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Dynamics of entanglement transfer from radiation modes to localized qubits

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Abstract. We address the dynamics of entanglement transfer from two radiation modes to a pair of localized qubits implemented as atoms flying through or trapped in separated cavities. We first generalize previous results to include radiation in entangled mixed states and to describe the effect of cavity mirror transmittance. Then we investigate the transfer process by Monte Carlo Wave Function approach, which allows us to solve the whole system dynamics including cavity mode and atomic decays. We focus on flying modes prepared in coherently correlated states and we find realistic conditions for efficient entanglement transfer out of the weak and strong coupling regimes in the perspective of quantum memories realization.

1 Introduction

The entanglement transfer from flying modes to localized qubits play an important role in quantum information processing. Indeed, a number of protocols, such as eavesdropping in quantum cryptography, quantum repeater, and linear optics quantum computing [1] would greatly benefit from a quantum memory with fidelity higher than in classical recording.

In recent years different schemes have been investigated from a theoretical point of view [2] and experimental realization have been also reported [3]. In this paper we focus to the case of two radiation modes carrying a certain amount of entanglement to be transferred to a pair atomic qubits, which represent the quantum memory storing quantum information. Deterministic trapping of atoms inside optical cavities [4] is a promising technique to realize this kind of qubit system. The process of entanglement transfer between the flying modes and two atoms placed in separated cavities may be described by different approaches. In [5] the coupling of two qubits with a broadband driving field was assumed through their small local environment, which isolates the qubits from uncontrollable environment, and the weak-coupling limit was investigated. A similar approach can be found in [6] where the cavity modes were assumed in equilibrium with an external two mode squeezed light whose bandwidth is larger than the cavity damping rate. The case of resonant Jaynes-Cummings interaction between atoms and cavities has been investigated in [7] for different types of two-mode radiation. In a recent paper [8] we investigated the entanglement transfer process including off-resonance Jaynes-Cummings interaction and different interaction times for the two atoms. We also showed that non-Gaussian states, such as two-mode coherently correlated states (TMC) [9], allow a larger entanglement transfer than for Gaussian ones such as the TWB states [10].

In [8] we assumed that the properties of the radiation modes are fully transferred to the cavity modes before the interaction with the atoms and we neglected any dissipative process

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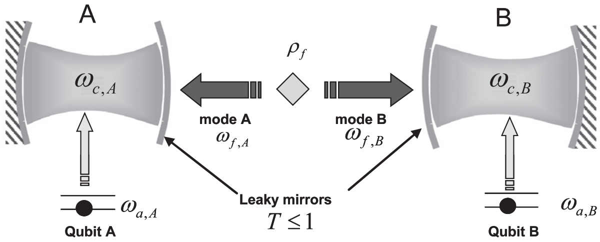

Fig. 1. Scheme for the entanglement transfer between a pair of freely propagating entangled radiation modes and two atoms placed in spatially separated cavities.

assuming the strong coupling regimes. This allowed us to model the whole transfer process only by unitary operations. A more realistic description of the coupling among flying and cavity modes may be effectively described by a beam-splitter-like interaction [11,12]. In this case one can take into account the effect of cavity mirror transmittance that is a partial entanglement transfer to the atoms. In this paper we first generalize the treatment in [8] to include the effect of cavity mirror transmittance and a two mode radiation prepared in a general mixed state. Then we provide a full a description of the entanglement transfer process by Monte Carlo Wave Function (MCWF) approach [13], which allows us to numerically solve the whole system dynamics including cavity mode and atomic dissipation. In addition, this approach can be applied in the case of atoms trapped inside the cavities simultaneously to the interaction with the driving radiation modes. We focus to TMC states and we investigate realistic conditions for efficient entanglement transfer out of the weak and strong coupling regimes in the perspective of quantum memories realization.

The paper is structured as follows. In section 2 we illustrate the model for entanglement transfer in the strong coupling regime both for pure and mixed radiation states, whereas section 3 is devoted to MCWF analysis of the full system dynamics. Section 4 closes the paper with some conclusive remarks.

2 Entanglement transfer in the strong coupling regime

In Fig. 1 we show the scheme for entanglement transfer between a two-mode radiation and a two-qubit system implemented as a pair of two-level atoms interacting with two modes of spatially separated cavities. We assume that the flying modes are prepared in a generic mixed state described by the density operator

$$\rho_f(x) = \sum_{n,m,s,t=0}^{\infty} c_{n,m,s,t}(x)|n,m\rangle\langle s,t| \tag{1}$$

where $\{|n,m\rangle = |n\rangle \otimes |m\rangle\}_{n,m=0}^{\infty}$ is the standard Fock basis in the Hilbert space of the two-mode radiation, the complex coefficients are given by $c_{n,m,s,t}(x) = \langle n,m|\rho_f(x)|s,t\rangle$, and x is a parameter used to specify the different preparations. The two modes are injected into spatially separated cavities (A and B); each radiation mode is coupled resonantly to a cavity mode, initially prepared in the vacuum state so that the initial statistical operator for the cavities is $\rho_c(0) = |0\rangle_{c,A}\langle 0| \otimes |0\rangle_{c,B}\langle 0|$.

The interaction between each flying mode and the corresponding cavity mode is described by a linear coupling (beam-splitter-like [14]) i.e. by means of the the unitary operators:

$$\hat{U}_{A/B}(\theta) = \exp\left[-\theta\left(\hat{f}^{\dagger}_{A/B}\hat{c}_{A/B} - \hat{f}_{A/B}\hat{c}^{\dagger}_{A/B}\right)\right] \tag{2}$$

where $\hat{f}_{A/B}$ ($\hat{f}^{\dagger}_{A/B}$) are the annihilation (creation) operators for the flying modes and $\hat{c}_{A/B}$ ($\hat{c}^{\dagger}_{A/B}$) for the cavity modes, respectively. The beam splitter parameter θ is related to the cavity mirror transmittance $T = \cos^2\theta$. If $T = 1$ the operator $\hat{U}_{A,B}(\theta)$ reduces to the identity and we are back to the situation investigated in [8] for the case of a pure state two-mode radiation. After the action of beam splitter, the statistical operator of the cavity mode and the driving radiation $\rho_{cf}(x,\theta)$ is given by:

$$\rho_{cf}(x,\theta) = \hat{U}_A(\theta)\hat{U}_B(\theta)\big[\rho_f(x)\otimes\rho_c(0)\big]\hat{U}_A^\dagger(\theta)\hat{U}_B^\dagger(\theta). \tag{3}$$

By taking the partial trace over the degrees of freedom of the flying we obtain the reduced two-mode density operator describing the cavity modes $\rho_c(x,\theta) = \mathrm{Tr}_f\{\rho_{cf}(x,\theta)\}$. The relation among the cavity matrix elements $b_{i,j,k,l}(x,\theta) = \langle i,j|\rho_c(x,\theta)|k,l\rangle$ and those of the input modes may be written as

$$\begin{aligned} b_{i,j,k,l}(x,\theta) = (\cos\theta)^{i+j+k+l} &\sum_{p,q=0}^{\infty} c_{i+p,j+q,k+p,l+q}(x)(\sin\theta)^{2(p+q)} \\ &\times\left[\frac{(i+p)!}{p!i!}\frac{(j+q)!}{q!j!}\frac{(k+p)!}{p!k!}\frac{(l+q)!}{q!l!}\right]^{1/2}. \end{aligned} \tag{4}$$

Once the state $\rho_c(x,\theta)$ has been established inside the cavities, we assume that a two-level atom is injected into each cavity. We assume that both atoms are prepared in the ground state so that the initial atomic statistical operator is $\rho_a(0) = |g\rangle_{a,A}\langle g|\otimes|g\rangle_{a,B}\langle g|$. We also assume that each atom interacts resonantly with the cavity mode for a time τ shorter than the cavity decay time γ^{-1}; therefore, we can describe the interaction by the standard Jaynes-Cummings (JC) unitary operators $\hat{U}_{A/B}^{(JC)}(\tau)$ [15]

$$\rho_{a,c}(\tau,x,\theta) = \hat{U}_A^{(JC)}\hat{U}_B^{(JC)}\left[\rho_c(x,\theta)\otimes\rho_a(0)\right]\hat{U}_A^{\dagger(JC)}\hat{U}_B^{\dagger(JC)}. \tag{5}$$

In order to assess the effectiveness of the entanglement transfer process we need to evaluate the entanglement properties of the two-atom subsystem after the interaction with the cavity modes. To this aim we need the reduced atomic density operator $\rho_a(\tau,x,\theta) = \mathrm{Tr}_c[\rho_{ac}(\tau,x,\theta)]$ obtained by tracing out the cavity modes. After lengthy but straightforward calculations we derive the density matrix elements, which are listed in the Appendix.

The state $\rho_a(\tau,x,\theta)$ is in general a mixed state whose purity is given by $\mu_a = \mathrm{Tr}_a[\rho_a^2]$. In order to quantify entanglement we evaluate the concurrence C [16], $C = \max\{0, \Lambda_1 - \Lambda_2 - \Lambda_3 - \Lambda_4\}$, where Λ_i are the square roots of the eigenvalues λ_i^R, selected in the decreasing order, of the non-hermitian matrix $R = \rho_a^{mb}(\rho_a^{mb})^*$ that is the atomic density matrix written in the magic basis [17]. From the concurrence one can also derive the entanglement of formation [18]:

$$\epsilon_F = -\frac{1-\sqrt{1-C^2}}{2}\log_2\frac{1-\sqrt{1-C^2}}{2} - \frac{1+\sqrt{1-C^2}}{2}\log_2\frac{1+\sqrt{1-C^2}}{2}.$$

2.1 Pure states

We first consider the flying modes prepared in a pure Bell-like state. In this case it is possible to derive analytical results. Then we consider the modes prepared in a TMC state, where the amount of entanglement transferred to the atoms may evaluated numerically.

2.1.1 Bell-like states

The flying modes are prepared in one of the Bell-like states given by

$$|\phi_B\rangle_f = c_{00}|00\rangle_f + c_{11}|11\rangle_f, \qquad |\psi_B\rangle_f = c_{01}|01\rangle_f + c_{10}|10\rangle_f \tag{6}$$

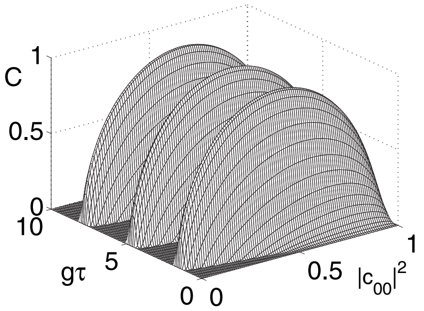

Fig. 2. Concurrence C vs. dimensionless interaction time $g\tau$ and probability $|c_{00}|^2$ for states $|\phi_B\rangle$ in the case of cavity mirror transmittance $T = 1$.

where $|c_{00}|^2 + |c_{11}|^2 = |c_{01}|^2 + |c_{10}|^2 = 1$. If $c_{ij} = \frac{1}{\sqrt{2}}$ $(i, j = 0, 1)$ we have the Bell states $|\Phi^+\rangle$ and $|\Psi^+\rangle$ that are maximally entangled. In Eqs. (20) and (22) of the Appendix we report the elements of the atomic density matrix $\rho_a(\tau, x, \theta)$ for both cases.

For states $|\phi_B\rangle_f$ we derive the following expression for the concurrence of the atomic subsystem:

$$C = \max\{0, 2|c_{11}|(1 - Y)(|c_{00}| - |c_{11}|Y)\} \tag{7}$$

where to simplify the notations we introduced $Y \equiv 1 - T\sin^2(g\tau)$ and g is the atom-cavity mode coupling constant (taken equal for both atoms). In Fig. 2 we show C as a function of dimensionless interaction time $g\tau$ and probability $|c_{00}|^2$ in the case of mirror transmittance $T = 1$. We see a periodic structure with regions of large values of C for $g\tau_{max}^{(k)} = (2k+1)\frac{\pi}{2}$, $(k = 1, 2, \ldots)$, and maxima corresponding to the Bell state $|\Phi^+\rangle$. For the sections corresponding to $g\tau_{max}^{(k)}$ we obtain $C = max\{0, 2|c_{00}||c_{11}|\}$. Therefore, only for interaction times $g\tau_{max}^{(k)}$ the entanglement properties of the two-mode radiation are fully transferred to the atoms. In Fig. 3(a) we show the effect of beam splitter transmittance $T < 1$ for fixed dimensionless interaction time $g\tau_{max} = \frac{\pi}{2}(Y = 1 - T)$. We see that the sections at fixed T values have a maximum of C that shifts toward states with larger probability $|c_{00}|^2$ for decreasing T. We can derive for the position of those maxima:

$$|c_{00}|^2_{max}(T) = \frac{1}{2}\left[1 - \frac{1-T}{\sqrt{1+(1-T)^2}}\right]. \tag{8}$$

The degree of mixedness of the two-atom subsystem is given by the purity μ_a:

$$\mu_a = \{|c_{00}|^2 + |c_{11}|^2[1 + 2Y^2 - 2Y]\}^2. \tag{9}$$

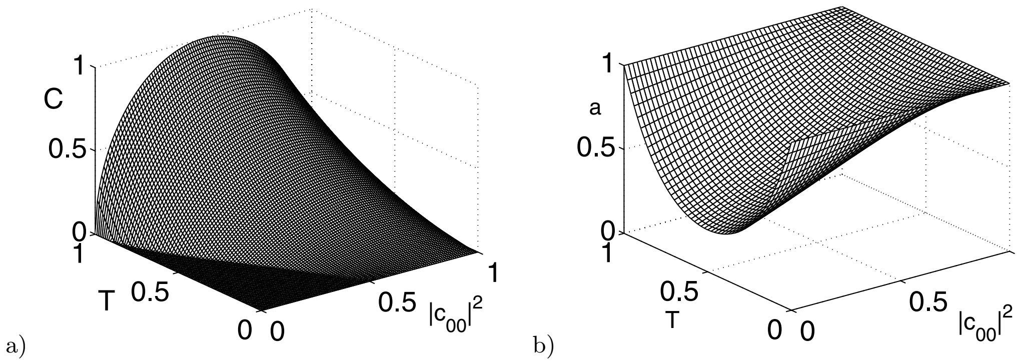

Fig. 3. Concurrence C (a) and purity μ_a (b) of the Bell-like states $|\phi_B\rangle$ vs. transmittance T and probability $|c_{00}|^2$. The dimensionless interaction time is $g\tau = \frac{\pi}{2}$.

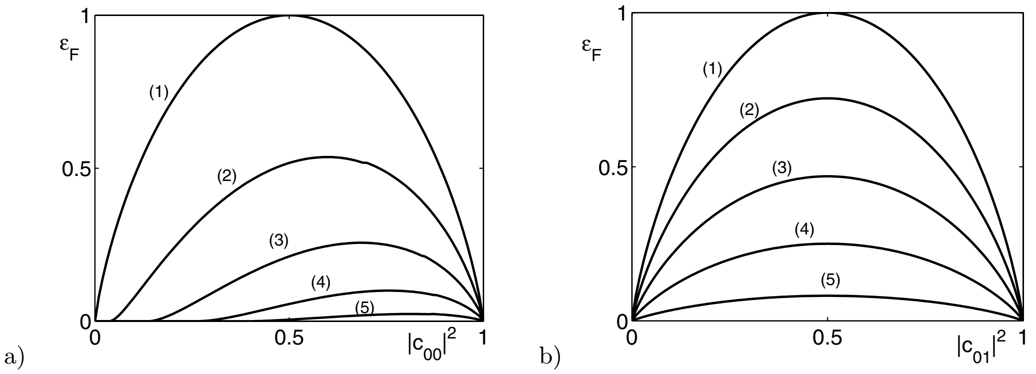

Fig. 4. Entanglement of formation ϵ_F vs. probability $|c_{10}|^2$ or $|c_{00}|^2$ for Bell-like states $|\phi_B\rangle$ (a) and $|\psi_B\rangle$ (b). Dimensionless interaction time $g\tau = \frac{\pi}{2}$ and transmittance T= 1.0 (1), 0.8 (2), 0.6 (3), 0.4 (4), 0.2 (5).

In Fig. 3(b) we show, for $g\tau_{max} = \frac{\pi}{2}$, the dependence of μ_a on transmittance T and probability $|c_{00}|^2$. We see a symmetric behavior with respect to the section at $T = 0.5$.

In the case of $|\psi_B\rangle$ the concurrence is given by

$$C = 2|c_{01}||c_{10}|(1-Y). \tag{10}$$

In the case $T = 1$ we obtain for the concurrence as a function of $g\tau$ and $|c_{01}|^2$ a behavior like that shown in Fig. 2 but with slightly larger regions around the values of interaction time $g\tau^{(k)}_{max}$. If $g\tau_{max} = \frac{\pi}{2}$ we have for every fixed value of transmittance $T < 1$ maximum entanglement transfer for the Bell state $|\Psi^+\rangle$ ($|c_{01}|^2 = 0.5$). Contrary to Bell-like states $|\phi_B\rangle$ the sections are symmetric with respect to $|c_{01}|^2 = 0.5$. The purity μ_a is given by:

$$\mu_a = 1 + 2Y^2 - 2Y \tag{11}$$

and we see that it is independent of the particular choice of probability $|c_{01}|^2$.

Finally we compare the effect of cavity mirror transmittance T for both types of Bell-like states by evaluating the entanglement of formation ϵ_F. In Fig. 4 we consider $g\tau_{max} = \frac{\pi}{2}$ and we compare different values of $T < 1$. We see that the effect of transmittance T on the entanglement transfer process is more relevant for the $|\phi_B\rangle$ states than for the $|\psi_B\rangle$ ones.

2.1.2 Two-mode coherently correlated states

In this section we consider a multiphoton two-mode radiation, the so-called two-mode coherently correlated states (TMC) also known as pair coherent states. They are an example of non-Gaussian CV states and can be obtained either by degenerate Raman processes [19] or, more realistically, by conditional measurements [20] and non degenerate parametric oscillators [21]. The Fock basis expansion of TMC is given by

$$|TMC\rangle = \frac{1}{\sqrt{I_0(2|x|)}} \sum_{n=0}^{\infty} \frac{x^n}{n!} |nn\rangle_f \tag{12}$$

where $x = |x|e^{\phi_x}$. The average number of photons of a TMC state that is given by $\langle N\rangle(x) = \frac{2|x|I_1(2|x|)}{I_0(2|x|)}$ where $I_0(y)$ and $I_1(y)$ are the 0-th and 1-st order modified Bessel functions of the first kind, respectively. For flying TMC at the input the matrix elements of the cavity field are given by

$$\begin{aligned} b_{i,j,k,k+i-l}(x) = &\frac{e^{i\phi_x(i-k)}}{I_0(2|x|)\sqrt{i!j!k!(k+j-i)!}} (\cos\theta)^{2(i+k)} (\sin\theta)^{2(i-j)} |x|^{(i+k)} \\ &\times \sum_{p=j-i}^{\infty} \frac{1}{p!(p-j+i)!} |x|^{2p} (\sin\theta)^{4p}. \end{aligned} \tag{13}$$

In Fig. 5 we show the concurrence C of the two-atom subsystem in the case of transmittance $T = 1$. We can see that regions of large values of ϵ_F correspond to $g\tau$ values very close to those obtained for the maxima in the case of Bell-like states. The reason of this analogy is that the photon statistics of TMC approaches that of a Bell state for small N.

In order to investigate the effect of transmittance $T < 1$ we fix e.g. the interaction time $g\tau = 1.56$, corresponding to a region of maximum entanglement transfer, and evaluate the entanglement of formation and the purity, shown in Fig. 6 for different values of T.

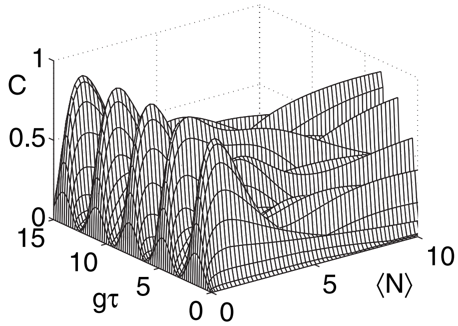

Fig. 5. Concurrence C vs. dimensionless interaction time $g\tau$ and cavity field mean photon number $\langle \hat{N} \rangle$ for TMC states and transmittance $T = 1$.

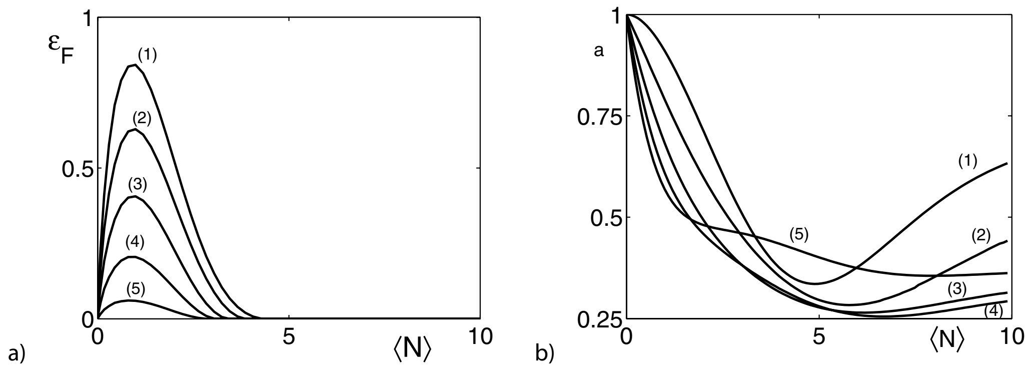

Fig. 6. Entanglement of formation ϵ_F (a) and purity μ_a (b) of atomic state for flying TMC states vs. mean photon number $\langle \hat{N} \rangle$ for $g\tau = 1.56$ and transmittance T= 1.0 (1), 0.8 (2), 0.6 (3), 0.4 (4), 0.2 (5).

2.2 Mixed states

In this section we consider the Werner states [22] as a basic example of two-mode radiation prepared in a mixed state. The statistical operator for these states is: $\rho_W = r|\Psi^+\rangle\langle\Psi^+| + \frac{1-r}{4}I$, where $0 \leq r \leq 1$ is a parameter related to the fidelity $F = \langle\Psi^+|\rho_W|\Psi^+\rangle$ by $r = \frac{4F-1}{3}$. If $r = 1$ we obtain the pure Bell state $|\Psi^+\rangle$ that is maximally entangled, and if $r = 0$ we obtain the identity that is maximally mixed and not entangled. It can be easily shown that the negativity of state ρ_W is given by $N(\rho_W) = max\{0, \frac{3r-1}{4}\}$ and the concurrence is simply $C = 2N(\rho_W)$. In Eq. (23) of Appendix A we report the atomic density matrix elements. For the concurrence we derive:

$$C = max\left\{0, (1-Y)\left[r - \sqrt{(1-r)\left(Y + \frac{1-r}{4}(1-Y)^2\right)}\right]\right\}. \tag{14}$$

In Fig. 7 we show the concurrence C as a function of dimensionless interaction time $g\tau$ and parameter r for maximum transmittance $T = 1$. We see a periodic structure for $g\tau^{\kappa}_{max} = (2k+1)\frac{\pi}{2}$, $(k = 1, 2, \ldots)$, and $r > \frac{1}{3}$.

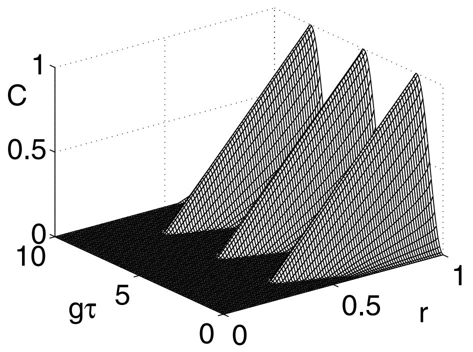

Fig. 7. Concurrence C of Werner states as a function of dimensionless interaction time $g\tau$ and parameter r for transmittance $T = 1$.

In Fig. 8(a) we can see the effect on the concurrence C of the transmittance $T < 1$ for interaction time $g\tau_{max} = \frac{\pi}{2}$. For the purity of the atomic density matrix we derive:

$$\mu_a = \frac{(1-r)^2}{4}(1-Y)^4 - (1-r)(1-Y)^3 + \frac{1}{2}(r^2 - r + 4)(1-Y)^2 + 2Y - 1. \tag{15}$$

In Fig. 8(b) we show, for $g\tau_{max} = \frac{\pi}{2}$, the dependence of μ_a on beam splitter transmittance $T < 1$ and the parameter r.

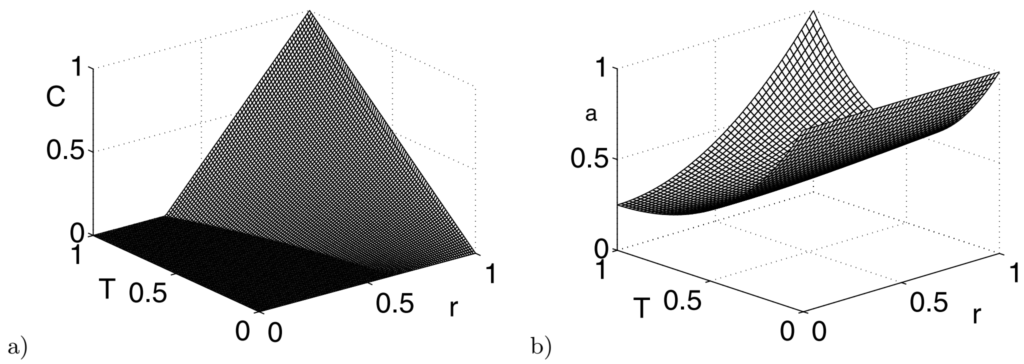

Fig. 8. Concurrence C (a) and purity μ_a(b) of Werner states vs. transmittance T and parameter r. The dimensionless interaction time is $g\tau = \frac{\pi}{2}$.

3 MCWF approach to full system dynamics

In the previous section we modeled the entanglement transfer process only by unitary operations because we assumed that the entanglement of the radiation modes was transferred to the cavity modes before the atomic injection and we neglected any cavity mode and atomic level decays (strong coupling regime). We found that for a large atomic entanglement transfer the cavity mirror transmittance should have large values, but this implies that the losses of photons cannot be in fact neglected. In the perspective of storage of quantum information the atoms should be trapped inside the cavities so that they simultaneously interact with the cavity modes and are driven by the external radiation modes. To this purpose we must carefully evaluate the effect of both the atomic and cavity mode decays and it is necessary to investigate the entanglement transfer in different coupling regimes. Therefore, we describe the whole entanglement transfer process from a dynamical point of view.

The Hamiltonian in the interaction picture and under full resonance conditions, $\omega_f = \omega_a = \omega_c$, can be written as $\hat{H}_i = \hat{H}_A + \hat{H}_B$, where:

$$\hat{H}_{A/B} = \hbar g \left[\hat{c}^{\dagger}_{A/B}\hat{\sigma}_{A/B} + \hat{c}_{A/B}\hat{\sigma}^{\dagger}_{A/B}\right] + i\hbar g_c(t)\left[\hat{c}_{A/B}\hat{f}^{\dagger}_{A/B} - \hat{c}^{\dagger}_{A/B}\hat{f}_{A/B}\right] \tag{16}$$

where $\hat{\sigma}^{\dagger}_{A/B}$ ($\hat{\sigma}_{A/B}$) are the raising (lowering) atomic operators. To simplify the model we assume that the cavity mode-external driving coupling frequency $g_{cf}(t)$ is constant for the interaction time τ_{cf} and vanishes for $t > \tau_{cf}$.

Taking into account the dissipative dynamics of both cavities and both atoms we must solve by the MCWF method [13] the following Master Equation for the whole system statistical operator $\rho(\tilde{t})$, written in the Lindblad form and for dimensionless time $\tilde{t} = gt$

$$\dot{\rho} = -\frac{i}{\hbar}(\hat{\mathcal{H}}_e\rho - \rho\hat{\mathcal{H}}^{\dagger}_e) + \sum_{i=1}^{4}\hat{C}_i\rho\hat{C}^{\dagger}_i \tag{17}$$

where the non-Hermitian effective Hamiltonian $\hat{\mathcal{H}}_e$ is given by

$$\hat{\mathcal{H}}_e = \frac{\hat{\mathcal{H}}_i}{g} - \frac{i\hbar}{2}\sum_{i=1}^{4}\hat{C}^{\dagger}_i\hat{C}_i. \tag{18}$$

Dissipation for cavities and atoms is described by standard Liouville super-operators:

$$\begin{aligned}\hat{\mathcal{L}}_{cA/B}\rho &= -\frac{\tilde{\kappa}}{2}(\hat{c}^{\dagger}_{A/B}\hat{c}_{A/B}\rho - \hat{c}_{A/B}\rho\hat{c}^{\dagger} + \rho\hat{c}^{\dagger}_{A/B}\hat{c}_{A/B})\\ \hat{\mathcal{L}}_{aA/B}\rho &= -\frac{\tilde{\gamma}}{2}(\hat{\sigma}^{\dagger}_{A/B}\hat{\sigma}_{A/B}\rho - 2\hat{\sigma}_{A/B}\rho\hat{\sigma}^{\dagger}_{A/B} + \rho\hat{\sigma}^{\dagger}_{A/B}\hat{\sigma}_{A/B})\end{aligned} \tag{19}$$

where $\tilde{\kappa} = \kappa/g$ and $\tilde{\gamma} = \gamma/g$ are dimensionless cavity and atomic decay rate, respectively. For the cavity mode we assumed the zero temperature limit since we are working in the optical regime. Therefore the collapse operators in (17) are $\hat{C}_{1,2} = \sqrt{\tilde{\kappa}}\hat{c}_{A/B}$ for the decays of cavity modes and $\hat{C}_{3,4} = \sqrt{\tilde{\gamma}}\hat{\sigma}_{A/B}$ for the decays of atomic upper levels. Using the MCWF method, the solution of Eq. (17) can be obtained by averaging over a suitable number N_{tr} of trajectories, i.e. stochastic evolutions of the whole system wave function $|\psi_j(\tilde{t})\rangle$ ($j = 1, 2, \ldots N_{tr}$), so that $\rho(\tilde{t}) \cong \frac{1}{N_{tr}}\sum_{i=j}^{N_{tr}}|\psi_j(\tilde{t})\rangle\langle\psi_j(\tilde{t})|$.

We consider the TMC states in the case $x = 0.75$, corresponding to the region of maxima in Fig. 5, and we investigate the system dynamics as a function of the dimensionless interaction time gt. To simulate different cavity mirror transmittances we consider different values of the dimensionless interaction time $\tilde{\tau}_{cf} = g\tau_{cf}$ and we assume, as an example, $\tilde{g}_{cf}(\tilde{t}) = 1$ for $\tilde{t} \le \tilde{\tau}_{cf}$. For the cavity and atomic decay rate we chose the values $\tilde{\kappa} = 0.1$ and $\tilde{\gamma} = 0.01$, respectively. In Fig. 9(a) we show the time evolution of the mean photon number $\langle\hat{N}\rangle_{fA}$ of the radiation mode A (it is the same for mode B). We see that increasing $\tilde{\tau}_{cf}$ the amount of photons transferred to the cavity modes increases and for $\tilde{\tau}_{cf} = 2.221$ almost all photons are injected inside the cavity (i.e. the transmittance of the cavity mirror is $T \cong 1$). In Fig. 9(b) we show the mean photon number of cavity mode A (it is the same for mode B) and we clearly see the effect of dissipative processes. In Fig. 9(c) we show the atomic probability $p_{g,A}$ to find atom A in the ground state (it is the same for atom B). Finally, in Fig. 9d we show the entanglement of formation ϵ_F of the two-atom subsystem. As expected, the amount of transferred entanglement increases for increasing values of the interaction time $\tilde{\tau}_{cf}$. In addition, the peaks of ϵ_F are located at gt values separated by π as in Fig. 5 in the "static" approach and for mean photon number $\langle\hat{N}\rangle \cong 1$. Their height progressively reduces for increasing interaction times due to the simultaneous effect of cavity and atomic dissipative processes.

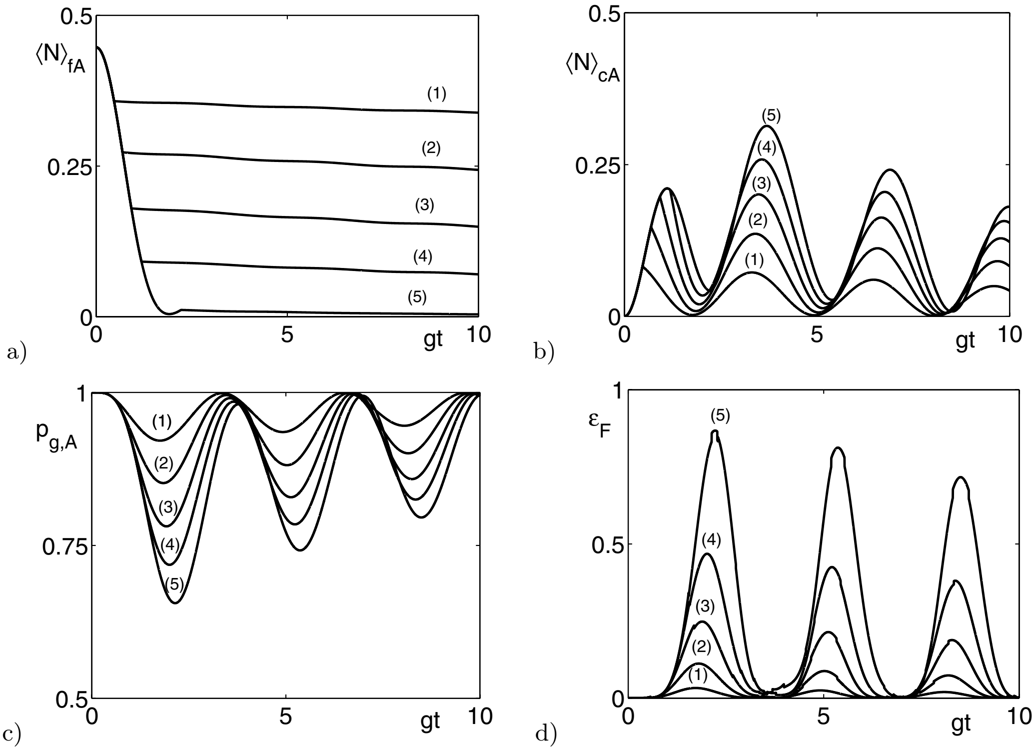

Fig. 9. Whole system dynamics for the TMC state with $x = 0.75$. We consider $\tilde{g}_{cf}(\tilde{t}) = 1$ for $\tilde{t} \leq \tilde{\tau}_{cf}$, $\tilde{\kappa} = 0.1$, $\tilde{\gamma} = 0.01$, and interaction times $\tilde{\tau}_{cf} = 0.468(1), 0.689\ (2), 0.921\ (3), 1.185(4), 2.221\ (5)$. (a) Mean photon number of radiation mode A $\langle \hat{N} \rangle_{fA}$, (b) mean photon number of cavity mode A $\langle \hat{N} \rangle_{cA}$, (c) atomic probability of atom A to be in the ground state $p_{g,A}$, (d) two-atoms subsystem entanglement of formation ϵ_F.

4 Conclusions

We have analyzed the process of entanglement transfer from two flying modes of the radiation field to a pair of localized qubits implemented as atoms interacting with two separated cavity modes. At first we have generalized our previous analytical results [8] in order to include the effect of cavity mirror transmittance and radiation modes prepared in a generic entangled mixed signal. To overcome the limitations of the above treatment, we investigated the whole transfer process by Monte Carlo Wave Function approach. We numerically solved the dynamics for the two tripartite systems each composed by one atom, a flying and a cavity mode, including dissipative processes. Upon focusing to the case of flying modes prepared in a coherently correlated state we have found realistic conditions for efficient entanglement transfer out of the weak and strong coupling regimes relevant for the implementation of quantum memories.

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Appendix: Atomic density matrix elements

For the scheme of Fig. 1 we consider a general mixed field state described by the statistical operator $\rho_f(x)$ and assume that both cavities are prepared in the vacuum state, both atoms are prepared in the ground state, and they are injected into the cavities after the two-mode radiation has been established inside each cavity. We describe the interaction between the flying modes and the cavity ones by beam splitters and the resonant interaction between atoms and cavity modes by the standard Jaynes-Cummings coupling. If x is the parameter characterizing the two mode radiation, θ the quantum beam-splitter parameter and τ the atomic interaction time, we derive the final atomic statistical operator $\rho_a(\tau,x,\theta) = Tr_c[\rho_{ac}(\tau,x,\theta)]$ corresponding to the following atomic density matrix elements in the standard basis $\{|ee\rangle_{AB}, |eg\rangle_{AB}, |ge\rangle_{AB}, |gg\rangle_{AB}\}$:

$$\rho_{a11}(\tau,x,\theta) = \sum_{i,j=0}^{\infty} b_{i+1,j+1,i+1,j+1}(x,\theta)\sin^2(g\tau\sqrt{i+1})\sin^2(g\tau\sqrt{j+1})$$
$$\rho_{a22}(\tau,x,\theta) = \sum_{i,j=0}^{\infty} b_{i+1,j,i+1,j}(x,\theta)\sin^2(g\tau\sqrt{i+1})\cos^2(g\tau\sqrt{j})$$
$$\rho_{a33}(\tau,x,\theta) = \sum_{i,j=0}^{\infty} b_{i,j+1,i,j+1}(x,\theta)\cos^2(g\tau\sqrt{i})\sin^2(g\tau\sqrt{j+1})$$
$$\rho_{a44}(\tau,x,\theta) = \sum_{i,j=0}^{\infty} b_{i,j,i,j}(x,\theta)\cos^2(g\tau\sqrt{i})\cos^2(g\tau\sqrt{j})$$
$$\rho_{a12}(\tau,x,\theta) = -i\sum_{i,j=0}^{\infty} b_{i+1,j+1,i+1,j}(x,\theta)\sin^2(g\tau\sqrt{i+1})\sin(g\tau\sqrt{j+1})\cos(g\tau\sqrt{j})$$
$$\rho_{a13}(\tau,x,\theta) = -i\sum_{i,j=0}^{\infty} b_{i+1,j+1,i,j+1}(x,\theta)\sin(g\tau\sqrt{i+1})\cos(g\tau\sqrt{i})\sin^2(g\tau\sqrt{j+1})$$
$$\rho_{a14}(\tau,x,\theta) = -\sum_{i,j=0}^{\infty} b_{i+1,j+1,i,j}(x,\theta)\sin(g\tau\sqrt{i+1})\cos(g\tau\sqrt{i})\sin(g\tau\sqrt{j+1})\cos(g\tau\sqrt{j})$$
$$\rho_{a23}(\tau,x,\theta) = \sum_{i,j=0}^{\infty} b_{i+1,j,i,j+1}(x)\sin(g\tau\sqrt{i+1})\cos(g\tau\sqrt{i})\sin(g\tau\sqrt{j})\cos(g\tau\sqrt{j+1})$$
$$\rho_{a24}(\tau,x,\theta) = -i\sum_{i,j=0}^{\infty} b_{i+1,j,i,j}(x,\theta)\sin(g\tau\sqrt{i+1})\cos(g\tau\sqrt{i})\cos^2(g\tau\sqrt{j})$$
$$\rho_{a34}(\tau,x,\theta) = -i\sum_{i,j=0}^{\infty} b_{i,j+1,i,j}(x,\theta)\cos^2(g\tau\sqrt{i})\sin(g\tau\sqrt{j+1})\cos(g\tau\sqrt{j})$$

and $\rho_{aji}(\tau,x,\theta) = \rho^*_{aij}(\tau,x,\theta)$. The relation between the coefficients $b_{i,j,k,l}(x,\theta)$ and those of the initial two-mode radiation is given in Eq. (4).

In the following we report the expressions of the atomic density matrix elements for the two-mode radiation types for which only the states $|0\rangle_f$,$|1\rangle_f$ are involved. In fact, in those cases it is possible to derive simple expressions for the eigenvalues of the non-hermitian matrix $R = \rho_a^{MB}(\rho_a^{MB})^*$.
For the Bell-like states $|\phi_B\rangle$ of Eq. (6) we have only the following non-vanishing elements of the atomic density matrix:

$$\begin{aligned} &\rho_{a11} = |c_{11}|^2(1-Y)^2 \qquad \rho_{a14} = -|c_{00}||c_{11}|e^{i\phi}(1-Y) \\ &\rho_{a22} = \rho_{a33} = |c_{11}|^2(1-Y)Y \qquad \rho_{a44} = |c_{00}|^2 + |c_{11}|^2Y^2, \end{aligned} \tag{20}$$

where to simplify the notation we introduced the quantity $Y \equiv 1 - T\sin^2(g\tau)$. The eigenvalues of the matrix R are

$$\lambda^R_{1,2} = \rho_{a22}\rho_{a33} \quad \lambda^R_{3,4} = (\sqrt{\rho_{a11}\rho_{a44}} \pm |\rho_{a14}|)^2. \tag{21}$$

For the Bell-like states $|\psi_B\rangle$ we find the atomic density matrix elements:

$$\begin{aligned} &\rho_{a22} = |c_{10}|^2(1-Y) \qquad \rho_{a33} = |c_{01}|^2(1-Y) \\ &\rho_{a44} = Y \qquad \rho_{a23} = |c_{01}||c_{10}|e^{i\phi}(1-Y) \end{aligned} \tag{22}$$

and the eigenvalues of R are given by $\lambda^R_{1,2} = 0$ and $\lambda^R_{3,4} = (\sqrt{\rho_{a22}\rho_{a33}} \pm |\rho_{a23}|)^2$.

For the field initially prepared in a Werner state we derive the atomic density matrix elements

$$\begin{aligned} &\rho_{a11} = \frac{1-r}{4}(1-Y)^2 \qquad \rho_{a22} = \rho_{a33} = \frac{1-Y}{4}[2-(1-r)(1-Y)] \\ &\rho_{a44} = Y + \frac{1-r}{4}(1-Y)^2 \qquad \rho_{a23} = \frac{r}{2}(1-Y) \end{aligned} \tag{23}$$

and the eigenvalues of R are $\lambda^R_{1,2} = \rho_{a11}\rho_{a44}$ and $\lambda^R_{3,4} = (|\rho_{a23}| \pm \rho_{a22})^2$.

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