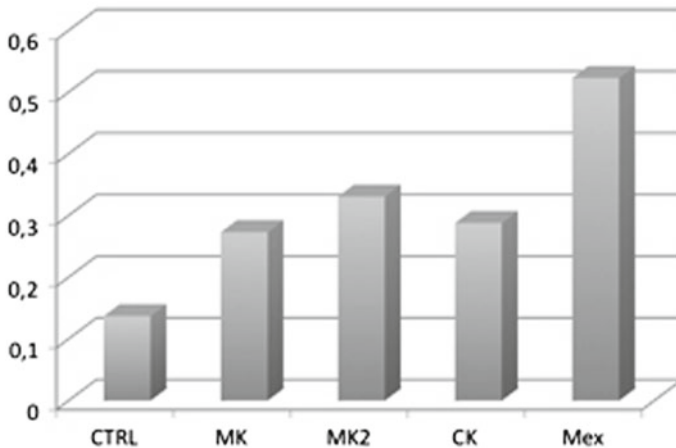


Fig. 14 Frequency of “7”

and Figs. 13 and 14 report the relative and absolute frequency of “7”, respectively. Finally, Fig. 15 illustrates the frequency of equilibrium play.

Pairwise comparisons between choice distributions in the first round reveal, as before, no statistically significant differences (Mann–Whitney  $U$  test; control-MK2  $p = .83$ ; mutual knowledge-MK2  $p = .73$ ; message-MK2  $p = .58$ ; common knowledge-MK2  $p = .67$ .) Instead, the share of observed “7” choices in MK2 in rounds 4–13 is significantly different from both the control ( $p = .006$ ) and the message treatment ( $p = .0001$ ) by a Fisher exact test. However, it is not different from the mutual knowledge ( $p = .9$ ) and from the common knowledge treatment ( $p = .77$ ). Restricting the analysis to the share of “7” choices in round 4 only (i.e., immediately



**Fig. 15** Frequency of equilibrium plays

after the announcement is publicly made in the common knowledge treatment), the only significant difference found is the one between MK2 and the control treatment ( $p = .05$ , Fisher's exact test, one-tailed).

Finally, the last comparison concerns the percentage of equilibrium play: all differences are highly significant by the same Fisher exact test (control-MK2  $p = .000$ ; mutual knowledge-MK2  $p = .02$ ; message-MK2  $p = .000$ ) *except* that between MK2 and the common knowledge treatment ( $p = .3$ ).

## 6 Conclusions and further work

To summarize, our results show that the “control” and “message” treatments envelope all others along the most important dimensions (average, minima, percentage of choices of 7, percentage of equilibrium play.) Thus, our experiment reveals two major findings: first, while group knowledge of advice in weak-link games does increase efficient play and coordination, levels of iteration of knowledge of advice do not matter with regards to the amount of the increase (in particular common knowledge does not significantly improve over other kinds of group knowledge); second, second-order mutual knowledge of advice has significantly different effects, as a fostering coordination device, depending on the specific mode with which subjects form expectations about the behavior of their fellow players.

Intuition, sometimes backed by formal results as in the analysis of the so-called *coordinated attack problem* (Fagin et al. 1996), has long been suggesting that the rational solution of coordination games required common-knowledge of the players' rationality and choices.<sup>20</sup> Our results to the effect that common knowledge of advice

<sup>20</sup> See however Parikh (2005) for an extensive discussion on the far reaching consequence of finite, indeed *small*, level knowledge and Hosni and Paris (2005) for an analysis of coordination which dispenses with common knowledge altogether.



does no better than mutual knowledge as a salience-generating device appears to be quite surprising and meaningful nonetheless. For instance, in the debate on whether conventions need to be common knowledge<sup>21</sup> weighs in favor of the position that they need not.

Our attempt at explaining this unexpected result is best framed in terms of the correlated unexpected finding, namely that second-order mutual knowledge of advice may lead to the selection of efficient equilibria more frequently than any other epistemic condition we investigated.

Why is our second-order mutual knowledge “message” condition so effective at promoting efficient coordination? To identify a plausible explanation, it is convenient to focus on the differences between the second-order mutual knowledge “message” condition and all other conditions in our experiment. To induce the second-order expectation, subjects are instructed to exchange messages in order to continue the experiment. Such a direct, first-person communication exchange is only present in the second-order “message” condition, therefore we surmise that it is an important factor in the explanation of our results. Indeed, computer-mediated message exchange is known to give rise to expectations of mutual commitment between players,<sup>22</sup> both in the sense that receiving the message makes the receiver believe that the sender will do her part to honor the advice, and in the sense that sending the message makes the sender believe that the receiver will expect that the sender ought to conform to the advice she sent. In short, first-person message exchange may elicit a norm of promise-keeping.

Thus, not only it is relevant what kind of beliefs players hold, but also how they arrive at holding such beliefs. In particular, beliefs formed through the embryonic communication used in our second-order “message” condition seem to foster compliance with the content of the exchanged message. Recall how in Bicchieri’s theory of social norms second-order expectations play a crucial role as *normative* expectations. While first-order, empirical expectations may not suffice to motivate compliance with the social norm, first- and second-order expectations, when aligned, may succeed in bringing about norm compliance. Our study is not concerned with social norms, however the coordination success observed in our second-order mutual knowledge condition, paired with the observation of high correlation between first-, second-order beliefs, and choices, highlights the relevance of second-order expectations for motivating action. In light of this observation, our hypothesis that subjects’ second-order beliefs have normative content calls for further research.

Our result that second-order mutual knowledge works better than first-order in fostering efficient equilibrium choices may lead to the generalization that there is a monotonic relationship between players’ abidance to advice and levels of mutual knowledge of advice. If that were the case, common knowledge of advice would result in highest level of efficient coordination. However, that is not what is observed in our common knowledge condition, lending support to the hypothesis formulated above that second-order expectations carry more weight because of their normative character. Our results, however, suggest not only that common knowledge of advice is not

<sup>21</sup> See for instance Binmore (2008) and Sillari (2008).

<sup>22</sup> For studies on the normative impact of (even very limited) computer-mediated communication in mixed-motive games see Bicchieri and Lev-On (2007) and Bicchieri et al. (2010), among others.

necessary to achieve efficient coordination, but also that it is not sufficient. This result bears on the literature on the relation between convention and common knowledge, showing that common knowledge is not the most effective device for creating salience and fostering efficient coordination, at least in the short run.

We would like to close this paper by insisting on our finding that coordination is more likely under mutual knowledge than common knowledge of advice. As we noted before the surprise might be accounted for by assuming a psychological relevance of the suggestion. If true this might suggest that belief formation mechanism weights more than the sole epistemic condition (common knowledge) towards the selection of the efficient equilibrium. This interpretation seems to be posing a challenge for Bayesian epistemology, insofar as the latter focusses only on the information revealed by the agent's (degrees of) belief. Yet the challenge is only apparent, for the surprising performance of mutual knowledge in the message treatment might well depend on the subjects' "updating" on the expectation of mutual commitment. This emerging notion of belief update, however, clearly deserves further investigation.

**Acknowledgments** Financial support from the Italian Ministry of Education, University and Research (MIUR, project PRIN 2008) is gratefully acknowledged. We thank Daniela Di Cagno, Massimo Egidi, Marco Tecilla, Mariagrazia Palmieri and Dearná Sulonjaku for their help in developing the experimental software and in implementing and conducting the experiments. We thank Cristina Bicchieri, Ryan Muldoon, the participants of the Pittsburgh-Tilburg Workshop on *Formal Epistemology Meets Experimental Philosophy* and particularly two anonymous referees for their helpful comments and suggestions. The usual disclaimer applies.

## A Appendix: Experimental instructions

### A.1 General instructions (common to all treatments)

#### A.1.1 Introduction

Welcome! You are about to participate in an experiment on decision making funded by the Ministry of Education, University and Research. Instructions are simple, and if you follow them correctly and make appropriate decisions you will be able to earn an appreciable sum of money that will be paid to you privately and in cash. All choices that you make will be stored and processed anonymously. All your earnings in the experiment will be expressed in Experimental Currency Units (ECUs). 1 ECU is worth 1 euro.

#### A.1.2 The choice task

The present experimental session is composed of 10 participants and will last a total of 13 periods, divided into 2 parts: the first part will last for 3 periods, and the remaining part will last for 10 periods. The choice task will be the same for all the 13 periods. You are now listening to the initial instructions, to be followed by further written instructions that will appear on your monitor. *At the beginning of each period the software will pair each of you with another participant from the room picked randomly.* You

and the other participant will both have to choose—independently from one another and without any possibility to communicate—one and only one number from 1 to 7 (included). *The euros you earn will depend on the number you have chosen and on the minimum number that has been chosen in the pair* (that is, the lower number between the one chosen by you and the one chosen by the participant with whom you are paired). Each of you has been given a paper copy of the double-entry table that will help you compute your earnings. *The table is identical for all participants*, and the same table will appear on your monitor every time you will have to make your choice for the round. Please have a look at the table now. The rows of the table report your possible choices in every period, i.e., the numbers from 1 to 7. The columns report the *minimum* number that has been chosen in the pair.

The table cells report the ECUs that you earn in correspondence of each possible outcome. The ECUs you earn are in the cell at the intersection between the row corresponding to the number you have chosen and the column corresponding to the minimum number chosen. For example, suppose you choose number 4 and the other participant chooses number 3: the minimum number in the pair is 3: therefore, your earnings are in the cell at the intersection between row 4 and column 3 (that is 8 ECUs).

Now, please pay attention to your monitor. The central part of the screen (green background) reports your earnings table, which is identical to the paper copy that you have. You will have to make your choice by typing the number chosen in the appropriate text box at the center of the blue square and by clicking on ‘OK’. When all participants have chosen, your monitor will show you the period outcome: the number you have chosen, the minimum number chosen and the ECUs you have earned. At any time you will be able to double-check that your earnings have been computed correctly by using the copy of the earnings table that has been given to you. By clicking on ‘Continue’ you will move to the next period. Please remember that at the beginning of each period the software will match you with a participant picked randomly; therefore, in general you will be paired with a different participant in each period, although it will happen that you will be paired with someone more than once. In any case you will never be told the identity of the participants with whom you are paired.

At the end of the third period the experiment will be interrupted and the monitor will show you further instructions for you to follow. At some point, **BEFORE YOU MAKE YOUR CHOICE FOR PERIOD 4**, the experimenter will ask you to take the blank sheet of paper that is on your desk and write the following 2 predictions:

- The *average number* that will be chosen by all participants in period 4, i.e. the arithmetic mean of all choices in period 4
- The *average prediction* of participants, i.e., the average of all participants’ forecasts regarding choices in period 4

This information will be asked only at the beginning of period 4, and you will be paid in the following way: for each prediction, you will earn 1 euro minus the arithmetic difference between your predicted value and the realized value, for a maximum of 2 euro in the case in which both your predictions are 100 % correct. The true realized values will be computed and publicly revealed at the end of the experiment.

### A.1.3 Your earnings

Your earnings will be computed as follows: at the end of the experiment, the software will randomly pick one among the 13 periods, and each of you will be paid according to the ECUs he/she has earned in that period. The final screenshot will inform you of which period has been extracted and of how many ECUs you have earned. This earnings will be summed to the earnings from your predictions and to the 2 euro show up fee. Each one of you will be paid privately in cash. After receiving your payment, you will be free to leave the lab.

We now ask you to respond to all the questions reported in the anonymous questionnaire that has just been handed to you, and to give the questionnaire back to the experimenter. In case of incorrect answers, the relevant part of the instructions will be repeated. During the experiment you are not allowed to communicate in any way. If any forms of verbal or non-verbal communication are detected, the session will be immediately interrupted and nobody will be paid.

If you have questions, please raise your hands now.

### A.2 Control questions (common to all treatments)

The present questionnaire is anonymous and only serves the purpose of making sure that all participants have understood the rules of the experiment before it starts.

- QUESTION 1. Imagine you have chosen number 1 in a period, and imagine that the participant with whom you are paired has chosen number 3. According to the table, how many ECUs have you earned? How many ECUs has he/she earned?
- QUESTION 2. Imagine you have chosen number 3 in a period, and imagine that the participant with whom you are paired has chosen number 2. According to the table, how many ECUs have you earned? How many ECUs has he/she earned?
- QUESTION 3. Your earnings will be determined by the ECUs that you have earned in one of the 13 periods that will be picked randomly by the software: TRUE or FALSE (mark with an X).
- QUESTION 4. You will always be paired with the same participant for all 13 periods: TRUE or FALSE (mark with an X).

### A.3 Treatment-specific instructions appearing on each subject's monitor at the start of round 4

Note that these instructions were not read aloud.

#### A.3.1 Control treatment

Before proceeding to make your choices for the remaining 10 periods, the experimenter will ask you to write down your predictions for period 4 on the blank sheet of paper that you can find on your desk. After all participants have written their predictions, the experiment will restart. The matching rules and the rules determining your earnings in each period remain invariant

- PREDICTION 1: the *average number* that will be chosen by all participants in period 4, i.e. the arithmetic mean of all choices in period 4.
- PREDICTION 2: the *average prediction* of participants, i.e., the average of all participants' forecast regarding choices in period 4.

Now please write your predictions.

### A.3.2 Mutual knowledge treatment

Before proceeding to make your choices for the remaining 10 periods, the experimenter will ask you to write down your predictions for period 4 on the blank sheet of paper that you can find on your desk. After all participants have written their predictions, the experiment will restart. The matching rules and the rules determining your earnings in each period remain invariant

#### SUGGESTION

**WE SUGGEST YOU TO CHOOSE NUMBER 7. Note, in fact, that if both participants choose number 7, they both earn their maximum earnings, that is 13 ECUs**

Now please write your predictions:

- PREDICTION 1: the *average number* that will be chosen by all participants in period 4, i.e. the arithmetic mean of all choices in period 4.
- PREDICTION 2: the *average prediction* of participants, i.e., the average of all participants' forecast regarding choices in period 4.

### A.3.3. Message treatment

Before proceeding to make your choices for the remaining 10 periods, follow the instructions that will appear on your monitor. The software will create the random pairings among participants. Then you will be shown a message and you will be asked to send the message to the participant with whom you have been paired for period 4.

BEFORE you make your choice the experimenter will ask you to write down your predictions for period 4 on the blank sheet of paper that you can find on your desk. After all participants have written their predictions, the experiment will restart. The matching rules and the rules determining your earnings in each period remain invariant. Now please follow the instructions on your monitor and then wait for the experimenter to tell you to write down your predictions.

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